DAE / IIA - 2020

MATH - 233 APPLIED MATHEMATICS - II

PAPER 'B' PART - A (OBJECTIVE)

Time: 30 Minutes Marks: 15

Q.1: *Encircle the correct answer.*

1. $\int\left(\frac{a+x}{x}\right)dx = ?$

[a] $a\,\ell nx + x$ [b] $\frac{(ax+b)^2}{2}$

[c] $\ell nx + a$ [d] $x + a$

2. $\int(\sec x)\,dx = ?$

[a] $\tan x$ [b] $\frac{\sec^2 x}{2}$

[c] $\ell n(\sec x + \tan x)$ [d] $\sec x \tan x$

3. ~~$\int e^{2x}\,dx = ?$~~

[a] $\frac{e^{2x}}{2}$ [b] $\frac{e^{x2}}{2}$ [c] $2e^{2x}$ [d] $\frac{e^{2x+1}}{2}$

4. ~~$\int\left(\frac{1}{1+x^2}\right)dx = ?$~~

[a] $\sin^{-1} x$ [b] $\cos^{-1} x$

[c] $\sec^{-1} x$ [d] $\tan^{-1} x$

5. $\int(x\sin x)\,dx = ?$

[a] $-x\cos x + \sin x$ [b] $\sin x$

[c] $x + \sin x$ [d] $\frac{x^2}{2}\cos x$

6. ~~$\int_1^3 \left(e^{2x}\right)dx = ?$~~

[a] $e^6 - e^2$ [b] $\frac{e^{2x}}{2}$

[c] $\frac{1}{2}\left(e^6 + e^2\right)$ [d] $\frac{1}{2}\left(e^6 - e^2\right)$

7. $\int_1^2 \left(3x^2\right)dx = ?$ [a] 7 [b] 8 [c] 6 [d] 9

8. An equation involving one or more derivative of a function is called:

[a] Quadratic [b] Linear

[c] Differential [d] Cubic

9. Solution of D.E. $\frac{dy}{dx} = 1$ is:

[a] $y = x$ [b] $y = x + c$

[c] $y = c$ [d] $y = x^2 + c$

10. Degree of D.E. $x\left(\frac{d^3y}{d^3}\right)^2 = 1$ is:

[a] 0 [b] 1 [c] 2 [d] 3

11. If a function $f(x)$ is periodic if $f(x) = f(\cdots)$:

[a] $x = T$ [b] $\pm\frac{x}{T}$ [c] $\pm xT$ [d] None

12. ~~If an odd function, then Fourier coefficient~~ 'a_0' ~~is;~~

[a] 0 [b] 1 [c] -1 [d] 2

13. Laplace transform of the function $f(t) = 1$ is:

[a] $\frac{1}{S^3}$ [b] $\frac{1}{S^2}$ [c] $\frac{1}{S}$ [d] $-\frac{1}{S}$

14. ~~$L^{-1}\left(\frac{S}{S^2+1}\right) = ?$~~

[a] $\sin t$ [b] $\cos t$ [c] $\sin\left(\frac{1}{t}\right)$ [d] $\cos\left(\frac{1}{t}\right)$

15. The series $\frac{a_0}{2} + \sum_{n=1}^{\infty}\left(a_n \cos nx + b_n \sin nx\right)$ is:

[a] Binomial [b] Fourier

[c] Arithmetic [d] Geometric

Answer Key

1	a	2	c	3	a	4	d	5	a
6	b	7	a	8	c	9	b	10	c
11	a	12	a	13	c	14	b	15	b

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DAE / IIA - 2020

MATH - 233 APPLIED MATHEMATICS - II

PAPER 'B' PART - B (SUBJECTIVE)

Time: 2:30 Hrs Marks: 60

Section - I

Q.1. Write short answersto any Eighteen (18) questions.

