(ii) $(mx + c)^2 = 4ax$ will be equal if $c = \frac{a}{m}$

(iii) $x^2 + (mx + c)^2 = a^2$ has equal roots if $c^2 = a^2 (1 + m^2)$.

Q4. If the roots of $(c^2 - ab)x^2 - 2(a^2 - bc)x + (b^2 - ac) = 0$ are equal then prove that $a^3 + b^3 + c^3 = 3abc$

Q5. Show that the roots of the following equations are real

(i) $x^2 - 2\left(m + \frac{1}{m}\right)x + 3 = 0$

(ii) $x^2 - 2ax + a^2 = b^2 + c^2$

(iii) $(b^2 - 4ac)x^2 + 4(a + c)x - 4 = 0$

Q6. Show that the roots of the following equations are rational

(i) $a(b - c)x^2 + b(c - a)x + c(a - b) = 0$

(ii) $(a + 2b)x^2 + 2(a + b + c)x + (a + 2c) = 0$

(iii) $(a + b)x^2 - ax - b) = 0$

(iv) $p x^2 - (p - q) x - q = 0$

Q7. For what value of 'K' the equation $(4-k) x^2 + 2(k+2) x + 8k + 1 = 0$ will be a perfect square.

(Hint : The equation will be perfect square if Disc. $b^2 - 4ac = 0$)

Answers 1.2

Q1. (i) Real, rational, unequal (ii) unequal, real and rational

(iii) ir-rational, unequal, real (iv) Real, unequal, ir-rational

Q2. (i) 1, $\frac{-11}{9}$ (ii) - 2 (iii) 1, -3 (iv) 2

Q7. 0, 3

1.10 Sum and Product of the Roots

(Relation between the roots and Co-efficient of $ax^2 + bx + c = 0$)

The roots of the equation $ax^2 + bx + c = 0$ are

$$\propto = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\beta = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

Sum of roots

Add the two roots

$$\propto + \beta = \frac{-b + \sqrt{b^2 - 4ac}}{2a} + \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-b+\sqrt{b^2-4ac}-b-\sqrt{b^2-4ac}}{2a}$$

$$= \frac{-b\ -b}{2a}$$

$$= \frac{-2b}{2a} \qquad = -\frac{b}{a}$$

Hence, sum of roots = $\propto + \beta = \dfrac{-\text{ Co-efficient of x}}{\text{Co-efficient of } x^2}$

Product of roots

$$\propto \beta = \left(\frac{-b+\sqrt{b^2-4ac}}{2a}\right) x \left(\frac{-b-\sqrt{b^2-4ac}}{2a}\right) =$$

$$= \frac{(-b)^2 - (\sqrt{b^2 - 4ac}\ )^2}{2\ a^2}$$

$$= \frac{b^2 - b^2 + 4ac}{4a^2}$$

$$= \frac{4ac}{4a^2}$$

$$a\beta = \frac{c}{a}$$

i.e. product of roots = $\propto \beta = \dfrac{-\text{ Constant term}}{\text{Co-efficient of } x^2}$

Example 1:

Find the sum and the Product of the roots in the Equation $2x^2 + 4 = 7x$

Solution:

$2x^2 + 4 = 7x$

$2x^2 - 7x + 4 = 0$

Here a = 2, b = -7, c = 4

Sum of the roots = $-\dfrac{b}{a} = -\left(-\dfrac{7}{2}\right) = \dfrac{7}{2}$

Product of roots = $\dfrac{c}{a} = \dfrac{4}{2} = 2$

Example 2:

Find the value of "K" if sum of roots of

$(2k - 1)x^2 + (4K - 1)x + (K + 3) = 0$ is $\dfrac{5}{2}$

Solution:

$(2k - 1)x^2 + (4K - 1)x + (K + 3) = 0$

Here $a = (2k - 1)$, $b = 4K - 1$, $c = K + 3$

Sum of roots $= -\frac{b}{a}$

$\frac{5}{2} = -\frac{(4K - 1)}{(2K - 1)}$ $\because$ Sum of roots $= \frac{5}{2}$

$5(2K - 1) = -2(4K - 1)$

$10K - 5 = -8K + 2$

$10K + 8K = 5 + 5$

$18K = 7$

$K = \frac{7}{18}$

Example 3:

If one root of $4x^2 - 3x + K = 0$ is 3 times the other, find the value of "K".

Solution:

Given Equation is $4x^2 - 3x + K = 0$

Let one root be α, then other will be 3α.

Sum of roots $= -\frac{a}{b}$

$\alpha + 3\alpha = -\frac{(-3)}{4}$

$4\alpha = \frac{3}{4}$

$\alpha = \frac{3}{16}$

Product of roots $= \frac{c}{a}$

$\alpha(3\alpha) = \frac{K}{4}$

$3\alpha^2 = \frac{K}{4}$

$K = 12\alpha^2$

Putting the value of $\alpha = \frac{3}{16}$ we have

$K = 12\left(\frac{3}{16}\right)^2$

$= \frac{12x9}{256} = \frac{27}{64}$

Exercise 1.3

Q1. Without solving, find the sum and the product of the roots of the following equations.

