Chapter 10
Area of triangles (Plane figures)

10.1 Introduction to Mensuration:

This is a branch of Applied Mathematics which deals with the calculation of length of Lines, areas and volumes of different figures.

Scope

It is widely applied in various branches of engineering. A chemical engineer has to find the volume or capacity of a plant, A civil engineer has to find the areas and volumes in embarkments, canal digging or dam works. An electrical engineer has to depend upon this branch of mathematics while calculating resistance or capacity of a conductor. In the same way a draftsman, a designer or an electrical supervisor very often uses mensuration in his work.

If we dismantle any complicated machine or its parts we will find the machine has been separated into different simple plane or solid figures like rings, cylinders, squares and prisms.

In this part of the text our aim is to enable the student to find areas, volume, and circumferences of different figures, so that when as a technician, he has to estimate the cost of material or to design a machine part, he may be able to understand the calculations needed to determine weight, strength and cost.

10.2 Plane Figures:

Plane figures are those figures which occupy an area with only two dimensions, room floors, grassy plots and tin sheets are examples of plane figure.

While the solid figures are those which occupy space with three dimensions, Shafts, Fly wheels, bolts, wooden boxes, and coal tar drum are examples of solids.

10.3 Triangle:

A triangle is a plane figure bounded by three straight lines. The straight lines AB, BC, CA which bound triangle ABC are called its sides. The side BC may be regarded as the base and AD as the height.

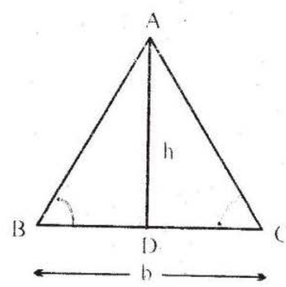


Fig. 10.1

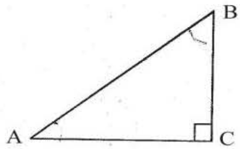
Kinds of Triangles

There are six types of triangles three of them are classified according to their sides and the remaining three are according to their angles.

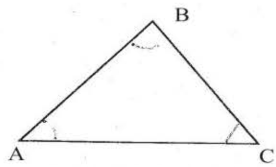
(a) Triangles classified to their angles:

(1) Right angled triangles

If one angle of a triangle is a right angle (90^{o}) then it is called a right angled triangle the side opposite to right side is called its by hypotenuse and remaining other two side an base and altitude.



Right angled Triangle



Acute angled Triangle

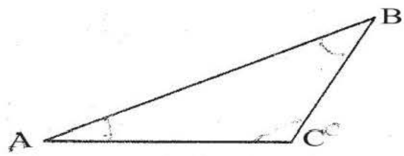
Fig. 10.2

(2) Acute Angled Triangles

If all the three angles of an triangle are acute (less then 90^{o}) then the triangle is called acute angled.

(3) Obtuse Angled Triangles

If one angle of a triangle is obtuse (greater than 90^{o}) the triangle is called obtuse angled.



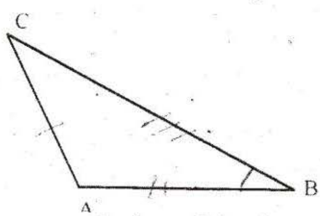
Obtuse angled Triangle

Fig. 10.3

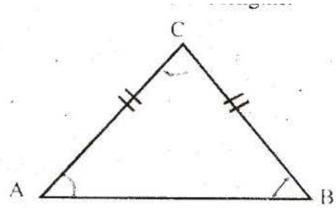
(b) Triangle Classified according to their sides:

(1) Scalene Triangle

A triangle in which all sides are of different lengths is called scalene triangle.



Scalence Triangle

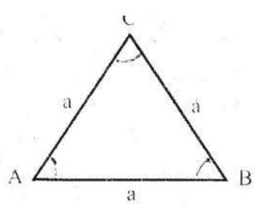


Isosceles Triangle

Fig. 10.4

(2) Isosceles Triangle

If two sides of triangle are equal, the triangle is called isosceles.



