DAE / IIA - 2020

MATH - 233 APPLIED MATHEMATICS - II

PAPER 'A' PART - A (OBJECTIVE)

Time : 30 Minutes Marks : 15

Q.1 : *Encircle the correct answer.*

1. Which one is the periodic function:

[a] x^2+1 [b] $2x$ [c] $\sin x$ [d] x^3+1

2. $\lim\limits_{x\to\infty}\left(1+\frac{2}{x}\right)^x = ?$ [a] e [b] e^{-1} [c] e^2 [d] 0

3. $\lim\limits_{x\to\frac{\pi}{3}}(\cos x) = ?$

[a] $\frac{\sqrt{3}}{2}$ [b] $\frac{1}{2}$ [c] 0 [d] $\frac{1}{\sqrt{2}}$

4. $\frac{d}{dx}\left(\frac{1}{x}\right) = ?$

[a] $\frac{1}{x^2}$ [b] $-\frac{1}{x^2}$ [c] $-\frac{1}{x^3}$ [d] $\frac{2}{x}$

5. $\frac{d}{dy}\left(\sqrt{1+y}\right) = ?$

[a] $\frac{1}{\sqrt{1+y}}$ [b] $\frac{1}{2\sqrt{1+y}}$

[c] $(1+y)^{1/3}$ [d] $\frac{-1}{2\sqrt{1+y}}$

6. mx^{m-1} is the differential w.r.t. x of:

[a] $m(m-1)x^{m-2}$ [b] $(m-1)x^{m-2}$

[c] x^m [d] mx^m

7. $\frac{d}{dx}(\sin ax+\cos ax) = ?$

[a] $a\cos x + a\sin x$

[b] $a\cos x - a\sin x$

[c] $\cos ax + \sin ax$

[d] $a\cos ax - \sin ax$

8. $\frac{d}{dx}\left(\sec^{-1}2x\right) = ?$

[a] $\frac{1}{x\sqrt{4x^2-1}}$ [b] $\frac{1}{x\sqrt{4x^2-1}}$

[c] $\frac{1}{x\sqrt{x^2-1}}$ [d] $\frac{1}{2x\sqrt{x^2-1}}$

9. $\frac{d}{dx}\left(e^{\sin x}\right) = ?$

[a] $e^{\cos x}$ [b] $\cos x e^{\sin x}$

[c] $\sin x e^{\sin x-1}$ [d] $\sin x e^{\sin x}$

10. $\frac{d}{dx}\left[\ell n\left(x^2+1\right)\right] = ?$

[a] $\frac{x}{x^2+1}$ [b] $\frac{2x}{x^2+1}$

[c] $\ell n(2x+1)$ [d] $2x\,\ell n(2x+1)$

11. If 2nd derivative is +ve at a point, then function is:

[a] Maximum [b] Minimum

[c] Point of inflection [d] None of these

12. If $\frac{dy}{dx}$ does not change sign before and after a point where it vanished then that is point of:

[a] Maxima [b] Minima

[c] Inflection [d] None of these

13. The result obtained from an experiment or a trial is called:

[a] Sample space [b] An event

[c] Out come [d] Population

14. If a fair coin is tossed, what is the probability of getting head is:

[a] $\frac{1}{3}$ [b] zero [c] $\frac{1}{2}$ [d] 1

15. A set of data is called:

[a] Continuous data

[b] Discontinuous

[c] Population [d] Sample

Answer Key

1	c	2	c	3	c	4	b	5	b
6	b	7	d	8	a	9	d	10	a
11	a	12	a	13	d	14	c	15	c

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DAE / IIA - 2020

MATH - 233 APPLIED MATHEMATICS - II

PAPER 'B' PART - B (SUBJECTIVE)

Time : 2 : 30 Hrs Marks : 60

Section - I

Q.1. Write short answers to any Eighteen (18) questions.

