8. $a = \sqrt{3}$ 9. 402271.44 10. 5.06 dm

11. 2 , 6 , 18 or 18 , 6 , 2 12. 2 , 6 , 18 , 54

2.11 Geometric Mean:

When three quantities are in G.P., the middle one is called the Geometric Mean (G.M.) between the other two. Thus G will be the G.M. between a and b if a, G, b are in G.P.

To Find G.M between a and b:

Let, G be the G.M. between a and b

Then a. G. b are in G.P

$$\therefore \quad \frac{G}{a} = \frac{b}{G} \Rightarrow G^2 = ab$$

$$G = \pm\sqrt{ab}$$

Hence the G.M. between two quantities is equal to the square root of their product.

Example 1:

Find the G.M. between 8 and 72.

Solution:

$$G = \pm\sqrt{ab}$$

$$G = \pm\sqrt{8 \text{ x } 72} = \pm\sqrt{8 \text{ x } 8 \text{ x } 9} = \pm 8\text{x } 3$$

$$G = \pm 24$$

2.12 n G.Ms Between a and b:

The numbers G_1, G_2, G_3 …… G_n are said to be n G.Ms between a and b if a, G_1, G_2, G_3 …… G_n, b are in G.P.

In order to obtain the G.M's between a and b, we use the formula $a_n = ar^{n-1}$ to find the value of r and then the G.M's can be computed.

To Insert n G.M's Between Two Numbers a and b

Let, G_1, G_2, G_3 …… G_n be n G.Ms between a and b

Here a = a, a_n = b, n = n + 2 , r = ?

$$a_n = ar^{n-1}$$

$$b = ar^{n-2-1} = ar^{n-1}$$

$$b/a = r^{n-1}$$

$$\Rightarrow r = (b/a)^{1/(n-1)}$$

So,

$$G_1 = ar = a(b/a)^{1/(n+1)}$$

$$G_2 = G_1r = a(b/a)^{1/(n+1)}(b/a)^{1/(n+1)} = a(b/a)^{2/(n+1)}$$

$$G_3 = G_2r = a(b/a)^{2/(n+1)}(b/a)^{1/(n+1)} = a(b/a)^{3/(n+1)}$$

$$G_n = a(b/a)^{n/(n+1)}$$

Example 2:

Find three G.M's between 2 and 32.

Solution:

Let, $G_1, G_2, G_3 \ldots\ldots G_n$ be n G.Ms between 2 and 32

Then 2, G_1, G_2, G_3 32 are in G.P.

Here a = 2, $a_n = 32$, r = ? n = 5

$$a_n = ar^{n-1}$$
$$32 = 2(r)^{5-1} = 2r^4$$
$$16 = r^4$$
$$2^4 = r^4$$
$$\Rightarrow r = 2$$

So,
$$G_1 = ar = 2(2) = 4$$
$$G_2 = G_1r = 4(2) = 8$$
$$G_3 = G_2r = 8(2) = 16$$

Thus three G.M's between 2 and 32 are 4, 8, 16.

Example 3:

Insert 6 G.M's between 2 and 256.

Solution:

Let, $G_1, G_2, G_3, G_4, G_5, G_6$ be six G.M's between 2 and 256.

Then 2, $G_1, G_2, G_3, G_4, G_5, G_6$ 256 are in G.P.

Here a = 2, $a_n = 32$, r = ? n = 5

$$a_n = ar^{n-1}$$
$$256 = 2(r)^{8-1} = 2r^7$$
$$128 = r^7$$
$$(2)^7 = r^2$$
$$\Rightarrow r = 2$$

So,
$$G_1 = ar = 2(2) = 4$$
$$G_2 = G_1r = 4(2) = 8$$
$$G_3 = G_2r = 8(2) = 16$$
$$G_4 = G_3r = 16(2) = 32$$
$$G_5 = G_4r = 32(2) = 64$$
$$G_6 = G_5r = 64(2) = 128$$

Hence, required G.M's are 4, 8, 16, 32, 64, 128.

Example 4:

The A.M between two numbers is 10 and their G.M is 8.

Determine the numbers.

Solution: $$\text{A.M} = \frac{a+b}{2} = 10$$

$$a + b = 20 \ldots\ldots\ldots (1)$$

$$\text{G.M.} = \sqrt{ab} = 8$$

$$\therefore \quad ab = 64 \ldots\ldots\ldots (2)$$

from (2) $b = \frac{64}{a}$, Put in (1)

$a + \frac{64}{2} = 20$

$a^2 + 64 = 20a$

$(a - 16)(a - 4) = 0$

$\Rightarrow a = 16$ or $a = 4$

When, $a = 16$, $b = \frac{64}{16} = 4$

When, $a = 4$, $b = \frac{64}{16} = 16$

Hence the numbers are 4 and 16.

Exercise 2.5

Q1. Find G.M between

(i) 4, 64 (ii) $\frac{1}{3}$, 243 (iii) $\frac{8}{9}, \frac{8}{9}$

Q2. Insert two G.M's between $\sqrt{2}$ and 2.

Q3. Insert three G.M's between 256 and 1.

Q4. Insert four G.M's between 9 and $\frac{1}{27}$.

Q5. Show that A.M of two unequal positive quantities is greater than this G.M.

Q6. For what value of n is $\frac{a^{n+1} + b^{n+1}}{a^n + b^n}$ the G.M between a and b, where a and b are not zero simultaneously.

Q7. Prove that the product of n G.M's between a and b is equal to the n power of the single G.M between them.

Q8. The A.M of two positive integral numbers exceeds their (positive)G.M by 2 and their sum is 20.Find the numbers.

Answers 2.5

Q1. (i) ± 16 (ii) ± 9 (iii) $\pm\frac{8}{9}$

Q2. $2^{2/3}, 2^{5/6}$ Q3. 64, 16, 4 Q4. 3, 1, $\frac{1}{3}, \frac{1}{9}$

Q6. $n = -\frac{1}{2}$ Q8. 16 , 4 or 4 , 16