Equilateral Triangle

Fig. 10.5

(3) Equilateral Triangle

If all the three sides of a triangle are equal in lengths, the triangle is called Equilateral.

Perimeter:

The perimeter of a closed plane figure is the total distance around the edges of the figure.

Perimeter of a triangle with sides a, b & c:
Perimeter of the triangle = (a + b + c) units

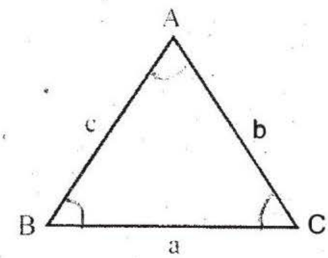


Fig. 10.6

10.4 Area of Triangles:

There are so many methods to find the area of triangle. We shall discuss them one by one.

(a) Area of a triangle in terms of its Height (altitude) and base:

Case I:When the triangle is Right angled:

Let ABC be a right angled triangle whose angle B is right angle. Side BC is the height (altitude) 'h' and side AB is the base 'b'.

Invert the same triangle in its new position ADC as shown in figure 10.7.

$$\text{Area of ABC} = \frac{1}{2}\text{area of rectangle BCDA.}$$

$$= \frac{1}{2}(AB)(BC)$$

$$= \frac{1}{2}bh$$

$$\text{Area} = \frac{1}{2}(\text{base})\ (\text{height})$$

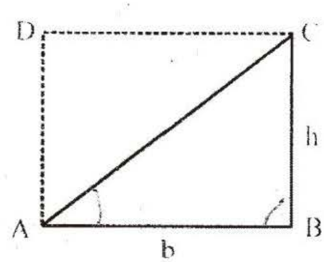


Fig. 10.7

Case II: When Triangle is acute-angled:

Let ABC be a triangle with its base 'b' and height h. When CD is perpendicular some to the base AB.

The area of ΔABC = Area of ΔADC + Area of ΔBDC

$$= \frac{1}{2}(AD)(CD) + \frac{1}{2}(DB)(CD)$$

$$= \frac{1}{2}(AD + DB)CD$$

$$= \frac{1}{2}(AB)(CD)$$

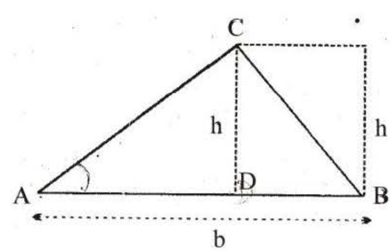


Fig. 10.8

$$= \frac{1}{2} b \times h$$

Hence Area $= \frac{1}{2}(\text{base x height})$

Case III: When Triangle is obtuse-angled

Let ABC be triangle whose obtuse angle is B . Also draw CD perpendicular to AB produced.

Then,

Area of ΔABC = Area of ΔADC – Area of ΔBDC

$$= \frac{1}{2}(AD)(CD) - \frac{1}{2}(BD)(CD)$$

$$= \frac{1}{2}(AD - BD)CD$$

$$= \frac{1}{2}(AB)(CD) = \frac{1}{2}(b)(h)$$

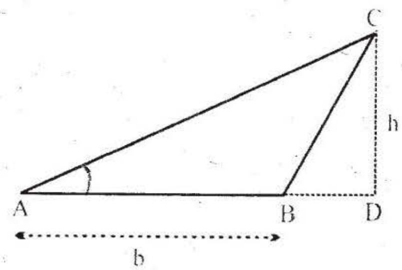


Fig. 10.9

Area $= \frac{1}{2}(\text{Base x height})$

Note : Hence from case I, II and III it is concluded that

Area of a triangle whose base and height is given $= \frac{1}{2}(\text{Base x height})$

Example 1:

Find the area of a triangle whose base is 12 cm. and hypotenuse is 20cm.

Soluyion :

Let ABC be a right triangle

Base = AB = 12cm.
Hieght = BC = h = ?
Hypotinuse = 20 cm.