1. Evaluate $\int\left(\sqrt{x}+\frac{1}{\sqrt{x}}\right)^2 dx$

Sol. $\int\left(\sqrt{x}+\frac{1}{\sqrt{x}}\right)^2 dx$

$$=\int\left[\left(\sqrt{x}\right)^2+2\left(\sqrt{x}\right)\left(\frac{1}{\sqrt{x}}\right)+\left(\frac{1}{\sqrt{x}}\right)^2\right]dx$$

$$=\int\left[x+2+\frac{1}{x}\right]dx=\boxed{\frac{x^2}{2}+2x+\ell nx+c}$$

2. $\int\left(3t-\sqrt{3t}+\frac{1}{\sqrt{t}}\right)dt$

Sol. $\int\left(3t-\sqrt{3t}+\frac{1}{\sqrt{t}}\right)dt$

$$=\int\left(3t-\sqrt{3}t^{1/2}+t^{-1/2}\right)dt$$

$$=\frac{3t^{1+1}}{1+1}-\sqrt{3}\frac{t^{\frac{1}{2}+1}}{\frac{1}{2}+1}+\frac{t^{-\frac{1}{2}+1}}{-\frac{1}{2}+1}+c$$

$$=\frac{3t^2}{2}-\sqrt{3}\frac{t^{3/2}}{3/2}+\frac{t^{1/2}}{1/2}+c$$

$$=\boxed{\frac{3}{2}t^2-\frac{2}{\sqrt{3}}t^{3/2}+2\sqrt{t}+c}$$

3. $\int\left(\sec^2 x\,\text{cosec}^2 x\right)dx$

Sol. $\int\left(\sec^2 x\,\text{cosec}^2 x\right)dx$

$$=\int\sec^2 x\left(1+\cot^2 x\right)dx$$

$$=\int\left(\sec^2 x+\sec^2 x\cot^2 x\right)dx$$

$$=\int\left(\sec^2 x+\frac{1}{\cos^2 x}\cdot\frac{\cos^2 x}{\sin^2 x}\right)dx$$

$$=\int\left(\sec^2 x+\text{cosec}^2 x\right)dx$$

$$=\boxed{\tan x-\cot x+c}$$

4. Find $\int\left(\frac{\sin x}{\sqrt{1+\cos x}}\right)dx$

Sol. $\int\left(\frac{\sin x}{\sqrt{1+\cos x}}\right)dx$

$$=\int(1+\cos x)^{-1/2}\sin x\,dx$$

$$=-\int(1+\cos x)^{-1/2}(-\sin x)\,dx$$

$$=-\frac{(1+\cos x)^{1/2}}{1/2}+c$$

$$=\boxed{-2\sqrt{1+\cos x}+c}$$

5. Evaluate $\int\left(\frac{x}{x^2+1}\right)dx$

Sol. $\int\left(\frac{x}{x^2+1}\right)dx$

$$=\frac{1}{2}\int\frac{2x}{x^2+1}dx$$

$$=\boxed{\frac{1}{2}\ell n\left(x^2+1\right)+c}$$

6. Find $\int\left(e^{3x}+e^{5x}\right)dx$

Sol. $\int\left(e^{3x}+e^{5x}\right)dx$

$$=\boxed{\frac{e^{3x}}{3}+\frac{e^{5x}}{5}+c}$$ {Using formula # 05 from page # 282}

7. Integrate $\int\left(e^x\sin e^x\right)dx$

Sol. $\int\left(e^x\sin e^x\right)dx$

$$= \int \sin e^{x}\left(e^{x}\right)dx$$

$$\boxed{\begin{aligned}&\text{Put } e^{x}=t\\&\frac{d}{dx}\left(e^{x}\right)=\frac{d}{dx}(t)\\&e^{x}=\frac{dt}{dx}\\&e^{x}dx=dt\end{aligned}}$$