(i) $x^2 - x + 1 = 0$ (ii) $2y^2 + 5y - 1 = 0$

(iii) $x^2 - 9 = 0$ (iv) $2x^2 + 4 = 7x$

(v) $5x^2 + x - 7 = 0$

Q2. Find the value of k, given that

(i) The product of the roots of the equation $(k + 1)x^2 + (4k + 3)x + (k - 1) = 0$ is $\frac{7}{2}$

(ii) The sum of the roots of the equation $3x^2 + k x + 5 = 0$ will be equal to the product of its roots.

(iii) The sum of the roots of the equation $4x^2 + kx - 7 = 0$ is 3.

Q3. (i) If the difference of the roots of $x^2 - 7x + k - 4 = 0$ is 5, find the value of k and the roots.

(ii) If the difference of the roots of $6x^2 - 23x + c = 0$ is $\frac{5}{6}$, find the value of k and the roots.

Q4. If α, β are the roots of $ax^2 + bx + c = 0$ find the value of

(i) $\alpha^3 + \beta^3$ (ii) $\frac{1}{\alpha^2} + \frac{1}{\beta^2}$ (iii) $\sqrt{\frac{\alpha}{\beta}} + \sqrt{\frac{\beta}{\alpha}} = 0$

(iv) $\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha}$ (v) $\frac{\alpha}{\beta} - \frac{\beta}{\alpha}$

(v)

Q5. If p, q are the roots of $2x^2 - 6x + 3 = 0$ find the value of $(p^3 + q^3) - 3pq(p^2 + q^2) - 3pq(p + q)$

Q6. The roots of the equation $px^2 + qx + q = 0$ are α and β,

Prove that $\sqrt{\frac{\alpha}{\beta}} + \sqrt{\frac{\beta}{\alpha}} + \sqrt{\frac{q}{p}} = 0$

Q7. Find the condition that one root of the equation $px^2 + qx + r = 0$ is square of the other.

Q8. Find the value of k given that if one root of $9x^2 - 15x + k = 0$ exceeds the other by 3. Also find the roots.

Q9. If α, β are the roots of the equation $px^2 + qx + r = 0$ then find the values of

(i) $\alpha^2 + \beta^2$ (ii) $(\alpha - \beta)^2$ (iii) $\alpha^3\beta + \alpha\beta^3$

Answers 1.3

Q1.(i) 1, 1 (ii) $-\frac{5}{2}, -\frac{1}{2}$ (iii) 0, - 9 (iv) $\frac{7}{2}, 2$ (v) $-\frac{1}{5}, -\frac{7}{5}$

Q2.(i) $\frac{7}{18}$ (ii) $-\frac{9}{5}$ (iii) - 12

Q3.(i) K =10, roots = 6, 1 (ii) $\alpha = \frac{7}{3}, \beta = \frac{3}{2}$; c = 21

Q4. (i) $\frac{-b^3+3abc}{a^3}$ (ii) $\frac{b^2-2ac}{c^2}$ (iii) $-\frac{b}{\sqrt{ac}}$ (iv) $\frac{3abc-b^3}{a^2c}$ (v) $\frac{-b\sqrt{b^2-4ac}}{ac}$

Q5. - 27 **Q7.** $Pr(p+r)+q^3 = 3pqr$ **Q8.** K = - 14, roots are $-\frac{2}{3}, \frac{7}{3}$

Q9. (i) $\frac{q^2-2pr}{p^2}$ (ii) $\frac{q^2-4pr}{p^2}$ (iii) $\frac{r(q^2-2pr)}{p^3}$

1.11 Formation of Quadratic Equation from the given roots :

Let α, β be the roots of the Equation $ax^2 + bx + c = 0$

The sum of roots $= \alpha + \beta = -\frac{b}{a}$ ……… (I)

Product of roots $= \alpha . \beta = \frac{c}{a}$ ……… (II)

The equation is $ax^2 + bx + c = 0$

Divide this equation by a $\Rightarrow x^2 + \frac{b}{a}x + \frac{c}{a} = 0$

Or $x^2 - \left(-\frac{b}{a}\right)x + \frac{c}{a} = 0$

From I and II this equation becomes

$$x^2 - (\alpha + \beta)x + \alpha\beta = 0$$

Or x^2 – (Sum of roots) x + Product of roots = 0

Or $x^2 - (S)x + (P) = 0$

is the required equation, where $S = \alpha + \beta$ and $P = \alpha\beta$

Alternate method:-

Let α, β be the roots of the equation $ax^2 + bx + c = 0$

i.e., $x = \alpha$ and $x = \beta$

$\Rightarrow$ $x - \alpha = 0$ and $x - \beta = 0$

$\Rightarrow$ $(x - \alpha)(x - \beta) = 0$

$$x^2 - \alpha x - \beta x + \alpha\beta = 0$$