Series:

The sum of the terms of a sequence is called as "series". For example: 1, 4, 9, 16, - - - - - - is a sequence.

Sum of the terms of sequence i.e., 1 + 4 + 9 + 16 - - - - - represent a series.

2.8 Arithmetic Series:

The sum of the terms of an Arithmetic sequence is called as Arithmetic series. For example:

7, 17, 27, 37, 47, - - - - - - - - - - is an A.P.

7 + 17 + 27 + 37 + 47 + - - - - - - - - is Arithmetic series.

The sum of n terms of an Arithmetic Sequence:

The general form of an arithmetic sequence is a, a + d, a + 2d, - - - - - - - - a + (n – 1)d.

Let Sn denoted the sum of n terms of an Arithmetic sequence.

Then $S_n = a + (a + d) + (a + 2d) + - - - - + [a + (n - 1)d]$

Let nth term $[a + (n - 1)d] = \ell$

The above series can be written as

$S_n = a + (a + d) + (a + 2d) + - - - - - - + \ell$

Or, $S_n = a + (a + d) + (a + 2d) + - - - - - - + (\ell - 2d) + (\ell - d) + \ell$ -----(I)

Writing 1 in reverse order, we have

$S_n = \ell + (\ell - d) + (\ell - 2d) + - - - (a + 2d) + (a + d) + a$...... (II)

Adding I and II

$2S_n = (a + \ell) + (a + \ell) + (a + \ell) + - - - - + (a + \ell)$

$2S_n = n(a + \ell)$

$S_n = \frac{n}{2}(a + \ell)$ But $\ell = a + (n - 1)d$

$S_n = \frac{n}{2}[a + (a + (n - 1)d)]$

$= \frac{n}{2}[a + a + (n - 1)d]$

$$\boxed{S_n = \frac{n}{2}[2a + (n - 1)d]}$$

is the formula for the sum of n terms of an arithmetic sequence.

Example 1:

Find the sum of the series 3 + 11 + 19 + - - - - to 16 terms.

Solution:

Here $a = 3,\ d = 11 - 3 = 8,\ n = 16$

Using formula $S_n = \frac{n}{2}[2a + (n-1)d]$

$$S_{16} = \frac{16}{2}[2(3) + (16-1)8]$$
$$= 8[6 + 15(8)]$$
$$= 8[6 + 120]$$
$$S_{16} = 8 \times 126 = 1008$$

Example 2:

Find the sum of all natural numbers from 1 to 500 which are divisible by 3.

Solution:

The sequence of numbers divisible by 3 is

3, 6, 9, 12, - - - - - 498 (which is in A.P.)

Here $a = 3$, $d = 6 - 3 = 3$, $n = ?$ $a_n = 498$

First we find n

For this using $a_n = a + (n-1)d$

$$498 = 3 + (n-1)(3)$$
$$498 = 3 + 3n - 3$$
$$3n = 498$$
$$n = 166$$

Now $a = 3$, $d = 3$, $n = 166$, $Sn = ?$

$$Sn = \frac{n}{2}[2a + (n-1)d]$$
$$S_{166} = \frac{166}{2}[2(3) + (166-1)3]$$
$$= 83[6 + 165(3)] = 83(6 + 495)$$
$$= 83 \times 501$$
$$S_{166} = 41583$$

Example 3:

` If the sum of n terms of an A.P. is $2n + 3n^2$. Find the nth term.

Solution:

We have $S_n = 2n + 3n^2$

$$S_{n-1} = 2(n-1) + 3(n-1)^2$$
$$= 2(n-1) + 3(n^2 - 2n + 1)$$
$$= 2n - 2 + 3n^2 - 6n + 3$$
$$S_{n-1} = 3n^2 - 4n + 1$$

nth term $= a_n = S_n - S_{n-1}$

$$= 2n + 3n^2 - (3n^2 - 4n + 1)$$
$$= 2n + 3n^2 - 3n^2 + 4n - 1$$
$$a_n = 6n - 1$$

Example 4:

The sum of three numbers in an A.P. is 12 and the sum of their cubes is 408. Find them.

Solution:

Let the required numbers be

$a - d, a, a + d$

According to 1st condition:

$$(a - d) + a + (a + d) = 12$$
$$a - d + a + a + d = 12$$
$$3a = 12$$
$$\boxed{a = 4}$$

According to 2nd given condition:

$$(a - d)^3 + a^3 + (a + d)^3 = 408$$
$$a^3 - d^3 - 3a^2d + 3ad^2 + a^3 + a^3 + d^3 + d^3 + 3a^2d + 3ad^2 = 408$$
$$3a^3 + 6ad^2 = 408$$
$$3(4)^3 + 6(4)\,d^2 = 408$$
$$24d^2 = 408 - 192$$
$$d^2 = 9$$
$$\Rightarrow \qquad d = \pm 3$$

When $a = 4$ $d = 3$ then number are $a - d, a, a + d$

i.e. $4 - 3, 4, 4 + 3$ i.e. $1, 4, 7$

when $a = 4, d = -3$ then numbers are $a - d, a, a + d$

$4 - (-3), 4, 4 + (-3)$

$4 + 3, 4, 4 - 3$ i.e., $7, 4, 1$

Hence the required numbers are 1, 4, 7 or 7, 4, 1

Note:

The problem containing three or more numbers in A.P. whose sum in given it is often to assume the number as follows.

