Note:- In geometric progression, the ratio between any two consecutive terms remains constant and is obtained by dividing the next term with the preceeding term, i.e., $r = \frac{a_n}{a_{n-1}}$, $n > 1$

2.10 nth term or General term(or, last term) of a Geometric Progression (G.P):

If a is the first term and r is the common ratio then the general form of G.P is a, ar, ar^2, ar^3, ……

If $a_1 = 1^{st}$ term $= a$

$a_2 = 2^{nd}$ term $= ar$

$a_3 = 3^{rd}$ term $= ar^2$

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a_n = nth term $= ar^{n-1}$

Which is the nth term of G.P in which:

a = 1st term

r = common ratio

n = number of terms

a_n = nth term = last term

Example 2:

Write the sequence in which

$a = 5,\ r = 3,\ n = 5$

$a_1 = a = 5$

$a_2 = ar = 5(3) = 15$

$a_3 = a_2r = 15(3) = 45$

$a_4 = a_3r = 45(3) = 135$

$a_5 = a_4r = 135(3) = 405$

Therefore, the required sequence is: 5, 15, 45, 135, 405

Example 3:

Find 4th term in the G.P. 5, 10, 20, ……

Solution:

$a = 5,\ r = \frac{10}{5} = 2,\ a_n = ?$

$a_n = ar^{n-1}$

$a_4 = a_4 = 5(2)^{4-1} = 5 \times 8 = 40$

Example 4:

Find n in the G.P. 4, –2, 1, …… if $a_n = \frac{1}{16}$

Solution: **Since** 4, –2, 1, ……

Here, $a = 4, r = -2, 4 = -\frac{1}{2}, a_n = \frac{1}{16}$

$a_n = ar^{n-1}$

$\frac{1}{16} = 4(-\frac{1}{2})^{n-1}$

let $\frac{1}{16} \times 4 = (-\frac{1}{2})^{n-1} = \frac{1}{64} = (-\frac{1}{2})^{n-1}$

$(-\frac{1}{2})^6 = (-\frac{1}{2})^{n-1}$

$\Rightarrow \quad n - 1 = 6 \quad$ or $\quad n = 6 + 1 = 7$

Example 5:

Find the G.P. of which the third term is 4 and 6th is –32.

Solution:

Here $a_3 = 4, \ a_6 = -32$

$a_n = ar^{n-1}$

$a_3 = ar^{3-1}, \ a_6 = ar^{6-1}$

$4 = ar^2$ …… (i) $\qquad -32 = ar^5$ …… (ii)

Dividing (i) by (ii)

$\frac{ar^2}{ar^5} = \frac{4}{-32}$ or $\frac{1}{r^3} = \frac{-1}{8}$

$r^3 = -8 = (-2)^3$

$\Rightarrow r = -2$

Example 6:

The population of a town increases at the rate of 10% annually. Its present population is 2,00,000 what will be its population at the end of 5 years?

Solution:

Let, present population = a = 2,00,000 (given)

The increase of population at the end of 1st year

$= a(10/100) = a(0.1)$

Total population at the end of 1st year $= a + a(0.1) = a(1.1)$

Total population at the end of 2nd year $= a(1.1)(1.1) = a(1.1)^2$

The population at the end of 5years is the 6^{th} terms of G.P

$a , a(1.1) , a(1.1)^2$ ……

Here $a = 2,00,000$, $r = 1.1$, $n = 6$, $a_6 = ?$

Since, $a_n = ar^{n-1}$

$a_6 = 2,00,000 (1.1)^5 = 2,00,000 (1.61051) = 322102$

Example 7:

The value of an auto mobile depreciate at the rate of 15% per year. What will be the value of an automobile 3 years hence which is now purchased for Rs. 45,000?

Solution:

a = 45,000 = Purchased value of automobile

The amount depreciate at the end of 1st year = a (15/100) = 0.15a

The value of automobile at the end of 1st year = a – 0.15a

= a(1 – 0.15) = a(0.85)

The value of automobile at the end of 2nd year = a(0.85)(1 – 0.15)

= a(0.85)(0.85)

= $a(0.85)^2$

The value of automobile at the end of 3rd year = $a(0.85)^3$

= $45,000(0.85)^3$

= 45,000 (0.614125)

= 27635.63 rupees

Exercise 2.4

Q.1 Write the next five terms of the following G.Ps.

(i) 2, 10, ……. (ii) 27, 9, 3, …….

(iii) $1, \frac{1}{2}, \frac{1}{4}, \ldots\ldots$

Q.2 Find the term indicated in each of the following G.Ps.

(i) $1, 3^3, 3^6, \ldots\ldots$, 6th term

(ii) i, –1, –i, 1, ……., 13th term

(iii) $\sqrt{2}, \sqrt{6}, 3\sqrt{2}, \ldots\ldots$, 15th term

(iv) $\frac{1}{3}, -\frac{1}{9}, \frac{1}{27} \ldots\ldots$, 6th term

Q.3 Find the nth term of the G.P.

(i) a = 8, r = $\frac{3}{2}$, n = 5

(ii) a = –1, r = –4, n = 6

(iii) a = 3, r = –2, n = 10

Q.4 Write down the finite geometric sequence which satisfies the given condition.

(i) a = 3, r = 5, n = 6

(ii) First term = 2, second term = – 6, n = 5

(iii) Third term = 9, sixth term = $\frac{1}{3}$, n = 8

(iv) Fifth term = 9, eight term = 243, n = 8

Q.5 If $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are G.P ,show that the common ratio is $\pm\sqrt{\frac{a}{b}}$

Q.6 If the second term of a G.P is 2 and the 11^{th} term is $\frac{1}{256}$, what is the first term? What is the nth term.

Q.7 Find the 10th term of a G.P if 2nd term 43 and 4th term is 9.

Q.8 What is the first term of a six term geometric progression in which the ratio is $\sqrt{3}$ and the sixth term is 27?

Q.9 A business concern pays profit at the rate of 15% compounded annually. If an amount of Rs. 2,00,000 is invested with the concern now, what total amount will become payable at the end of 5 years?

Q.10 A rubber bell is dropped from a height of 16dm, it continuously rebounds to $\frac{3}{4}$ of the distance of its previous fall. How high does it rebound its fourth time?

Q.11 Find three consecutive numbers in G.P whose sum is 26 and their product is 216.

Q.12 If the sum of the four numbers consecutive numbers of a G.P is 80 and A.M between second and fourth of them is 30.Find the terms.

Answers 2.4

1. (i) 50, 250, 1250, 6250, 31250 (ii) $1, \frac{1}{3}, \frac{1}{9}, \frac{1}{27}, \frac{1}{81}$
   (iii) $\frac{1}{8}, \frac{1}{16}, \frac{1}{32}, \frac{1}{64}, \frac{1}{128}$

2. (i) $(27)^5$ (ii) i (iii) $\sqrt{2}(3)^7$ (iv) $-\frac{1}{729}$

3. (i) $\frac{81}{2}$ (ii) 1024 (iii) −1536

4. (i) 3, 15, 75, 375, 1875, 9375
   (ii) 2, −6, 18, −54, 162
   (iii) $81, 27, 9, 3, 1, \frac{1}{3}, \frac{1}{9}, \frac{1}{27}$
   (iv) $\frac{1}{9}, \frac{1}{3}, 1, 3, 9, 27, 81, 243$

6. $a = 4, a_n = 4\left(\frac{1}{2}\right)^{n-1}$ 7. 243