1. If $f(x) = 2x\sqrt{1-x^2}$, then find $f(\sin\theta)$

Sol. As, $f(x) = 2x\sqrt{1-x^2}$

Put $x = \sin\theta$, we have :

$$f(\sin\theta) = 2\sin\theta\sqrt{1-\sin^2\theta}$$

$$= 2\sin\theta\sqrt{\cos^2\theta}$$

$$= 2\sin\theta\cos\theta = \boxed{\sin 2\theta}$$

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2. Find the value of

$$\lim_{x\to 2}\frac{x^3-8}{x^2-3x+2}$$

Sol. $\lim\limits_{x\to 2}\dfrac{x^3-8}{x^2-3x+2}\left(\dfrac{0}{0}\right)$ form

$$= \lim_{x\to 2}\frac{(x)^3-(2)^3}{x^2-2x-x+2}$$

$$= \lim_{x\to 2}\frac{(x-2)(x^2+2x+4)}{x(x-2)-1(x-2)}$$

$$= \lim_{x\to 2}\frac{(x-2)(x^2+2x+4)}{(x-2)(x-1)}$$

$$= \lim_{x\to 2}\frac{(x^2+2x+4)}{(x-1)}$$

$$= \frac{(2)^2+2(2)+4}{2-1}$$

$$= \frac{4+4+4}{1} = \boxed{12}$$

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3. ~~Find the value of~~ ~~$\lim\limits_{x\to 0}\left(1+\dfrac{x}{3}\right)^{1/x}$~~.

Sol. $\lim\limits_{x\to 0}\left(1+\dfrac{x}{3}\right)^{1/x}$

$$= \lim_{x\to 0}\left(1+\frac{x}{3}\right)^{\frac{3}{x}\times\frac{1}{3}}$$

$$= \left[\lim_{x\to 0}\left(1+\frac{x}{3}\right)^{3/x}\right]^{1/3} = \boxed{e^{1/3}}$$

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4. ~~Evaluate:~~ ~~$\lim\limits_{x\to\frac{\pi}{2}}\dfrac{\sec x}{\tan x}$~~

Sol. $\lim\limits_{x\to\frac{\pi}{2}}\dfrac{\sec x}{\tan x}\left(\dfrac{\infty}{\infty}\right)$ form

$$= \lim_{x\to\frac{\pi}{2}}\left(\frac{1}{\cos x}\times\frac{\cos x}{\sin x}\right)$$

$$= \lim_{x\to\frac{\pi}{2}}\left(\frac{1}{\sin x}\right)$$

$$= \frac{1}{\sin\frac{\pi}{2}} = \frac{1}{\sin 90^\circ} = \frac{1}{1} = \boxed{1}$$

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5. Differentiate $\left(ax^p+bx^q\right)^{p/q}$ w.r.t. 'x'.

Sol. $\dfrac{d}{dx}\left[\left(ax^p+bx^q\right)^{p/q}\right]$

$$= \frac{p}{q}\left(ax^p+bx^q\right)^{\frac{p}{q}-1}\left[\frac{d}{dx}\left(ax^p+bx^q\right)\right]$$

$$= \boxed{\frac{p}{q}\left(ax^p+bx^q\right)^{\frac{p}{q}-1}\left(apx^{p-1}+bqx^{q-1}\right)}$$

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6. Differentiate $\dfrac{x}{x^2+1}$ w.r.t. 'x'.

Sol. Differentiate w.r.t. 'x' :

$\dfrac{d}{dx}\left(\dfrac{x}{x^2+1}\right)$ {using Quotient Rule}

$$= \frac{(x^2+1)\left(\frac{d}{dx}(x)\right) - x\left(\frac{d}{dx}(x^2+1)\right)}{(x^2+1)^2}$$

$$= \frac{(x^2+1)(1) - x(2x+0)}{(x^2+1)^2}$$

$$= \frac{x^2+1-2x^2}{(x^2+1)^2} = \boxed{\frac{1-x^2}{(x^2+1)^2}}$$

7. **Find $\frac{dy}{dx}$ If $xy + y^2 = 2$.**

Sol. $xy + y^2 = 2$

Differentiate both sides w.r.t. 'x':

