Q2.(i) $\frac{7}{18}$ (ii) $-\frac{9}{5}$ (iii) - 12

Q3.(i) K =10, roots = 6, 1 (ii) $\alpha = \frac{7}{3}, \beta = \frac{3}{2}$; c = 21

Q4. (i) $\frac{-b^3+3abc}{a^3}$ (ii) $\frac{b^2-2ac}{c^2}$ (iii) $-\frac{b}{\sqrt{ac}}$ (iv) $\frac{3abc-b^3}{a^2c}$ (v) $\frac{-b\sqrt{b^2-4ac}}{ac}$

Q5. - 27 **Q7.** $Pr(p+r)+q^3 = 3pqr$ **Q8.** K = - 14, roots are $-\frac{2}{3}, \frac{7}{3}$

Q9. (i) $\frac{q^2-2pr}{p^2}$ (ii) $\frac{q^2-4pr}{p^2}$ (iii) $\frac{r(q^2-2pr)}{p^3}$

1.11 Formation of Quadratic Equation from the given roots :

Let α, β be the roots of the Equation $ax^2 + bx + c = 0$

The sum of roots $= \alpha + \beta = -\frac{b}{a}$ ……… (I)

Product of roots $= \alpha . \beta = \frac{c}{a}$ ……… (II)

The equation is $ax^2 + bx + c = 0$

Divide this equation by a $\Rightarrow x^2 + \frac{b}{a}x + \frac{c}{a} = 0$

Or $x^2 - \left(-\frac{b}{a}\right)x + \frac{c}{a} = 0$

From I and II this equation becomes

$$x^2 - (\alpha + \beta)x + \alpha\beta = 0$$

Or x^2 – (Sum of roots) x + Product of roots = 0

Or $x^2 - (S)x + (P) = 0$

is the required equation, where $S = \alpha + \beta$ and $P = \alpha\beta$

Alternate method:-

Let α, β be the roots of the equation $ax^2 + bx + c = 0$

i.e., $x = \alpha$ and $x = \beta$

$\Rightarrow x - \alpha = 0$ and $x - \beta = 0$

$\Rightarrow (x - \alpha)(x - \beta) = 0$

$$x^2 - \alpha x - \beta x + \alpha\beta = 0$$

$x^2 - (\alpha + \beta)x + \alpha\beta = 0$

Or x^2 – (Sum of roots) x + Product of roots = 0

Or $x^2 - S x + P = 0$

is the required equation, where $S = \alpha + \beta$ and $P = \alpha\,\beta$

Example 4:

Form a quadratic Equation whose roots are $3\sqrt{5}, -3\sqrt{5}$

Solution:

Roots of the required Equation are $3\sqrt{5}$ and $-3\sqrt{5}$

Therefore S = Sum of roots = $3\sqrt{5} - 3\sqrt{5}$

$S = 0$

P = Product of roots = $(3\sqrt{5})(-3\sqrt{5}) = -9\,(5)$

$P = -45$

Required equation is

x^2 – (Sum of roots) x + (Product of roots) = 0

Or $x^2 - Sx + P = 0$

$x^2 - 0(x) + (-45) = 0$

$x^2 - 0 - 45 = 0$

$x^2 - 45 = 0$

Example 5:

If α, β are the roots of the equation $ax^2 + bx + c = 0$, find the equation whose **roots are** $\dfrac{\alpha}{\beta}, \dfrac{\beta}{\alpha}$**.**

Solution:

Because α, β are the roots of the Equation $ax^2 + bx + c = 0$

The sum of roots = $\alpha + \beta = -\dfrac{b}{a}$

Product of roots = $\alpha\,\beta = \dfrac{b}{a}$

Roots of the required equation are $\dfrac{\alpha}{\beta}, \dfrac{\beta}{\alpha}$

Therefore ,

$$S = \text{sum of roots of required equation} = \frac{\alpha}{\beta} + \frac{\beta}{\alpha}$$

$$= \frac{\alpha^2 + \beta^2}{\alpha\beta} \because (\alpha + \beta)^2 = \alpha^2 + \beta^2 + 2\alpha\beta$$

$$= \frac{(\alpha + \beta)^2 - 2\alpha\beta}{\alpha\beta} = \frac{\left(-\frac{b}{a}\right)^2 - 2\frac{c}{a}}{\alpha\beta}$$

$$= \frac{\frac{b^2}{a^2} - \frac{2c}{a}}{\frac{c}{a}} = \frac{b^2 - 2ac}{a^2} x \frac{a}{c}$$

$$S = \frac{b^2 - 2ac}{ac}$$

P = Product of roots of required equation $= \frac{\alpha}{\beta} \cdot \frac{\beta}{\alpha} = \frac{\alpha\beta}{\beta\alpha}$

$P = 1$

Required equation is: $x^2 - Sx + P = 0$

$$x^2 + \left(\frac{b^2 - 2ac}{ac}\right) x + 1 = 0$$

$$acx^2 - (b^2 - 2ac)x + ac = 0$$

Exercise 1.4

Q1. Form quadratic equations with the following given numbers as its roots.