By pythagoruse Theorem

$$AB^2 + BC^2 = AC^2$$

$$BC^2 = AC^2 - AB^2$$

$$h^2 = 20^2 - 12^2$$

$$= 400 - 144$$

$$= 256$$

$$h = 16 \text{ cm.}$$

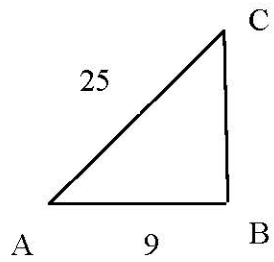


Fig. 10.10

Now , Area of triangle ABC $= \frac{1}{2}$(Base x height)

$= \frac{1}{2}(12 \times 16)$

$=$ 96 sq.cm

Example 2:

The discharge through a triangular notch is 270 cu cm/sec. Find the maximum depth of water if the velocity of water is 10 cm/sec and the width of water surface is 16cm.

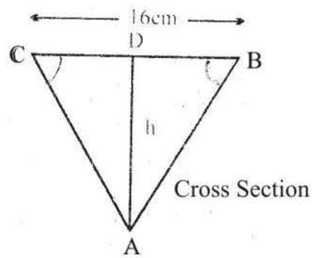


Fig. 10.11

Solution:

Given that

Discharge = 720 cu. cm/sec

Velocity of water = 10 cm/sec

Width of water = 16 cm.

Let 'h' be the depth of water

Area of cross-section = Area of ΔABC

$= \frac{1}{2}(BC)(AD) = \frac{1}{2}(16)(h) = 8h$

Discharge = (Area of cross-section) velocity

720 = 8h(10)

h = 9cm

(b) Area of the Triangle when two adjacent sides and their included angle is given:

Let, ABC be a triangle with two sides b, c and included angel A are given. Draw CP perpendicular to AB.

Area of ΔABC $= \frac{1}{2}$ base $\times$ height

$$= \frac{1}{2} AB \times CP$$

Since $\frac{CP}{AC} = \text{Sin } A$

$CP = AC \text{ Sin } A$

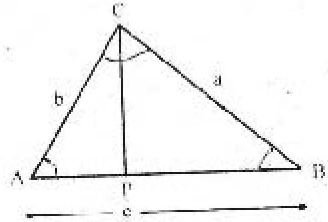


Fig. 10.12

Area of ΔABC $= \frac{1}{2} AB \times CP$

$= \frac{1}{2} AB \times AC \ \text{Sin } A$

$= \frac{1}{2} b \times c \text{ Sin } A$

$$A = \frac{1}{2} bc \text{ Sin } A = \frac{1}{2} ac \text{ Sin } B = \frac{1}{2} ab \text{ Sin } C$$

i.e Half of the product of two sides with sine of the angle between them.

Example 2:

Find the area of a triangle whose two adjacent sides are 17.5cm and 25.7cm respectively, and their included angle is 57°.

Solution:

Let two adjacent sides be a = 17.5 cm
b = 25.7 cm and included angle $\theta = 57°$

Area of ΔABC $= \frac{1}{2} ab \text{ Sin } \theta$

$= \frac{1}{2}(17.5)(25.7) \text{ Sin } 57°$

$= (224.88) \times (.84) = 188.89$ sq. cm

(c) Area of an equilateral triangle:

A triangle in which all the sides are equal, all the angles are also equal that is 60 degree. If each side of the triangle is 'a'.

Draw AP perpendicular to BC.