$$\frac{d}{dx}(xy + y^2) = \frac{d}{dx}(2)$$

$$\left(\frac{d}{dx}(x)\right).y + x\left(\frac{d}{dx}(y)\right) + 2y\frac{d}{dx}(y) = 0$$

$$1.y + x\frac{dy}{dx} + 2y\frac{dy}{dx} = 0$$

$$\frac{dy}{dx}(x+2y) = -y \Rightarrow \boxed{\frac{dy}{dx} = -\frac{y}{x+2y}}$$

8. **If $y = \frac{1}{(x-3)(x+2)}$, find $\frac{dy}{dx}$**

Sol. $y = \frac{1}{(x-3)(x+2)}$

Differentiate both sides w.r.t. 'x':

$$\frac{d}{dx}(y) = \frac{d}{dx}\left(\frac{1}{(x-3)(x+2)}\right) \left\{\text{using Quotient Rule}\right\}$$

$$\frac{dy}{dx} = \frac{(x-3)(x+2)\left(\frac{d}{dx}(1)\right) - (1)\left(\frac{d}{dx}((x-3)(x+2))\right)}{((x-3)(x+2))^2}$$

$$\frac{dy}{dx} = \frac{(x-3)(x+2)(0) - \left[\left(\frac{d}{dx}(x-3)\right)(x+2) + (x-3)\left(\frac{d}{dx}(x+2)\right)\right]}{[(x-3)(x+2)]^2}$$

$$\frac{dy}{dx} = \frac{0 - [(1-0)(x+2) + (x-3)(1+0)]}{[(x-3)(x+2)]^2}$$

$$\frac{dy}{dx} = \frac{-(x+2+x-3)}{[(x-3)(x+2)]^2}$$

$$\frac{dy}{dx} = \frac{-(2x-1)}{[(x-3)(x+2)]^2}$$

$$\boxed{\frac{dy}{dx} = \frac{1-2x}{[(x-3)(x+2)]^2}}$$

9. **Differentiate $\sin\sqrt{x} + \sqrt{\sin x}$ w.r.t. 'x'.**

Sol. $\frac{d}{dx}\left(\sin\sqrt{x} + \sqrt{\sin x}\right)$

$$= \cos\sqrt{x}\frac{d}{dx}\left(\sqrt{x}\right) + \frac{1}{2}(\sin x)^{\frac{1}{2}-1}\frac{d}{dx}(\sin x)$$

$$= \cos\sqrt{x}.\frac{1}{2}(x)^{-1/2} + \frac{1}{2}(\sin x)^{-1/2}\cos x$$

$$= \boxed{\frac{\cos\sqrt{x}}{2\sqrt{x}} + \frac{\cos x}{2\sqrt{\sin x}}}$$

10. **Find the derivative of $x^2 \tan x$.**

Sol. $\frac{d}{dx}(x^2\tan x)$ {using Product Rule}

$$= \left(\frac{d}{dx}(x^2)\right)\tan x + x^2\left(\frac{d}{dx}(\tan x)\right)$$

$$= \boxed{2x\tan x + x^2\sec^2 x}$$

11. ~~Differentiate $\sec^{-1}(\sin x)$ w.r.t. 'x'.~~

Sol. $\frac{d}{dx}\left(\sec^{-1}(\sin x)\right)$

$$= \frac{1}{\sin x\sqrt{\sin^2 x - 1}} \cdot \frac{d}{dx}(\sin x)$$

$$= \frac{1}{\sin x\sqrt{\sin^2 x - 1}}(\cos x)$$

$$= \boxed{\frac{\cot x}{\sqrt{\sin^2 x - 1}}}$$

12. **Find $\frac{dy}{dx}$ If $y = \cos(\ell n\, x)$.**

Sol. $y = \cos(\ell n\, x)$

Differentiate both sides w.r.t. 'x':

$$\frac{d}{dx}(y) = \frac{d}{dx}\left(\cos(\ell n\, x)\right)$$

$$\frac{dy}{dx} = -\sin(\ell n\, x)\left(\frac{d}{dx}(\ell n\, x)\right)$$

$$\frac{dy}{dx} = -\sin(\ell n\, x).\frac{1}{x}$$

$$\boxed{\frac{dy}{dx} = -\frac{\sin(\ell n\, x)}{x}}$$

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13. ~~Find $\frac{d}{dx}\left(e^{2x}\cos 2x\right)$~~