(i) $2, -3$ (ii) $3 + i\ , 3 - i$ (iii) $2+\sqrt{3},\ 2-\sqrt{3}$

(iv) $-3+\sqrt{5},\ -3-\sqrt{5}$ (v) $4 + 5i\ ,\ 4 - 5i$

Q2. Find the quadratic equation with roots

(i)Equal numerically but opposite in sign to those of the roots of the equation $3x^2 + 5x - 7 = 0$

(ii)Twice the roots of the equation $5x^2 + 3x + 2 = 0$

(iii)Exceeding by '2' than those of the roots of $4x^2 + 5x + 6 = 0$

Q3. Form the quadratic equation whose roots are less by '1' than those of $3x^2 - 4x - 1 = 0$

Q4. Form the quadratic equation whose roots are the square of the roots of the equation $2x^2 - 3x - 5 = 0$

Q5. Find the equation whose roots are reciprocal of the roots of the equation $px^2 - qx + r = 0$

Q6. If $\propto, \beta$ are the roots of the equation $x^2 - 4x + 2 = 0$ find the equation whose roots are

(i) $\alpha^2\ ,\ \beta^2$ (ii) $\alpha^3\ ,\ \beta^3$ (iii) $\propto + \frac{1}{\propto}, \beta + \frac{1}{\beta}$

(iv) $\alpha + 2\ , \beta + 2$

Q7. If α , β are the roots of $ax^2 + bx + c = 0$ form an equation whose roots are

(i) $\frac{\alpha}{\beta}, \frac{\beta}{\alpha}$ (ii) $\frac{\alpha^2}{\beta}, \frac{\beta^2}{\alpha}$ (iii) $\frac{\alpha+1}{\alpha}, \frac{\beta+1}{\beta}$

Answers 1.4

Q1. (i) $x^2 + x - 6 = 0$ (ii) $x^2 - 6x + 10 = 0$
(iii) $x^2 - 4x + 1 = 0$ (iv) $x^2 + 6x + 4 = 0$ (v) $x^2 - 5x + 41 = 0$

Q2. (i) $3x^2 - 5x - 7 = 0$ (ii) $5x^2 - 6x + 8 = 0$
(iii) $4x^2 - 11x + 12 = 0$

Q3. $3x^2 + 2x - 2 = 0$ **Q4.** $4x^2 - 29x + 25 = 0$

Q5. $rx^2 - qx + p = 0$ **Q6.** (i) $x^2 - 12x + 4 = 0$ (ii) $x^2 - 40x + 8 = 0$
(iii) $2x^2 - 12x + 17 = 0$ (iv) $x^2 - 8x + 14 = 0$

Q7. (i) $acx^2 - (b^2 - 2ac)x + ac = 0$ (ii) $a^2cx^2 + (b^3 - 3abc)x + ac^2 = 0$
(iii) $cx^2 - (2c - b)x + (a - b + c) = 0$

Summary

Quadratic Equation:

An equation of the form $ax^2 + bx + c = 0$, $a \neq 0$, where $a, b, c \in R$ and x is a variable, is called a quadratic equation.

If α, β are its roots then

$$\alpha = \frac{-b+\sqrt{b^2-4ac}}{2a}, \ \beta = \frac{-b-\sqrt{b^2-4ac}}{2a}$$

Nature of Roots:

(i) If $b^2 - 4ac > 0$ the roots are real and distinct.
(ii) If $b^2 - 4ac = 0$ the roots are real and equal.
(iii) If $b^2 - 4ac < 0$ the roots are imaginary.
(iv) If $b^2 - 4ac$ is a perfect square, roots will be rational, otherwise irrational.

Relation between Roots and Co-efficients

If α and β be the roots of the equation $ax^2 + bx + c = 0$

Then sum of roots $= \alpha + \beta = \frac{-b}{a}$

Product of roots $= \alpha\beta = \frac{c}{a}$

Formation of Equation

If α and β be the roots of the equation $ax^2 + bx + c = 0$ then we have

x^2 – (sum of roots)x + (product of roots) = 0