(vii). $\left\{-\frac{1}{2}, 3\right\}$ (viii). {-a, -b} (ix). $\{-\frac{b}{a}, \frac{c}{b}\}$

(x). $\{-1, -\frac{b+c}{a+b}\}$ (xi). $\{\frac{a+b}{ab}, \frac{2}{a+b}\}$ (xii). $\{\frac{13}{4}, \frac{1}{2}\}$

Q.2. (i). {2, 4} (ii). $\{2, -\frac{16}{3}\}$ (iii). $\{\frac{1 \pm \sqrt{113}}{2}\}$

(iv). {– a, – b} (v). $\{3, \frac{1}{3}\}$ (vi). {-1, 25}

(vii). $\{3b, -\frac{b}{2}\}$ (viii) { (a + b), (a – b)}

Q.3. (i). $\{\frac{3}{2}, -3\}$ (ii). $\{\frac{1 \pm \sqrt{53}}{2}\}$ (iii). $\{\frac{-11 \pm \sqrt{13}}{6}\}$

(iv). $\{\frac{25}{2}, -\frac{1}{2}\}$ (v). $\{m - n, -2(m - n)\}$ (vi) $\left\{-1, \frac{-1}{m}\right\}$

(vii). $\{-\frac{2}{a}, \frac{3}{b}\}$ (viii). $\{-b, a\}$ (ix) $\left\{\frac{-6+\sqrt{3}}{3}, \frac{-6-\sqrt{3}}{3}\right\}$

Q.4. 7 , - 8 Q.5. 8.465 seconds Q.6. 10.6 m

1.8 Classification of Numbers

1. The Set N of Natural Numbers:
Whose elements are the counting, or natural numbers:
N = {1, 2, 3, - - - - - - - - }

2. The Set Z of Integers:
Whose elements are the positive and negative whole numbers and zero:
Z = {- - - - - - , -2, -1, 0, 1, 2, - - - - - - }

3. The Set Q of Rational Numbers: Whose elements are all those numbers that can be represented as the quotient of two integers $\frac{a}{b}$, where b ≠ 0. Among the elements of Q are such numbers as $-\frac{3}{4}, \frac{18}{27}, \frac{5}{1}, -\frac{9}{1}$. In symbol

$$Q = \left\{\frac{a}{b} \mid a, b \in Z, b \neq 0\right\}$$

Equivalently, rational numbers are numbers with terminating or repeating decimal representation, such as

1.125, 1.52222, 1.56666, 0.3333

4. The Set Q′ of Irrational Numbers:

Whose elements are the numbers with decimal representations that are non-terminating and non-repeating. Among the elements of this set are such numbers as $\sqrt{2}, -\sqrt{7}, \pi$.

An irrational number cannot be represented in the form $\frac{a}{b}$, where $a, b \in Z$. In symbols,

$Q' = \{\text{irrational numbers}\}$

5. The Set R of Real Numbers:

Which is the set of all rational and irrational numbers:

$$R = \{x \mid x \in Q \cup Q'\}$$

6. The set I of Imaginary Numbers:

Whose numbers can be represented in the form x + yi, where x and y are real numbers, $y \neq 0$ and $i = \sqrt{-1}$

$$I = \{x + yi \mid x, y \in R, y \neq 0, i = \sqrt{-1}\}$$

If x = 0, then the imaginary number is called a pure imaginary number.

An imaginary number is defined as, a number whose square is a negative i.e, $\sqrt{-1}, \sqrt{-3}, \sqrt{-5}$

7. The set C of Complex Numbers:

Whose members can be represented in the form x + y i, where x and y real numbers and $i = \sqrt{-1}$:

$$C = \{x + yi \mid x, y \in R, i = \sqrt{-1}\}$$

With this familiar identification, the foregoing sets of numbers are related as indicated in Fig. 1.