If the required numbers in A.P are odd i.e. 3, 5, 7 etc. Then take 'a'(first term) as the middle number and d as the common difference.

Thus three numbers are $a - d, a, a + d$. If the required numbers in A.P are even i.e. 2, 4, 6, etc. then take $a - d, a + d$ as the middle numbers and 2d as the common difference.

Thus four numbers are $a - 3d, a - d, a + d, a + 3d$ and six numbers are:

$a - 5d, a - 3d, a - d, a + d, a + 3d, a + 5d$ etc.

Example 5:

A man buys a used car for \$600 and agrees to pay \$100 down and \$100 per month plus interest at 6 percent on the outstanding indebtedness until the car paid for. How much will the car cost him?

Solution:

The rate of 6 percent per year is 0.5 percent per month.

Hence, when the purchaser makes his first payment, he will owe 1 month's interest.

The interest on $500 = (500)(0.005) = $2.50

The purchaser will pay in the second month = $102.50

Since the purchaser pays $100 on the principal, his interest from month to month is reduced by 0.5 percent of $100, which is $0.50 per month.

The final payment will be $100 plus interest on 100 for 1 month, which is = $100.50

Hence his payments on $500 constitute an arithmetic progression

$$102.50 + ------ + 100.50$$

Here $a = 102.40$, $\ell = 100.50$ and $n = 5$

Therefore by the formula

$$S = \frac{n}{2}(a + \ell)$$

$$= \frac{5}{2}(102.50 + 100.50)$$

$$= \frac{5}{2}(203) = \$507.50$$

Thus, the total cost of the car will be $ 607.50

Exercise 2.3

Q.1 Sum the series:

(i) $5 + 8 + 11 + 14 + \ldots$ to n terms.

(ii) $51 + 50 + 49 + \ldots + 21$.

(iii) $5 + 3 + 1 - 1 - \ldots$ to 10 terms.

(iv) $\frac{1}{1-\sqrt{x}} + \frac{1}{1-x} + \frac{1}{1+\sqrt{x}} + \text{--------}$ to n terms

Q.2 The nth term of a series in $4n + 1$. Find the sum of its 1st n terms and also the sum of its first hundred terms.

Q.3 Find the sum of the first 200 odd positive integers.

Q.4 Find the sum of all the integral multiples of 3 between 4 and 97

Q.5 How many terms of the series:

(i) $-9 - 6 - 3 \ldots$ amount to 66?

(ii) $5 + 7 + 9 + \ldots$ amount to 192?

Q.6 Obtain the sum of all the integers in the first 1000 positive integers which are neither divisible by 5 nor 2.

Q.7 The sum of n terms of a series is $7n^2 + 8n$. Show that it is an A.P and find its common difference.

Q.8 Sum the series
$1 + 3 - 5 + 7 + 9 - 11 + 13 + 15 - 17$ …… to 3n terms.

Q.9 If S_1, S_2, S_3 be sums to n, 2n, 3n terms of an arithmetic progression, Show that $S_3 = 3(S_2 - S_1)$.

Q.10 The sum of three numbers in A.P. is 24, and their product is 440. Find the numbers.

Q.11 Find four numbers in A.P. whose sum is 24 and the sum of whose square is 164.

Q.12 Find the five number in A.P. whose sum is 30 and the sum of whose square is 190.0

Q.13 How many bricks will be there in a pile if there are 27 bricks in the bottom row, 25 in the second row, etc., and one in the top row?

Q.14 A machine costs Rs. 3200, depreciates 25 percent the first year, 21 percent of the original value the second year, 17 percent of the original value of the third year, and so on for 6 years. What is its value at the end of 6 years.

Answers 2.3

Q.1 (i) $S_n = \frac{n}{2}[3n + 7]$ (ii) $n = 31, S_n = 1116$

(iii) $S_{10} = 40$ (iv) $\frac{n}{2}\left[\frac{2+(3-n)\sqrt{x}}{1-x}\right]$

Q.2 $S_n = n(2n + 3), S_{100} = 20300$ Q.3 $S_{200} = 40,000$

Q.4 1581 Q.5 (i) $n = 11$ (ii) $n = 12$

Q.6 2000,000 Q.7 $d = 14$ Q.8 $S_{3n} = n\,(3n - 4)$

Q.10 5, 8, 11 or 11, 8, 5 Q.11 3, 5, 7, 9 or 9, 7, 5, 3

Q.12 4, 5, 6, 7, 8 or 8, 7, 6, 5, 4 Q.13 196 Q.14 Rs. 320.00

2.9 Geometric Sequence or Progression (G.P):

A geometric progression is a sequence of numbers each term of which after the first is obtained by multiplying the preceding term by a constant number called the common ratio. Common ratio is denoted by 'r'.

Example:

(i) 2, 4, 8, 16, 32, …… is G.P
because each number is obtained by multiplying the preceding number by 2.

(ii) 2, 4, 8, ……

(iii) 4, 12, 36, ……