$$= \int (\sin t)\,dt$$

$$= -\cos t + c$$

$$= \boxed{-\cos e^{x} + c}$$

8. **Find** $\int \frac{\left(\tan^{-1}x\right)^{3}}{1+x^{2}}dx$

Sol. $\int \frac{\left(\tan^{-1}x\right)^{3}}{1+x^{2}}dx$

$$= \int \left(\tan^{-1}x\right)^{3}.\left(\frac{1}{1+x^{2}}\right)dx$$

$$\boxed{\begin{aligned}&\text{Put } \tan^{-1}x=t\\&\frac{d}{dx}\left(\tan^{-1}x\right)=\frac{d}{dx}(t)\\&\frac{1}{1+x^{2}}=\frac{dt}{dx}\\&\left(\frac{1}{1+x^{2}}\right)dx=dt\end{aligned}}$$

$$= \int t^{3}\,dt$$

$$= \frac{t^{4}}{4}+c$$

$$= \boxed{\frac{1}{4}\left(\tan^{-1}x\right)^{4}+c}$$

9. **Evaluate** $\int (x\ell nx)\,dx$

Sol. $\int (x\ell nx)\,dx$

$$= \int \ell nx.x\,dx$$

Integrating by parts:

taking $u=\ell nx$ & $v=x$

$$= \ell nx\int x\,dx - \int\left\{\frac{d}{dx}(\ell nx)\int x\,dx\right\}dx$$

$$= \ell nx.\frac{x^{2}}{2} - \int \frac{1}{x}.\frac{x^{2}}{2}dx$$

$$= \frac{x^{2}}{2}\ell nx - \frac{1}{2}\int x\,dx$$

$$= \frac{x^{2}}{2}\ell nx - \frac{1}{2}.\frac{x^{2}}{2}+c$$

$$= \boxed{\frac{x^{2}}{2}\ell nx - \frac{1}{4}x^{2}+c}$$

10. **Find** $\int \frac{1}{25+x^{2}}\,dx$

Sol. $\int \frac{1}{25+x^{2}}\,dx = \int \frac{1}{(5)^{2}+(x)^{2}}\,dx$

$$= \boxed{\frac{1}{5}\tan^{-1}\left(\frac{x}{5}\right)+c}$$ {Using formula # 17 from page # 282}

11. **Evaluate** $\int_{-3}^{-1}\frac{dx}{(x-1)^{2}}$

Sol. $\int_{-3}^{-1}\frac{dx}{(x-1)^{2}}$

$$= \int_{-3}^{-1}(x-1)^{-2}\,dx$$

$$= \left[\frac{(x-1)^{-1}}{-1}\right]_{-3}^{-1}$$ {using Rule-I}

$$= \left[\frac{1}{-(x-1)}\right]_{-3}^{-1}$$

$$= \frac{1}{-(-1-1)} - \frac{1}{-(-3-1)}$$

$$= \frac{1}{-(-2)} - \frac{1}{-(-4)}$$

$$= \frac{1}{2}-\frac{1}{4} = \frac{2-1}{4} = \boxed{1/4}$$

12. **Evaluate** $\int_0^{\pi/4}\left(1+\sec^2 x\right)dx$

Sol. $\int_0^{\pi/4}\left(1+\sec^2 x\right)dx$

$=\left[x+\tan x\right]_0^{\pi/4}$ {Using formula # 01 & 13 from page # 282}

$=\left[\frac{\pi}{4}+\tan\left(\frac{\pi}{4}\right)\right]-\left[0+\tan(0)\right]$

$=\left[\frac{\pi}{4}+\tan(45°)\right]-\left[0+\tan(0°)\right]$

$=\frac{\pi}{4}+1-0-0$ {using calculator tan45° = 1 & tan0° = 0}

$=\boxed{\frac{\pi+4}{4}}$

13. **Find area bounded by y = 3x, $y = x^2$ between x = 1 and x = 3.**

Sol. $\text{Area}=\int_a^b\left[f(x)-g(x)\right]dx$

$A=\int_1^3\left(3x-x^2\right)dx$

$A=\left[3\frac{x^2}{2}-\frac{x^3}{3}\right]_1^3$

$A=\left(\frac{3}{2}(3)^2-\frac{(3)^3}{3}\right)-\left(\frac{3}{2}(1)^2-\frac{(1)^3}{3}\right)$

