Chapter 11
Area of Quadrilateral

11.1 Quadrilateral

A plane figure bounded by four sides is known as a quadrilateral. The straight line joining the opposite corners is called its diagonal. The diagonal divides the quadrilateral in to two triangles.

Types of Quadrilateral

The following types of quadrilateral are

(1) Square (2) Rectangle (3) Parallelogram
(4) Rhombus (5) Trapezoid (6) Cyclic quadrilateral

11.2 Area of Quadrilateral

1. Square

A square is a plane figure of four sides in which all sides are equal, the opposite sides are parallel and diagonals are also equal. The angle between the adjacent sides is a right angle.

Let ABCD be a square whose each side has length equal to 'a' and AC is a diagonal which divides the square ABCD into equal right triangles, named ΔABC & ΔACD. Therefore,

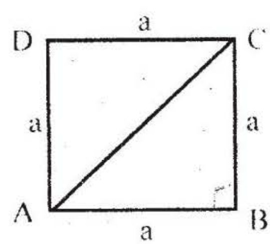


Fig. 11.1

Area of the square ABCD = Area of ΔABC + Area of ΔACD

$$= \frac{1}{2}(AB)(BC) + \frac{1}{2}(AD)(DC) = \frac{1}{2}(a)(a) + \frac{1}{2}(a)(a)$$

$$= \frac{1}{2}a^2 + \frac{1}{2}a^2 = a^2$$

Area of square = $(\text{Side})^2$

Length of square = $\sqrt{\text{Area of square}}$

i.e. side = $\sqrt{\text{Area}}$

Perimeter of square = 4a

Example 1:

A square lawn whose area is 2.5 sq. km has to be enclosed within iron railing. Find the length of the railing and its cost at Rs 10.50 per meter.

Solution:

Given that

Area of square lawn = 2.5 sq. km

$= 2.5 \times (1000)^2$ sq. m

A $= 2500000$ sq. m

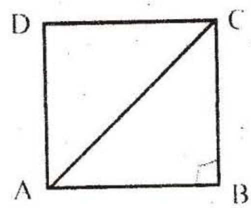


Fig. 11.2

One side of lawn = $\sqrt{2500000} = 1581$m

Perimeter of square lawn = 4 x 1581= 6324m

Cost for 1meter = 10.50 rupees

Cost for 6324 meters = 10.50 x 6324 = 66402 rupees

Example 2:

The diagonal of a square plate is 19 cm. Find the length of the plate and its area.

Solution:

Let ABC be a square plate.

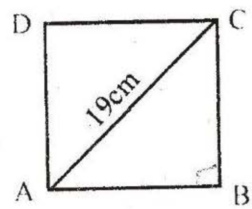


Fig. 11.3

Diagonal = AC = 19cm

Let a = length of square plate in right ΔABC

$(AB)^2 + (BC)^2 = (AC)^2 \Rightarrow a^2 + a^2 = (19)^2 \Rightarrow 2a^2 = 361$

$a^2 = 180.5 \Rightarrow a = 13.43$ cm

2. Rectangle

A rectangle is a four sided figure whose opposite sides are parallel and equal in length, diagonals are equal and angles between adjacent sides are right angles.

Also diagonals of rectangle bisect each other

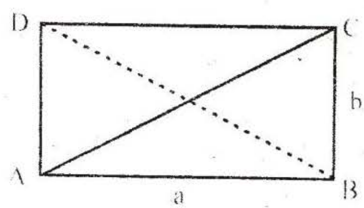


Fig. 11.4

Let ABCD be rectangle having side AB = a and BC = b and the diagonal AC divides the rectangle into two right triangles. ΔABC and ΔADC.

Area of rectangle ABCD = Area of ΔABC + Area of ΔADC.

$$= \frac{1}{2}(AB)(BC) + \frac{1}{2}(DC)(AD) = \frac{1}{2}ab + \frac{1}{2}ab$$

Area of rectangle ABCD = ab

$\therefore$ Area = length x width

Note: Length = $\frac{\text{Area}}{\text{Width}}$ Width = $\frac{\text{Area}}{\text{length}}$

Perimeter of rectangle ABCD = AB + BC + CD + AD

$= 2a + 2b = 2(a + b)$

Perimeter = 2 (length + breadth)

Example 3:

The area of a rectangle is 20 sq. cm and one of its sides is 4cm long. Find the breadth and the perimeter of the rectangle.