Then, $BP = PC = \frac{a}{2}$

Since, $|AB|^2 = |BP|^2 + |AP|^2$

$|AP|^2 = |AB|^2 - |BP|^2$

$= a^2 - \left(\frac{a}{2}\right)^2$

$$= a^2 - \frac{a^2}{4}$$

$$= \frac{3a^2}{4}$$

AP $$= \frac{\sqrt{3}}{2}a$$

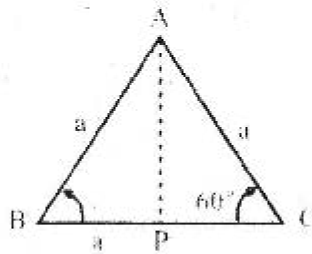


Fig. 10.14

Area of triangle = $\frac{1}{2}$x (base) (height)

$$= \frac{1}{2}a \times \frac{\sqrt{3}}{2}a$$

$$\boxed{A = \frac{\sqrt{3}}{4}a^2}$$

Example 3:

A triangle blank of equal sides is to punch in a copper plate, the area of the blank should be 24 sq. cm find the side.

Solution:

Area of triangular blank of equal sides = 24 sq. cm

Area of triangle of equal sides = $\frac{\sqrt{3}}{4}a^2$

$$24 = \frac{\sqrt{3}}{4}a^2$$

$$\sqrt{3}a^2 = 96$$

$$a^2 = 50.40$$

a = 7.4 cm each side

(d) Area of Triangle when all sides are given:

Let ABC be a triangle whose sides are a, b and c respectively, then

Area $= \sqrt{s(s-a)(s-b)(s-c)}$, where $s = \frac{a+b+c}{2}$

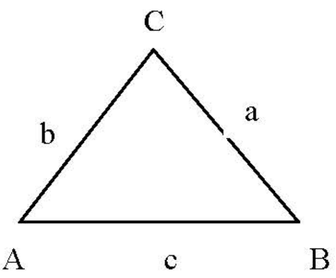


Fig.10.14

Which is called **Hero's Formul**

Corollary : For equilateral triangle ,

$$a = b = c$$

$$s = \frac{a + a + a}{2} = \frac{3a}{2}$$

$$s - a = s - b = s - c = \frac{3a}{2} - a = \frac{a}{2}$$

$$\text{Area of equilateral triangle} = \sqrt{\frac{3a}{2}\left(\frac{a}{2}\right)\left(\frac{a}{2}\right)\left(\frac{a}{2}\right)}$$

$$A = \frac{\sqrt{3}}{2}a^2$$

Example 4: Find the area of a triangle whose sides are 51, 37 and 20cm respectively.

Solution : Here, a =51 cm, b =37 cm c = 20cm

$$s = \frac{a + b + c}{2} = \frac{51 + 37 + 20}{2} = 54$$

$$\text{Area by Hero's Formula} = \sqrt{S(s-a)(s-b)(s-c)}$$

$$= \sqrt{54\,(3)\,(17)\,(34)} \qquad = 306 \text{ sq.unit}$$

Example 5:

Find the area of a triangle whose sides are 51, 37 and 20 cm respectively.

Solution:

Let a = 51cm, b = 37cm, and c = 20cm

Be the given sides triangle

$$S = \frac{a + b + c}{2} = \frac{51+37+20}{2} = \frac{108}{2} = 54$$

$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{54(54-51)(54-37)(54-20)} = \sqrt{54(3)(17)(34)}$$

$$= 306 \text{ sq. cm}$$

Example 6:

The sides of a triangle are 13, 12 and 9cm respectively. Find the distance of the longest side from the opposite vertex.

Solution:

Let

Given sides be

a = 13cm, b = 12cm, c = 9cm

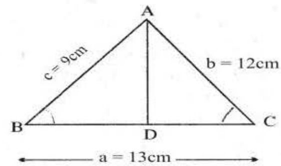


Fig. 10.17

$$S = \frac{a+b+c}{2} = \frac{13+12+9}{2} = \frac{34}{2} = 17$$

Fig.10.15

$$\text{Area of } \Delta ABC = \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{17(17-13)(17-12)(17-9)} = \sqrt{17(4)(5)(8)}$$

$$\text{Area} = \sqrt{2720} = 52.15 \text{ sq. cm.}$$

Let h = distance of the longest side from opposite vertex

$$\text{Also Area of } \Delta ABC = \frac{1}{2}(\text{base})(\text{height}) = \frac{1}{2}(a)(h)$$

$$52.15 = \frac{1}{2}(\text{longest sides})(\text{required distance})$$

$$52.15 = \frac{1}{2}(13)(h)$$

$$h = \frac{2x52.15}{13} = 8.02\text{cm}$$

Exercise 10

Q.1. The hypotenuse of a right triangle is 10 cm and its height is twice of its base. Find the area of the triangle.