Sol. $\frac{d}{dx}\left(e^{2x}\cos 2x\right)$

$$= \left(\frac{d}{dx}\left(e^{2x}\right)\right)\cos 2x + e^{2x}\left(\frac{d}{dx}(\cos 2x)\right)$$

$$= e^{2x}\left(\frac{d}{dx}(2x)\right)\cos 2x + e^{2x}(-\sin 2x)\left(\frac{d}{dx}(2x)\right)$$

$$= e^{2x}(2)\cos 2x - e^{2x}\sin 2x(2)$$

$$= \boxed{2e^{2x}(\cos 2x - \sin 2x)}$$

---

14. **Is the following function even, odd or neither $f(x) = x\sqrt{x^2-1}$**

Sol. As, $f(x) = x\sqrt{x^2-1}$

Replace x by $-x$, we have :

$$f(-x) = -x\sqrt{(-x)^2-1}$$

$$f(-x) = -x\sqrt{x^2-1}$$

$$f(-x) = -f(x)$$

Hence $f(x)$ is an **odd** function.

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15. **If $y = \ell n\, x$, find y_2**

Sol. $y = \ell n\, x$

Differentiate both sides w.r.t. 'x':

$$\frac{d}{dx}(y) = \frac{d}{dx}(\ell n\, x)$$

$$y_1 = \frac{1}{x}$$

Differentiate both sides w.r.t. 'x':

$$\frac{d}{dx}(y_1) = \frac{d}{dx}\left(\frac{1}{x}\right)$$

$$y_2 = \frac{d}{dx}\left(x^{-1}\right) = -1(x)^{-2} = \boxed{\frac{-1}{x^2}}$$

---

16. **Find the slope of tangent to the curve $y = \cos^2 x$ at $x = \frac{\pi}{4}$.**

Sol. $y = \cos^2 x$

Differentiate both sides w.r.t. 'x':

	At $x = \pi/4$
$\frac{d}{dx}(y) = \frac{d}{dx}(\cos^2 x)$	
$\frac{dy}{dx} = 2\cos x(-\sin x)$	$\frac{dy}{dx} = -\sin 2\left(\pi/4\right)$
$\frac{dy}{dx} = -2\sin x\cos x$	$\frac{dy}{dx} = -\sin\left(\pi/2\right)$
$\frac{dy}{dx} = -\sin 2x$	$\frac{dy}{dx} = \boxed{-1}$

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17. **Find the critical values for 'x' of the function:**

$y = x^3 - 3x^2 - 24x + 10$

Sol. Let $y = x^3 - 3x^2 - 24x + 10$

Differentiate both sides w.r.t. 'x':

$$\frac{d}{dx}(y) = \frac{d}{dx}\left(x^3 - 3x^2 - 24x + 10\right)$$

$$\frac{dy}{dx} = 3x^2 - 3(2x) - 24(1) + 0$$

$$\frac{dy}{dx} = 3x^2 - 6x - 24$$

For critical values, put $\frac{dy}{dx} = 0$

$3x^2 - 6x - 24 = 0$

Dividing each on 3, we get:

$\frac{3x^2}{3} - \frac{6x}{3} - \frac{24}{3} = \frac{0}{3}$

$x^2 - 2x - 8 = 0$

$x^2 - 4x + 2x - 8 = 0$

$x(x-4) + 2(x-4) = 0$

$(x-4)(x+2) = 0$

$x - 4 = 0 \quad | \quad x + 2 = 0$

$\boxed{x = 4} \quad | \quad \boxed{x = -2}$

18. If $y = \sqrt{\sin x + \sqrt{\sin x + \sqrt{\sin x + \ldots \infty}}}$ prove that: $(2y-1)\frac{dy}{dx} = \cos x$

Sol.