Solution: Given that

Area of rectangle = 20 sq. cm

One of its side i.e. length = 4cm

$\therefore$ Breadth of rectangle = ?

Since area = length x breadth

20 = 4 x breadth

Also perimeter of rectangle = 2(length + breadth)

= 2(4 + 5) = 18cm

Example 4:

A rectangle field is 13m long and 10 m wide it has a cement path 3.5m wide around it. What is the area of the cement path?

Solution:

Given that

Width of cement path = 3.5cm

Length and width of rectangular field are 13m & 10m respectively.

Therefore

Area of field = $(13 \times 10)m^2 = 130m^2$

Length of outer rectangle = 13 + 3.5 + 3.5 = 20m

& width of outer rectangle = 10 + 3.5 + 3.5 = 17m

$\therefore$ Area of outer rectangle = $(20 \times 17)m^2 = 340m^2$

Hence, Area of cement path = Area of outer rectangular – Area of inner rectangular field

$= 340m^2 - 130m^2 = 210m^2$

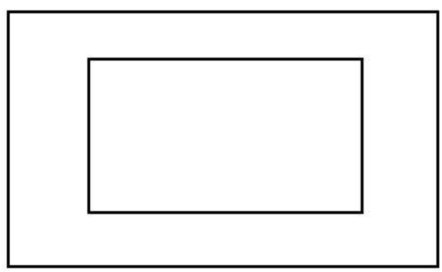


Fig.11.5

3. Parallelogram

A parallelogram is a quadrilateral whose opposite sides are equal in length and parallel, however, its diagonals are unequal and bisect each other.

Area of Parallelogram

Area of a parallelogram can be calculated in two ways

(i) When the base and height are given

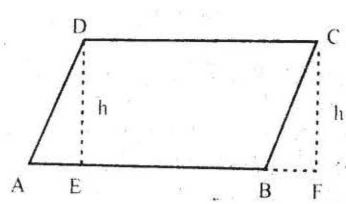


Fig. 11.6

Let ABCD be a parallelogram whose base AB=a and Height DE=h.

$\therefore$ Area of parallelogram ABCD

= Area of rectangle DEFC

= (Length)(breadth) = (EF)(DE)

= (DC)(DE) $\therefore$ EF = CD

= (AB)(DE) & CD = AB

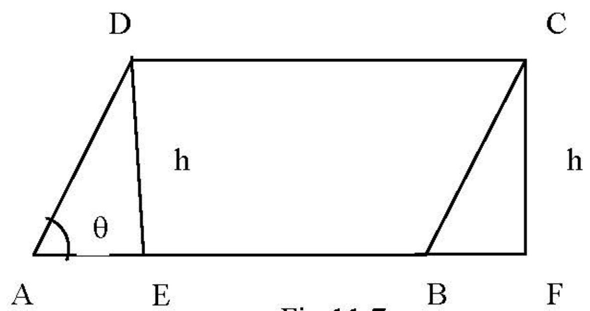
= a h

Area of parallelogram = (Base)(height)

(ii) When the two adjacent sides and included angle given

Let ABC be the parallelogram with AB = b, AD = c

As adjacent sides are b, c and θ be the include angle. Draw DE $\perp$r on AB from vertex D.



Fig,11.7

Thus ΔAED becomes a right triangle.

Area of parallelogram ABCD = (AB)(DE) -------- (1)

In right ΔAED, $\sin\theta = \frac{DE}{AD}$ $\Rightarrow$ DE = AD $\sin\theta$

DE = C $\sin\theta$

From (1)

$\therefore$ Area of ||gm (parallelogram) ABCD = bc $\sin\theta$

Area of ||gm (product of adjacent sides) $\sin\theta$

Example 5:

Find the area of a parallelogram whose base is 24cm and height 13 cm respectively.