$A=\left(\frac{27}{2}-9\right)-\left(\frac{3}{2}-\frac{1}{3}\right)$

$A=\left(\frac{27-18}{2}\right)-\left(\frac{9-2}{6}\right)$

$A=\frac{9}{2}-\frac{7}{6}=\frac{27-7}{6}$

$A=\frac{20}{6}=\boxed{\frac{10}{3}\text{ sq. unit}}$

14. **Evaluate** $\int_1^{\sqrt{3}}\frac{1}{1+x^2}dx$

Sol. $\int_1^{\sqrt{3}}\frac{1}{1+x^2}dx$

$=\left[\tan^{-1}x\right]_1^{\sqrt{3}}$

$=\tan^{-1}\left(\sqrt{3}\right)-\tan^{-1}(1)$

$=60°-45°$

$=\frac{\pi}{3}-\frac{\pi}{4}=\frac{4\pi-3\pi}{12}=\boxed{\frac{\pi}{12}}$

15. **Evaluate** $\int_0^2\left(\frac{x^3}{x+1}\right)dx$

Sol. $\int_0^2\left(\frac{x^3}{x+1}\right)dx$

$=\left[x^2-x+1-\frac{1}{x+1}\right]_0^2$

$=\left[\frac{x^3}{3}-\frac{x^2}{2}+x-\ell n(x+1)\right]_0^2$

$=\left[\frac{2^3}{3}-\frac{2^2}{2}+2-\ell n(2+1)\right]$

$-\left[\frac{0^3}{3}-\frac{0^2}{2}+0-\ell n(0+1)\right]$

$=\left(\frac{8}{3}-2+2-\ell n(3)\right)$

$-\left(0-0+0-\ell n(1)\right)$

$=\boxed{\frac{8}{3}-\ell n(3)}$

16. **Evaluate** $\int \text{cosec}^2x\,\sqrt{\cot x}\,dx$

Sol. $\int \text{cosec}^2x\,\sqrt{\cot x}\,dx$

$=-\int(\cot x)^{1/2}\left(-\text{cosec}^2x\right)dx$

$=-\frac{(\cot x)^{3/2}}{3/2}+c$ {using Rule-I}

$=\boxed{-\frac{2}{3}(\cot x)^{3/2}+c}$

17. **Evaluate** $\int \frac{\sin x}{a+b\cos x}\,dx$

Sol. $\int \frac{\sin x}{a+b\cos x}\,dx$

Put $a+b\cos x = t$

$\frac{d}{dx}(a+b\cos x)=\frac{d}{dx}(t)$

$0+b(-\sin x)=\frac{dt}{dx}$

$-b\sin x dx = dt$

$\sin x dx = -\frac{1}{b}dt$

$$= \int \frac{-\frac{1}{b}dt}{t} = -\frac{1}{b}\int \frac{1}{t}\,dt$$

$$= -\frac{1}{b}\,\ell n(t)+c$$

$$= \boxed{-\frac{1}{b}\,\ell n(a+b\cos x)+c}$$

18. **Evaluate** $\int \frac{\sqrt{1+\ell n x}}{x}\,dx$

Sol. $\int \frac{\sqrt{1+\ell n x}}{x}\,dx$

$$= \int \sqrt{1+\ell n x}\left(\frac{1}{x}\right)dx$$

$$= \frac{(1+\ell n x)^{3/2}}{3/2}+c$$

$$= \boxed{\frac{2}{3}(1+\ell n x)^{3/2}+c}$$

19. **Find the general solution of** $\mathbf{xdy = 3ydx}$

Sol. $xdy = 3ydx$

$$\frac{1}{y}dy = \frac{3}{x}dx$$

Integrating both sides, we have:

$$\int \frac{1}{y}dy = \int \frac{3}{x}dx$$

$$\ell n y = 3\ell n x + \ell n c$$

$$\ell n y = \ell n x^3 + \ell n c$$

$$\ell n y = \ell n\left(cx^3\right) \Rightarrow \boxed{y = cx^3}$$

20. **Write down the formula for extended rule of integration.**

Sol. $\int (fg)dx$

$$= fg_1 - f'g_2 + f''g_3 + \ldots + (-1)^n \int f^n g_n dx$$

21. **Find the order and degree of differential equation**

$$\left[\frac{d^2y}{dx^2}\right]^3 - \left[\frac{d^3y}{dx^3}\right]^2 = y$$

Sol. $\boxed{\text{Oder} = 3 \ \ \& \ \text{Degree} = 2}$

22. **Solve the differential equation**

$$\left(e^x + e^{-x}\right)\frac{dy}{dx} = \left(e^x - e^{-x}\right)$$

Sol. $\left(e^x + e^{-x}\right)\frac{dy}{dx} = \left(e^x - e^{-x}\right)$

$$dy = \left(\frac{e^x - e^{-x}}{e^x + e^{-x}}\right)dx$$

Integrating both sides, we have:

$$\int 1dy = \int \left(\frac{e^x - e^{-x}}{e^x + e^{-x}}\right)dx$$

$$\boxed{y = \ell n\left(e^x + e^{-x}\right) + c}$$

23. **If a function is even integrable on $[-\pi, \pi]$ then which co-efficient exist.**

Sol. a_0 and a_n exists and $b_n = 0$.

24. **Let $f(t) = 2\sin wt$. Find $L\{f(t)\}$**

Sol. $L\{f(t)\} = L\{2\sin wt\}$

$= 2L\{\sin wt\}$

$= 2\left(\frac{w}{s^2+w^2}\right) = \boxed{\frac{2w}{s^2+w^2}}$

25. **Find the Laplace transform of $3t+4$**

Sol. $L\{3t+4\}$

$= 3L\{t\} + 4L\{1\}$

$= 3\left(\frac{1}{s^2}\right) + 4\left(\frac{1}{s}\right) = \boxed{\frac{3}{s^2} + \frac{4}{s}}$

26. **What is the main use of Laplace transformation?**

Sol. The Laplace transform is used to solve differential equations.

27. ~~What is the inverse transformation of~~ $\frac{1}{s+a}$?

Sol. $L^{-1}\left\{\frac{1}{s+a}\right\}$

$= L^{-1}\left\{\frac{1}{s-(-a)}\right\} = \boxed{e^{-at}}$

Section - II

Note: Attemp any three (3) questions $3 \times 8 = 24$

Q.2.[a] Evaluate $\int (\sin^4 x)\, dx$

Sol. See example # 19 of Chapter 07.

[b] Evaluate $\int \left(\frac{1}{\sqrt{x+a}+\sqrt{x+b}}\right) dx$

Sol. See Q.15 of Ex# 7.1 (Page # 287)

Q.3.[a] ~~Evaluate~~ ~~$\int \frac{dx}{(x^2+a^2)^2}$.~~

Sol. See Q.1(ix) of Ex# 8.2 (Page # 333)

[b] ~~Evaluate $\int (e^{ax} \cos bx)\, dx$.~~

Sol. See Q.5(v) of Ex# 8.3 (Page # 357)

Q.4[a] Evaluate $\int_1^3 \frac{dx}{x^2-16}$

Sol. See Q.1(x) of Ex# 9.1 (Page # 378)

[b] ~~Find the area bounded by the curves: $y = x^3$ and $y = 4x^2$.~~

Sol. See Q.3 of Ex# 9.2 (Page # 388)

Q.5.[a] Find the general solution of $(x+1)\frac{dy}{dx} = x(y^2+1)$

Sol. See Q.11 of Ex# 10 (Page # 416)

[b] Show that $y = ce^{x^2}$ is the solution of differential equation $\frac{1}{x}\frac{dy}{dx} - 2y = 0$

Sol. See Q.9 of Ex# 10 (Page # 424)

Q.6[a] Find $L\{t^2 - 2t\}$

Sol. See Q.1(v) of Ex# 12 (Page # 465)

[b] ~~Find $L^{-1}\left\{\frac{1}{s(s^2+1)}\right\}$~~

Sol. See Q.6(v) of Ex# 12 (Page # 472)

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