Q.6 $a = -25$ Q.7 $a_{20} = 41$

Q.8 $a = -3, d = 2$ Q.9 $10, \frac{25}{2}, 15$

Q.10 9, 13, 17, - - - - - - and the difference between consecutive terms is equal. So the sequence is an A.P.

2.6 Arithmetic Means (A.Ms):

If a, A, b are three consecutive terms in an Arithmetic Progression, Then A is called the Arithmetic Mean (A.M) of a and b.

i.e. if a, A, b are in A.P. then

$$A - a = b - A$$
$$A + A = a + b$$
$$2A = a + b$$
$$A = \frac{a+b}{2}$$

The arithmetic mean of two numbers is equal to one half the sum of the two numbers.

Example 1:

Find the A.M. between $\sqrt{5} - 4$ and $\sqrt{5} + 4$

Solution:

Here $a = \sqrt{5} - 4$, $b = \sqrt{5} + 4$

$$\text{A.M.} = \quad A = \frac{a+b}{2}$$

$$A = \frac{\sqrt{5} - 4 + \sqrt{5} + 4}{2}$$

$$= \frac{2\sqrt{5}}{2} = \sqrt{5}$$

2.7 n Arithmetic Means between a and b:

The number A_1, A_2, A_3 - - - - - An are said to be n arithmetic means between a and b if a, A, A_2, A_3, - - - - - - An, b are in A.P. We may obtain the arithmetic means between two numbers by using $a_n = a + (n-1)d$ to find d, and the means can then be computed.

Example 2:

Insert three A.M's between – 18 and 4.

Solution:

Let A_1, A_2, A_3 be the required A.M's between –18 and 4, then

$-18, A_1, A_2, A_3, 4$ are in A.P.

Here $a = -18$, $n = 5$, $a_5 = 4$, $d = ?$

Using $a_n = a + (n-1)d$

$$a_5 = -18 + (5 - 1)d$$
$$4 = -18 + 4d$$
$$4d = 4 + 18$$
$$4d = 22$$
$$d = \frac{11}{2}$$

Therefore A_1 = 2nd term $= a + d$

$$= -18 + \frac{11}{2} = \frac{-25}{2}$$

A_2 = 3rd term $= A_1 + d$ $= \frac{-25}{2} + \frac{11}{2}$

$$= \frac{-25 + 11}{2}$$

$$A_2 = \frac{-14}{2} = -7$$

Thus the required A.M's are $\frac{-25}{2}, -7, \frac{-3}{2}$

Example 3:

Insert n A.M's between a and b.

Solution:

Let, $A_1, A_2, A_3, \ldots\ldots A_n$ be the n. A.M's between a and b.

Then $a, A_1, A_2, A_3, \ldots\ldots b$, are in A.P.

Let, d be the common difference

So, $a = a$, $n = n + 2$, $d = ?$ $a_n = b$

$$a_n = a + (n - 1)d$$
$$b = a + (n + 2 - 1)d$$
$$b = a + (n + 1)d$$
$$b - a = (n + 1)d$$
$$d = \frac{b - a}{n + 1}$$
$$A_1 = a + d = a + \frac{b - a}{n + 1} = \frac{a(n + 1) + b - a}{n + 1} = \frac{an + a + b - a}{n + 1}$$
$$A_1 = \frac{an + b}{n + 1}$$
$$A_2 = A_1 + d = \frac{an + b}{n + 1} + \frac{b - a}{n + 1} = \frac{an + a + b - a}{n + 1}$$

$$A_2 = \frac{(n-1)a + 2b}{n+1}$$

$$A_n = \frac{[n-(n-1)]a + nb}{n+1} = \frac{(n-n+1)a + nb}{n+1} \;....\; \frac{a + nb}{n+1}$$

Thus n A.M's between a and b are:

$$\frac{an + b}{n+1}, \frac{(n-1)a + 2b}{n+1}, \frac{(n-2)a + 3b}{n+1} \;....\; \frac{a + nb}{n+1}$$

Exercise 2.2

Q.1 Find the A.M. between

(i) 17 and -3 (ii) -5 and 40

(iii) $2 + \sqrt{3}$ and $2 - \sqrt{3}$ (iv) $x + b$ and $x - b$

Q.2 Insert two A.M's between -5 and 40.

Q.3 Insert four A.M's between $\frac{\sqrt{2}}{2}$ and $\frac{3\sqrt{2}}{2}$

Q.4 Insert five A.M's between 10 and 25.

Q.5 Insert six A.M's between 12 and -9.

Q.6 If 5 , 8 are two A.M's between a and b , find a and b

Q.7 Find the value of n so that $\frac{a^{n+1} + b^{n+1}}{a^n + b^n}$ may be the A.M's between a and b.

Q.8 Find the value of x if $x + 1$, $4x + 1$ and $8x - 1$ are the consecutive terms of an arithmetic progression.

Q.9 Show that the sum of n A.M's between a and b is equal to n times their single A.M.

Answer 2.2

Q.1 (i) 7 (ii) $\frac{35}{2}$ (iii) 2 (iv) x

Q.2 10, 25 Q.3 $\frac{7\sqrt{2}}{10}, \frac{9\sqrt{2}}{10}, \frac{11\sqrt{2}}{10}, \frac{13\sqrt{2}}{10}$

Q.4 $\frac{25}{2}, 15, \frac{35}{2}, 20, \frac{45}{2}$ Q.5 9, 6, 3, 0 -3, -6

Q.6 a= 2 , b = 11 Q.7 n = 0 Q.8 x = 2