Solution:

Here, b = 24cm, h = 13 cm

Area parallelogram = b x h = 24 x 13 = 312 sq. cm

Example 6:

Find the area of a parallelogram whose two adjacent sides of which are 70 cm and 80cm and their included angle 60^o.

Solution:

Here, b = 80, c = 70, $\theta = 60^o$

Area $= bc \sin\theta$
$= 80 \times 70 \times \sin 60^\circ$
$= 5600\ (.866) = 4849$ sq. cm

4. Rhombus

A rhombus is a quadrilateral having all sides equal with unequal diagonal, which bisect each other. If a square is pressed from two opposite corners the rhombus is formed.

Area of Rhombus

(a) If side a and the included angle θ is given

Let ABCD be a rhombus and length of each side be 'a' and θ be the included angle.

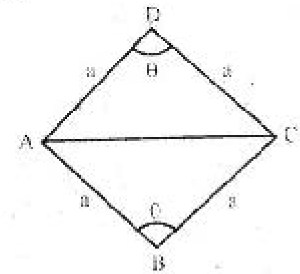


Fig. 11.9

Since diagonal AC divides the rhombus into two equal triangles ABC and ACD.

Therefore

Area of rhombus ABCD $= 2(\text{area of } \Delta ABC)$
$= 2(\frac{1}{2} a\ a.\ \sin\theta) = a^2 \sin\theta$

Area $= (\text{one side})^2 \sin\theta$

Note : When diagonal of Rhombus d_1 and d_2 are given the side 'a' can be calculated as $a = \sqrt{(\frac{d_1}{2})^2 + (\frac{d_2}{2})^2}$

(b) Area of rhombus, when two diagonals are given

Let $AC = d_1$ and $BD = d_2$ be the two diagonals.

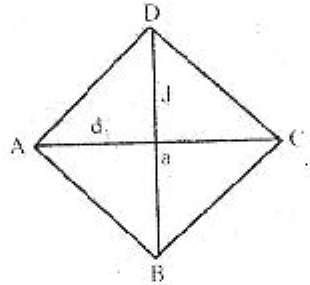


Fig.11.9
Fig. 11.10

Since diagonals of rhombus divide into four equal triangles.
Therefore,

$$\text{Area of rhombus} = 4(\text{area of one triangle})$$

$$= 4\left(\frac{1}{2} \times \frac{AC}{2} \times \frac{BD}{2}\right)$$

$$= \frac{AC \times BD}{2} \Rightarrow A = \frac{d_1 \times d_2}{2}$$

$$\text{Area of rhombus} = \frac{1}{2}(\text{product of two diagonals})$$

Example 7:

The length of each side of a rhombus is 120cm and two of its opposite angles are each 60^{o} find the area.

Solution:

Here each side a = 120cm
Angle, $\theta = 60^{o}$

$$\text{Area} = a \times a \times \sin\theta$$
$$= 120 \times 120 \times \sin 60$$
$$= 14400 \times 0.866$$
$$= 12470.4 \text{ sq. cm.}$$

Example 8:

The diagonals of a rhombus are 56 and 33 cm respectively. Find the length of side and the area of the rhombus.

Solution:

Let, ABCD is a rhombus with diagonal BD = 56cm = d_1
AC = 33 cm = d_2
Since diagonals of rhombus intersect at right angle triangle
∴ ABO in right angle triangle

$$\text{Where } OB = \frac{BD}{2} = \frac{56}{2}$$

$$OB = 28$$

$$OA = \frac{AC}{2} = \frac{33}{2} = 16.5$$

$$\therefore |AB|^2 = |AO|^2 + |BO|^2$$
$$= (16.5)^2 + (28)^2$$
$$AB = 32.5 \text{ cm}$$

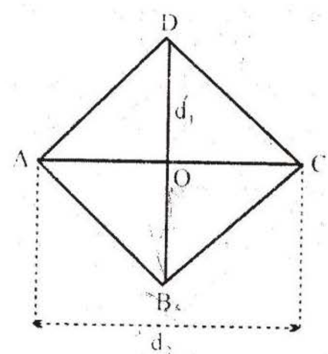


Fig. 11.11

$$\text{Area of Rhombus} = \frac{d_1 \times d_2}{2}$$

$$= \frac{56 \times 33}{2} = 924 \text{ sq. cm}$$

5. Trapezoid or Trapezium

A trapezoid is a quadrilateral which has two sides parallel and other two are unparallel. The parallel sides of trapezoid are called base. The altitude is the perpendicular distance between these bases.