Q.2. From a point within an Equilateral triangle perpendicular are drawn to the three sides are 6, 7 and 8 cm respectively. Find the area of triangle.

Q.3. The sides of triangular pond are 242, 1212 and 1450m. Find the total amount of antiseptic medicine needed for spraying when one gallon of the medicine is sufficient for 100 square meter of water surface of the pond.

Q.4. The sides of a triangle are 21, 20 and 13cm respectively, find the area of the triangles into which it is divided by the perpendicular upon the longest side from the opposite angular point.

Q.5. From a point within a triangle, it is found that the three sides subtend equal angles. From this point, three line are drawn to meet the opposite edges. If these lines measure 5, 6 and 7cm respectively. Find the area of the triangle.

Q.6. The sides of a triangular lawn are proportional to the numbers 5, 12 and 13. The cost of fencing it at the rate of Rs 2 per meter is Rs 120. Find the sides. Also find the cost of turfing the lawn at 25 paisa per square meter.

Q.7. Find the area of the triangle whose sides are in the ratio 9 : 40 : 41 and whose perimeter in 180 meters.

Q.8. The corresponding bases of two triangles, giving equal altitude, are 8cm and 10cm. the area of the smaller is 108 sq.cm. Find the area of the larger triangle and altitude of each

Answers 10

Q1. 19.98 sq.cm **Q2.** 254.60 sq. cm **Q3.** 290.40 gallons
Q4. 8.32cm, 24.96cm **Q5.** 46.33 sq. cm
Q6. 10m, 24m, 26m, Rs. 30 **Q.7** 720sq.cm **Q.8.** 135sq.cm, 27cm

Summary

(1) **Plane Figures**

Plane figures are those figures which occupy an area with only two dimensions e.g. room floor, grassy plots, tin sheets.

(2) **Triangle**

A triangle is plane figure bounded by three straight lines.

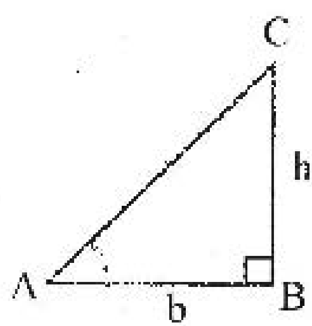


Fig. 10.18

Kinds of Triangles

(i) Obtuse angled triangle (in which one angle >90°)
(ii)Right angled triangle (in which one angle in 90°)
(iii)Acute angled triangle (in which one angle less than 90°)
(iv)Iosceles triangle (two side equal)
(v) Equilateral triangle (three sides equal)
(vi)Scalene triangle (all sides are different)

(3)Area of Triangle in terms of its height (altitude) and base

Area = $\frac{1}{2}$(base) height

$$\text{Area} = \frac{1}{2}bh$$

(4) Area of an equilateral triangle with side 'a' is

$$\text{Area} = \frac{\sqrt{3}}{4}a^2$$

(5) Area of triangle when two adjacent sides and their included angle is given.

$$\text{Area} = \frac{1}{2}bc\,\sin\theta$$

Which is also called Snell's Formula.

(6) Area of a triangle when all sides are given

$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}$$

$$\text{When } S = \frac{a+b+c}{2}$$

It is called Hero's Formula

Perimeter of triangle = (a + b + c) units

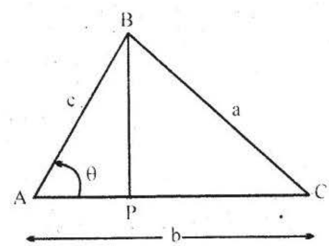


Fig. 10.19