As, $y = \sqrt{\sin x + \sqrt{\sin x + \sqrt{\sin x + \ldots \infty}}}$

Squaring both sides:

$y^2 = \left[\sqrt{\sin x + \sqrt{\sin x + \sqrt{\sin x + \ldots \infty}}}\right]^2$

$y^2 = \sin x + \sqrt{\sin x + \sqrt{\sin x + \ldots \infty}}$

$y^2 = \sin x + y \left\{ \because \text{Given that } \sqrt{\sin x + \sqrt{\sin x + \sqrt{\sin x + \ldots \infty}}} \right\}$

$y^2 - y = \sin x$

Differentiate both sides w.r.t. 'x':

$\frac{d}{dx}(y^2 - y) = \frac{d}{dx}(\sin x)$

$2y\frac{dy}{dx} - \frac{dy}{dx} = \cos x$

$\boxed{(2y-1)\frac{dy}{dx} = \cos x}$ **Proved.**

19. If displacement $s = \sin 2t$, find the velocity at $t = \frac{\pi}{6}$.

Sol. $s = \sin 2t$

Differentiate both sides w.r.t. 't':

$v = \frac{ds}{dt} = \frac{d}{dt}(\sin 2t)$ | At $t = \frac{\pi}{6}$

$v = \cos 2t\left(\frac{d}{dt}(2t)\right)$ | $v = 2\cos 2\left(\frac{\pi}{6}\right)$

$v = \cos 2t(2)$ | $v = 2\left(\frac{1}{2}\right) = \boxed{1}$

$v = 2\cos 2t$

20. **Define decreasing function.**

Sol. A function $f(x)$ is said to be Decreasing function of 'x' if as x increases $f(x)$ decrease or if as x decrease $f(x)$ increase.

21. ~~Find the standard deviation from the values~~ **2, 3, 5, 8, 11.**

Sol. 2, 3, 5, 8, 11.

x	x^2
2	4
3	9
5	25
8	64
11	121
$\Sigma x = 29$	$\Sigma x^2 = 223$

$S.D. = \sqrt{\frac{\Sigma x^2}{n} - \left(\frac{\Sigma x}{n}\right)^2}$

$\sigma = \sqrt{\left(\frac{223}{5}\right) - \left(\frac{29}{5}\right)^2}$

$\sigma = \sqrt{44.6 - 33.64}$

$\sigma = \sqrt{10.96} = \boxed{3.31}$

22. **If a die is rolled once, what is the probability of getting an odd number?**

Sol. $S=\{1, 2, 3, 4, 5, 6\}$, $n(S)=6$

Let A be event that odd number appear.

$A=\{1, 3, 5\}$, $n(A)=3$

$$P(A)=\frac{n(A)}{n(S)}$$

$$P(A)=\frac{3}{6}=\boxed{\frac{1}{2}}$$

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23. **Find the median of 4, 3, 5, 2, 11.**

Sol. Write the value in ascending order

2, 3, 4, 5, 11

Here n = 5

$$\text{Median}=\left(\frac{n+1}{2}\right)\text{th value}$$

$$=\left(\frac{5+1}{2}\right)\text{th}=3\text{th value}$$

$\therefore$ The median is the third value

which is $=\boxed{4}$

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24. **Find mean of the data.**

x	1	3	5	7	9
f	2	7	11	5	4

Sol.

x	f	fx
1	2	2
3	7	21
5	11	55
7	5	35
9	4	36
	$\Sigma f=30$	$\Sigma fx=156$

$$\text{A.M.}=\frac{\Sigma fx}{\Sigma f}=\frac{156}{30}=\boxed{5.2}$$

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25. **Define sample space and give example.**