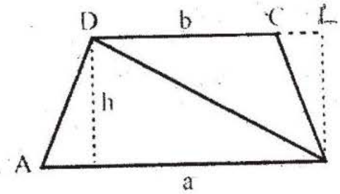


Fig. 11.12

Let, ABCD be a trapezoid whose sides AB and CD are parallel, AD and BC and unparallel sides.
Let AB = a, CD = b, and DP = h
BL= h, Since the diagonal BD divide the trapezoid into two triangles ABD & BCD.
Therefore,

$$\text{Area of Trapezium} = \text{Area } \Delta ABD + \text{Area } \Delta BCD$$

$$= \frac{1}{2} AB \text{ x } DP + \frac{1}{2} \text{ x } DC \text{ x } BL$$

$$= \frac{1}{2} ah + \frac{1}{2} bh$$

$$A = \frac{1}{2}(a+b)h$$

$$\text{Area of Trapezium} = \frac{\text{Sum of parallel sides}}{2} \text{ x height}$$

Example 9:

Find the area of a trapezium whose parallel sides are 57 cm and 85 cm and perpendicular distance between them is 4cm.

Solution:

Here, a = 85cm
b = 57cm
h = 4cm

$$\text{Area of Trapezium} = \frac{a+b}{2} \text{ x } h$$

$$= \frac{85+57}{2} \text{ x } 4 = 142 \text{ x } 2$$

$$A = 284 \text{ sq. cm}$$

Area of any quadrilateral

It may be found as the sum of two triangles formed by joining one of the diagonals of the quadrilateral.

OR

Area of any quadrilateral can be calculated by dividing it into two triangles.

Example 10:

Find the area of the quadrilateral ABCD in which the sides AB, BC, CD, DA and diagonal AC are 25, 60, 52, 39 and 65cm. respectively.

Solution:

Area $\Delta ABC = \sqrt{S(S-a)(S-b)(S-c)}$

Where, $2S = a + b + c$

$2S = 25 + 60 + 65$

$S = 75$

Area= $\sqrt{75(75-25)(75-60)(75-65)} = 750$ Sq. cm

Area = $\Delta ACD = \sqrt{S(S-a)(S-b)(S-c)}$

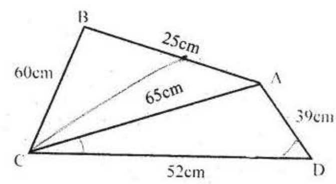


Fig. 11.15

Where,

$2S = a + b + c$

$2S = 52 + 39 + 65$

$S = 78$

$A = \sqrt{78(78-52)(78-39)(78-65)}$

$A = 1014$ Sq. cm

Area of quadrilateral ABCD = $\Delta ABC + \Delta ACD$

$= 750 + 1014 = 1764$ Sq. cm

Area of Cyclic Quadrilateral

A four sided figure which is in a circle is called a cyclic quadrilateral. Its area is given by the formula.

$A = \sqrt{(S-a)(S-b)(S-c)(S-d)}$

Where,

$2S = a + b + c + d$

and a, b , c, d are sides

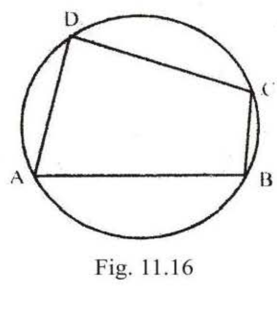


Fig. 11.16

Example 11:

In a circular grassy plot a quadrilateral shape with its corners touching the boundary of the plot is to be paved with bricks. Find the area of the pavement in square meter if sides of the quadrilateral are 36, 77, 75, and 40 meter respectively.

