Chapter 2
Sequences and Series

2.1 Introduction:

The INVENTOR of chess asked the King of the Kingdom that he may be rewarded in lieu of his INVENTION with one grain of wheat for the first square of the board, two grains for the second, four grains for the third, eight grains for the fourth, and so on for the sixty four squares. Fortunately, this apparently modest request was examined before it was granted. By the twentieth square, the reward would have amounted to more than a million grains of wheat; by the sixty-fourth square the number called for would have been astronomical and the bulk would have for exceeded all the grains in the kingdom.

The basis of this story a sequence of numbers that have a mathematical relationship --- has a great many important applications. Many of them are beyond the scope of this book, but we shall explore the means of dealing with a number of practical, and often entertaining, problems of this type.

2.2 Sequences:

A set of numbers arranged in order by some fixed rule is called as sequences.

For example

(i) 2, 4, 6, 8, 10, 12, 14 - - - - - - - - - - - -

(ii) 1, 3, 5, 7, 9, - - - - - - - - - - - -

(iii) $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16},$ - - - - - - - - - - - -

In sequence $a_1, a_2, a_3, \ldots\ldots a_n, \ldots\ldots$ a_1 is the first term, a_2 is the second term, a_3 is the third and so on.

A sequence is called **finite sequence** if it has finite terms e.g., 2, 4, 6, 8, 10, 12, 14, 16.

A sequence is called **infinite sequence** if it has infinite terms, e.g., , 4, 6, 8, 10, 12, 14, - - - - - - - -

2.3 Progression:

If a sequence of number is such that each term can be obtained from the preceding one by the operation of some law, the sequence is called a progression.

Note:- Each progression is a sequence but each sequence may or may not be a progression

2.4 Arithmetic Sequence:

A sequence in which each term after the first term is obtained from the preceding term by adding a fixed number, is called as an arithmetic sequence or Arithmetic Progression, it is denoted by A.P.

e.g., (i) 2, 4, 6, 8, 10, 12, - - - - - - - - -

(ii) 1, 3, 5, 7, 9, 11, - - - - - - - - - -

Common Difference:

The fixed number in above definition is called as common difference. It is denoted by d. it is obtained by subtracting the preceding terms from the next term i.e; $a_n - a_{n-1}$; $n>1$.

For example

2, 4, 6, 8, 4, - - - - - - - -

d = Common difference = $a_2 - a_1 = 4 - 2 = 2$

Or d = Common difference = $a_3 - a_2 = 6 - 4 = 2$

The General Form of an Arithmetic Progression:

Let "a" be the first term and "d" be the common difference, then General form of an arithmetic progression is

$a, a + d, a + 2d,$ - - - - - - - - - - $a + (n - 1)d$

2.5 nth term or General term(or, last term)of an Arithmetic Progression:

If "a" be the first term and "d" be the common difference then

a_1 = first term = $a = a + (1 - 1)d$

a_2 = 2nd term = $a + d = a + (2 - 1)d$

a_3 = 3rd term = $a + 2d = a + (3 - 1)d$

a_4 = 4th term = $a + 3d = a + (4 - 1)d$

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a_n = nth term = $a + (n - 1)d$

$$\boxed{a_n = \text{nth term} = a + (n - 1)d}$$

in which $a = 1^{st}$ term

d = common difference

n = number of terms

Example 1:

Find the 7th term of A.P. in which the first term is 7 and the common difference is – 3.

Solution:

a_7 = 7th term = ?

$a_1 = 7$

$d = 3$

Putting these values in $a_n = a + (n - 1)d$

$a_7 = 7 + (7 - 1)(-3)$

$$= 7 + 6(-3)$$
$$= 7 - 18$$
$$\boxed{a_7 = -11}$$

Example 2:

Find the 9th term of the A.P. $-\frac{5}{4}, -\frac{1}{4}, \frac{3}{4}, \ldots\ldots$

Solution:

$$a_1 = -\frac{5}{4}$$
$$d = -\frac{1}{4} - \left(-\frac{5}{4}\right) = \frac{-1}{4} + \frac{5}{4}$$
$$= \frac{-1+5}{4} = \frac{4}{4} = 1$$

9th term $=$ $a_9 = ?$
$$a_n = a + (n-1)d$$
$$a_9 = -\frac{5}{4} + (9-1).1$$
$$= \frac{-5}{4} + 8$$
$$a_9 = \frac{-5+32}{4} = \frac{27}{4}$$

Example 3:

Find the sequence whose general term is $\frac{n(n-1)}{2}$

Solution:

Here $a_n = \frac{n(n-1)}{2}$

Put n = 1, $a_1 = \frac{1(1-1)}{2} = \frac{0}{2} = 0$

Put n = 2, $a_2 = \frac{2(2-1)}{2} = \frac{2(1)}{2} = 1$

Put n = 3, $a_3 = \frac{3(3-1)}{2} = \frac{3(2)}{2} = 3$

Put n = 4, $a_4 = \frac{4(4-1)}{2} = \frac{4(3)}{2} = 6$

Put n = 5, $a_5 = \frac{5(5-1)}{2} = \frac{5(4)}{2} = 10$

Therefore the required sequence is 0, 1, 3, 6, 10, - - - - - - - -

Exercise 2.1

Q.1 Find the terms indicated in each of the following A.P.

(i) 1, 4, 7, - - - - - - - - 7th term

(ii) 7, 17, 27, - - - - - - - - 13th term

(iii) $\frac{1}{6}, \frac{1}{4}, \frac{1}{3},$ - - - - - - - - 20th term

Q.2 Find the first four terms of A.P. in which first term is 7 and common difference is -4.

Q.3 Find the number of terms in an A.P. in which $a_1 = 5$, $d = 25$ and $a_n = 130$.

Q.4 (i) Which term in the arithmetic progression 4, 1, -2 …. is -77?

(ii) Which term in the arithmetic progression 17, 13, 9 …. is -19?

Q.5 Find the 7th term of an A.P. whose 4th term is 5 and the common difference is -2.

Q.6 What is the first term of the eight term A.P. in which the common difference is 6 and the 8th term is 17.

Q.7 Find the 20th term of the A.P. whose 3rd term is 7 and 8th term is 17.

Q.8 If the 12th term of an A.P. is 19 and 17th term in 29, Find the first term and the common difference.

Q.9 The 9th term of an A.P. is 30 and the 17th term is 50. Find the first three terms.

Q.10 Find the sequence whose nth term is 4n + 5. Also prove that the sequence is in A.P.

Q.11 If $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in A.P, show that $b = \frac{2ac}{a+c}$

Q.12 If $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in A.P, show that the common difference is $\frac{a-c}{2ac}$

Answers 2.1

Q.1 (i) $a_7 = 19$ (ii) $a_{13} = 127$ (iii) $a_{20} = \frac{7}{4}$

Q.2 7, 3, -1, -5 Q.3 $n = 6$

Q.4 (i) $n = 28$ (ii) $n = 10$ Q.5 $a_{17} = -21$