Sol. A set of all possible outcomes of an experiment is called sample space. It is denoted by S. For example when a die is rolled then $S=\{1, 2, 3, 4, 5, 6\}$

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26. **Find $\frac{dy}{dx}$ at the given point if:**

$y=x+2x^{-1}$ **at** $x=2$

Sol. $y=x+2x^{-1}$

$$\frac{d}{dx}(y)=\frac{d}{dx}\left(x+2x^{-1}\right)$$

$$\frac{dy}{dx}=1+2(-1)x^{-2}$$

$$\frac{dy}{dx}=1-\frac{2}{x^2}$$

$$\frac{dy}{dx}=\frac{x^2-2}{x^2}$$

At $x=2$

$$\frac{dy}{dx}=\frac{(2)^2-2}{(2)^2}=\frac{4-2}{4}=\frac{2}{4}=\boxed{\frac{1}{2}}$$

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27. ~~**Find** $\frac{d}{dx}\left(a^{x^2}\right)$~~

Sol. $\frac{d}{dx}\left(a^{x^2}\right)$

$$=a^{x^2}\,(\ell n\ a)\left(\frac{d}{dx}\left(x^2\right)\right)$$

$$=a^{x^2}\,(\ell n\, a)\,(2x)$$

$$=\boxed{2x(\ell n\, a)a^{x^2}}$$

Section - II

Note : Attemp any three (3) questions $3 \times 8 = 24$

Q.2.(a) Show that $x.\dfrac{a^x+1}{a^x-1}$ is an Even function of x.

Sol. See $Q.12(ii)$ of $Ex\#1.1$ $(Page\ \#10)$

(b) ~~Evaluate~~: $\lim\limits_{x\to 0}\dfrac{\tan x-\sin x}{x^3}$

Sol. See $Q.1(xi)$ of $Ex\#1.3$ $(Page\ \#28)$

Q.3.(a) Find the derivative w.r.t. 'x'

$$\frac{\sqrt{x}}{\sqrt{x+1}}$$

Sol. See $Q.4(vii)$ of $Ex\#2.2$ $(Page\ \#59)$

(b) ~~Differentiate~~

$$\frac{x^2+a^2}{x^2-a^2} \text{ w.r.t. } \frac{x-a}{x+a}$$

Sol. See $Q.1(vii)$ of $Ex\#2.4$ $(Page\ \#88)$

Q.4(a) If $xy=\cos(x+y)$, show that $\dfrac{dy}{dx}+\dfrac{y+\sin(x+y)}{x+\sin(x+y)}=0$

Sol. See $Q.5[a]$ of $Ex\#3.1$ $(Page\ \#120)$

(b) If $y=x\tan^{-1}\left(\dfrac{x}{y}\right)$, then prove that: $\dfrac{dy}{dx}=\dfrac{y}{x}$

Sol. See $Q.4(i)$ of $Ex\#3.2$ $(Page\ \#132)$

Q.5(a) ~~Find the derivative w.r.t. 'x'~~

$$(x)^{\sin x}.$$

Sol. See $Q.3(iii)$ of $Ex\#3.3$ $(Page\ \#150)$

(b) Find the relative maxima and minima of function

$$y=\frac{1}{4}x^4-\frac{3}{2}x^2$$

Sol. See $Q.3(iii)$ of $Ex\#4.2$ $(Page\ \#200)$

Q.6(a) Calculate Arithmetic Mean and Mode.

Class Interval	Frequency
1 – 3	12
4 – 6	5
7 – 9	20
10 – 12	22
13 – 15	14
16 – 18	17

Sol. See $Q.8$ of $Ex\#5.1$ $(Page\ \#235)$

(b) If two dice are rolled. What is the probability that:

(i) A sum greater then 9 appears.

(ii) A sum divisible by 5 appears.

Sol. See $Q.5$ of $Ex\#6.1$ $(Page\ \#258)$

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