Solution:

Since this is cyclic quadrilateral, so, a = 36, m. b = 77m, c = 75m, d= 40m

$\therefore$ Area = $\sqrt{(S-a)(S-b)(S-c)(S-d)}$

Where

$$S = \frac{a+b+c+d}{2} = \frac{36+77+75+40}{2} = 114$$

$$\text{Area} = \sqrt{(114-36)(114-77)(114-75)(114-40)}$$
$$= \sqrt{78 \times 37 \times 39 \times 74}$$
$$= 2886 \text{ Sq. m}$$

Exercise 11

Q1.. A lawn is in the shape of a rectangle of length 60m and width 40m outside the lawn there is a foot path of uniform width 1m, boarding the lawn, Find area of the path.

Q2. A track round the inside of a rectangular grassy plot 40m by 30m occupies 600 sq. m show that the width of the track is 5m.

Q3. A wire rectangle is pressed at the corners to form a parallelogram. The included angle is reduced to 60°. Find the reduction in area if the original size of the rectangle is 16 x 12cm.

Q4. The perimeter of a rhombus is 146cm and one of its diagonal is 55cm. Find the other diagonal and area.

Q5. The diagonals of a rhombus are 80 cm and 60 cm respectively. Find the area and length of each side.

Q6. The difference between two parallel sides of a trapezoid is 8 cm. The perpendicular distance between them is 24m and the area of the trapezoid is 312 square meter. Find the two parallel sides.

Q7. The altitude of a triangle is 15cm and its base is 40cm find the area of trapezoid formed by a line parallel to the base of the triangle and 6cm from the vertex.

Q8. A swimming pool is in the form of an isosceles trapezoid. The length of its parallel banks are 72m and 45m and the perpendicular distance between them is 60m. Find the area of its water surface. Also find out the number of tiles required to line its bottom of each tile is 1.5m x 1.5m.

Q9. In a quadrilateral the diagonal is 125cm and the two perpendiculars on it from the other two angles are 19cm and 25cm respectively, find the area.

Q10. Two triangles are cut from a 14ft. square piece of sheet metal one has a base of 2.5ft. an altitude of 14ft. and the other has a base of 5ft. and an altitude of 7.5ft. Find area of sheet metal left in the piece.

Answers 11

Q1. 2604 sq.m	Q3. 26 sq.cm	Q4. 48 cm; 1320 sq. cm
Q5. 2400 sq.cm, 50cm	Q6. 17 m, 9m	Q7. 252sq.cm
Q8. 3510sq.m,1560	Q9. 2750sq.cm	Q10. 107.75 sq.ft.

Summary

1. Area of square with side 'a' = a^2 perimeter of square = 4a.
   Length of square = $\sqrt{\text{area of square}}$
2. Area of rectangle with length a and breadth b is ab.
   Perimeter of rectangle = 2(a + b)
3. **(i) Area of parallelogram when base and height are given**
   Area of ||gm = base x height
   (ii) Area of parallelogram when two adjacent sides and included angle are given
   If b and c are the adjacent sides and θ is the included angle.
   Then
   Area of ||gm (parallelogram) = bcsin θ
   i.e. Area of ||gm = (Product of adjacent sides) Sin θ
4. **Area of Rhombus**
   (i) If side a and the included angle θ
   Then
   Area of rhombus = $a^2 \sin\theta$
   (ii) If d_1 and d_2 are the two diagonals of rhombus
   Then
   Area of rhombus = $\frac{d_1 \text{ x } d_2}{2}$
5. **Area of Trapezoid**
   If a and b are parallel sides and h is the perpendicular distance between these parallel sides.
   Then
   $$\text{Area of trapezoid} = \frac{\text{Sum of parallel sides}}{2} \text{x height}$$
   $$\text{Area} = (\frac{a+b}{2}) \text{x h}$$
6. **Area of any quadrilateral**
   Area of any quadrilateral can be calculated by dividing it into two triangles.
   i.e. if ABCD is any quadrilateral
   Then
   Area of quadrilateral ABCD = Area of ABC + Area of ΔACD
7. **Area of Cyclic quadrilateral**
   $$A = \sqrt{(S-a)(S-b)(S-c)(S-d)}$$
   Where $S = \frac{a+b+c+d}{2}$ and, a, b, c, d sides