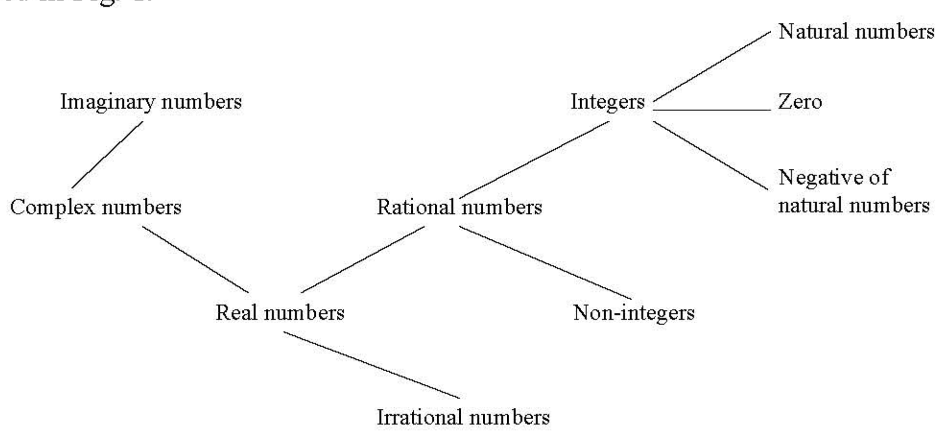

Fig. 1

Hence, it is clear that $N \subseteq Z \subseteq Q \subseteq R \subseteq C$

1.9 Nature of the roots of the Equation $ax^2 + bx + c = 0$

The two roots of the Quadratic equation $ax^2 + bx + c = 0$ are:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

The expression $b^2 - 4ac$ which appear under radical sign is called the Discriminant (Disc.) of the quadratic equation. i.e., Disc = $b^2 - 4ac$

The expression $b^2 - 4ac$ discriminates the nature of the roots, whether they are real, rational, irrational or imaginary. There are three possibilities.

(i) $b^2 - 4ac < 0$ (ii) $b^2 - 4ac = 0$ (iii) $b^2 - 4ac > 0$

(i) If $b^2 - 4ac < 0$, then roots will be imaginary and unequal.
(ii) If $b^2 - 4ac = 0$, then roots will be real, equal and rational.
(This means the left hand side of the equation is a perfect square).
(iii) If $b^2 - 4ac > 0$, then two cases arises:
(a) $b^2 - 4ac$ is a perfect square, the roots are real, rational and unequal.
(This mean the equation can be solved by the factorization).
(b) $b^2 - 4ac$ is not a perfect square, then roots are real, irrational and unequal.

Example 1:

Find the nature of the roots of the given equation

$9x^2 + 6x + 1 = 0$

Solution:

$9x^2 + 6x + 1 = 0$

Here $a = 9$, $b = 6$, $c = 1$

Therefore, Discriminant $= b^2 - 4ac$
$= (6)^2 - 4(9)(1)$
$= 36 - 36$
$= 0$

Because $b^2 - 4ac = 0$

∴ roots are equal, real and rational.

Example 2:

Find the nature of the roots of the Equation

$3x^2 - 13x + 9 = 0$

Solution:

$3x^2 - 13x + 9 = 0$

Here $a = 3$, $b = -13$, $c = 9$

Discriminant $= b^2 - 4ac$
$= (-13)^2 - 4(3)(9)$
$= 169 - 108 = 61$

Disc = $b^2 - 4ac = 61$ which is positive

Hence the roots are real, unequal and irrational.

Example 3:

For what value of "K" the roots of $Kx^2 + 4x + (K - 3) = 0$ are equal.

Solution:

$Kx^2 + 4x + (K - 3) = 0$

Here $a = K$, $b = 4$, $c = K - 3$

Disc $= b^2 - 4ac$
$= (4)^2 - 4(K)(K - 3)$
$= 16 - 4K^2 + 12K$

The roots are equal if $b^2 - 4ac = 0$

i.e. $16 - 4K^2 + 12K = 0$

$4K^2 - 3K - 4 = 0$

$K^2 - 4K + K - 4 = 0$

$K(K - 4) + 1(K - 4) = 0$

Or $K = 4, -1$

Hence roots will be equal if $K = 4, -1$

Example 4:

Show that the roots of the equation

$2(a + b)x^2 - 2(a + b + c)x + c = 0$ are real

Solution: $2(a + b)x^2 - 2(a + b + c)x + c = 0$

Here, $a = 2(a + b)$, $b = -2(a + b + c)$, $c = c$

Discriminant $= b^2 - 4ac$

$= [-2(a + b + c)]^2 - 4[2(a + b)c]$

$= 4(a^2 + b^2 + c^2 + 2ab + 2bc + 2ac) - 8(ac + bc)$

$= 4(a^2 + b^2 + c^2 + 2ab + \cancel{2bc} + \cancel{2ac} - \cancel{2ac} - \cancel{2bc})$

$= 4(a^2 + b^2 + c^2 + 2ab)$

$= 4[(a^2 + b^2 + 2ab) + c^2]$

$= 4[(a + b)^2 + c^2]$

Since each term is positive, hence

Disc > 0 Hence , the roots are real.

Example 5:

For what value of K the roots of equation $2x^2 + 5x + k = 0$ will be rational.

Solution:

$2x^2 + 5x + k = 0$

Here, $a = 2$, $b = 5$, $c = k$

The roots of the equation are rational if

Disc $= b^2 - 4ac = 0$

So, $5^2 - 4(2)k = 0$

$25 - 8k = 0$

$k = \frac{25}{8}$ Ans

Exercise 1.2

Q1. Find the nature of the roots of the following equations

(i) $2x^2 + 3x + 1 = 0$ (ii) $6x^2 = 7x + 5$

(iii) $3x^2 + 7x - 2 = 0$ (iv) $\sqrt{2}\,x^2 + 3x - \sqrt{8} = 0$

Q2. For what value of K the roots of the given equations are equal.

(i) $x^2 + 3(K + 1)x + 4K + 5 = 0$ (ii) $x^2 + 2(K - 2)x - 8k = 0$

(iii) $(3K + 6)x^2 + 6x + K = 0$ (iv) $(K + 2)x^2 - 2Kx + K - 1 = 0$

Q3. Show that the roots of the equations

(i) $a^2(mx + c)^2 + b^2x^2 = a^2 b^2$ will be equal if $c^2 = b^2 + a^2m^2$

(ii) $(mx + c)^2 = 4ax$ will be equal if $c = \dfrac{a}{m}$

(iii) $x^2 + (mx + c)^2 = a^2$ has equal roots if $c^2 = a^2 (1 + m^2)$.

Q4. If the roots of $(c^2 - ab)x^2 - 2(a^2 - bc)x + (b^2 - ac) = 0$ are equal then prove that $a^3 + b^3 + c^3 = 3abc$

Q5. Show that the roots of the following equations are real

(i) $x^2 - 2\left(m + \dfrac{1}{m}\right)x + 3 = 0$

(ii) $x^2 - 2ax + a^2 = b^2 + c^2$

(iii) $(b^2 - 4ac)x^2 + 4(a + c)x - 4 = 0$

Q6. Show that the roots of the following equations are rational

(i) $a(b - c)x^2 + b(c - a)x + c(a - b) = 0$

(ii) $(a + 2b)x^2 + 2(a + b + c)x + (a + 2c) = 0$

(iii) $(a + b)x^2 - ax - b) = 0$

(iv) $p x^2 - (p - q) x - q = 0$

Q7. For what value of 'K' the equation $(4-k) x^2 + 2(k+2) x + 8k + 1 = 0$ will be a perfect square.

(Hint : The equation will be perfect square if Disc. $b^2 - 4ac = 0$)

Answers 1.2

Q1. (i) Real, rational, unequal (ii) unequal, real and rational

(iii) ir-rational, unequal, real (iv) Real, unequal, ir-rational

Q2. (i) $1, \dfrac{-11}{9}$ (ii) -2 (iii) $1, -3$ (iv) 2

Q7. 0, 3

1.10 Sum and Product of the Roots

(Relation between the roots and Co-efficient of $ax^2 + b x + c = 0$)

The roots of the equation $ax^2 + bx + c = 0$ are

$$\propto = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\beta = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

Sum of roots

Add the two roots

$$\propto + \beta = \frac{-b + \sqrt{b^2 - 4ac}}{2a} + \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$