DAE / IIA - 2020

MATH - 212 APPLIED MATHEMATICS - II

PART - A (OBJECTIVE)

Time : 30 Minutes   Marks : 20

Q.1 : *Encircle the correct answer.*

1. If $f(x)=3^x-1$, then $f(3)=?$

[a] 27 [b] 8
[c] 26 [d] 16

2. $\lim_{\theta\to\frac{\pi}{2}}\frac{\sin\theta}{\theta}=?$

[a] 1 [b] $\frac{\pi}{2}$
[c] $\frac{2}{\pi}$ [d] $\frac{1}{2}$

3. mx^{m-1} is the differential w.r.t. x of:

[a] $m(m-1)x^{m-2}$
[b] $(m-1)x^{m-2}$
[c] x^m [d] mx^m

4. If $u=t^2-3$, then $\frac{du}{dt}=?$:

[a] $2t$ [b] $2t-3$
[c] t^{-2} [d] $2t^{-2}$

5. $\frac{d}{dx}(\sec 3x)=?$

[a] $3\sec 3x\tan 3x$
[b] $\sec 3x\tan 3x$
[c] $3\sec 3x\cot 3x$
[d] $-3\sec 3x\tan 3x$

6. ~~$\frac{d}{dx}(\tan^{-1}x^2)=?$~~

[a] $\frac{1}{1+x^2}$ [b] $\frac{1}{1-x^2}$
[c] $\frac{2x}{1+x^4}$ [d] $\frac{2x}{1-x^4}$

7. ~~$\frac{d}{dx}(e^{3x})=$~~

[a] e^{3x-1} [b] e^{x-1}
[c] $3e^{3x}$ [d] $3xe^{3x}$

8. A function is maximum at a point if its 2^{nd} derivative is:

[a] +ve [b] −ve
[c] zero [d] None of these

9. $\int(x^{n+1})dx=?$

[a] $\frac{x^{n+2}}{n+2}$ [b] $\frac{x^{n+1}}{n+2}$
[c] $(n+1)x^n$ [d] $\frac{x^2}{n}$

10. $\int(\tan x\sec^2 x)dx=?$

[a] $\ell n\tan x$ [b] $\frac{\tan^2 x}{2}$
[c] $\frac{\sec^2 x}{3}$ [d] $\sec x\tan x$

11. ~~$\int\left(\frac{1}{\sqrt{1-x^2}}\right)dx=?$~~

[a] $\sin^{-1}x$ [b] $\cos^{-1}x$
[c] $\sec^{-1}x$ [d] $\tan^{-1}x$

12. $\int\left(\frac{\sec x\tan x}{3+\sec x}\right)dx=?$

[a] $\sec x+\tan x$
[b] $3+\sec x$
[c] $\ell n(3+\sec x)$
[d] $\ell n(\sec x+\tan x)$

13. $\int_0^3\left(\frac{2x}{x^2+1}\right)dx=?$

[a] $\ell n2$ [b] $\ell n10-\ell n6$
[c] $\ell n(x^2+1)$ [d] $\ell n10+\ell n6$

14. $\int_0^{\pi/4}\left(\sec^2 x\right)dx = ?$

[a] 1 [b] 2

[c] 0 [d] 3

15. Ratio formula for y-coordinate is:

[a] $\frac{x_1r_2 + x_2r_1}{r_1 + r_2}$ [b] $\frac{y_1r_2 + y_2r_1}{r_1 + r_2}$

[c] $\frac{x - y}{2}$ [d] $\frac{y_1r_2 - y_2r_1}{r_1 + r_2}$

16. When two lines are parallel:

[a] $m_1 = m_2$ [b] $m_1m_2 = -1$

[c] $m_1m_2 = 1$ [d] $m_1 = -m_2$

17. Slope of the line $\frac{x}{a} + \frac{y}{b} = 1$ is:

[a] $\frac{a}{b}$ [b] $\frac{b}{a}$

[c] $-\frac{b}{a}$ [d] $-\frac{a}{b}$

18. y – intercept of the line $3x + 4y - 12 = 0$:

[a] −4 [b] 3

[c] 4 [d] −3

19. Straight line from center to the circumference is:

[a] Circle [b] Radius

[c] Diameter [d] Circumference

20. Radius of the circle $x^2 + y^2 = 1$ is:

[a] 1 [b] 0

[c] 2 [d] −1

Answer Key

1	c	2	c	3	c	4	a	5	a
6	c	7	c	8	b	9	a	10	b
11	a	12	c	13	a	14	d	15	b
16	a	17	c	18	d	19	b	20	a

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DAE / IIA - 2020

MATH - 212 APPLIED MATHEMATICS - II

PART - B (SUBJECTIVE)

Time : 2 : 30 Hrs Marks : 60

Section - I

Q.1: Write short answers to any Twenty Five (25) of the follwing questions. **25 × 2 = 50**

1. If $f(x) = 2x\sqrt{1 - x^2}$, find $f(\sin\theta)$

Sol. As, $f(x) = 2x\sqrt{1 - x^2}$

Put $x = \sin\theta$, we have:

$$f(\sin\theta) = 2\sin\theta\sqrt{1 - \sin^2\theta}$$

$$= 2\sin\theta\sqrt{\cos^2\theta}$$

$$= 2\sin\theta\cos\theta = \boxed{\sin 2\theta}$$

2. Find $\lim_{x\to 1}\frac{x^3 - 1}{x - 1}$

Sol. $\lim_{x\to 1}\frac{x^3 - 1}{x - 1}$ $\left(\frac{0}{0}\right)$ Form

$$= \lim_{x\to 1}\frac{(x)^3 - (1)^3}{x - 1}$$

$$= \lim_{x\to 1}\frac{\cancel{(x-1)}\left(x^2 + x + 1\right)}{\cancel{(x-1)}}$$

$$= \lim_{x\to 1}\left(x^2 + x + 1\right) = (1)^2 + 1 + 1 = \boxed{3}$$

3. Is the function $f(x) = \frac{x}{x^2 + 1}$ even, odd or neither?

Sol. As, $f(x) = \frac{x}{x^2 + 1}$

Replace x by −x, we have:

$$f(-x) = \frac{-x}{(-x)^2 + 1}$$

$$f(-x) = -\frac{x}{x^2 + 1}$$

$$f(-x) = -f(x)$$

Hence $f(x)$ is an $\boxed{\text{odd}}$ function.

4. **Evaluate** $\lim\limits_{x\to\infty}\left(1+\frac{2}{x}\right)^x$

Sol. $\lim\limits_{x\to\infty}\left(1+\frac{2}{x}\right)^x$

$$=\lim_{x\to\infty}\left(1+\frac{2}{x}\right)^{\frac{x}{2}\times 2}$$

$$=\left(\lim_{x\to\infty}\left(1+\frac{2}{x}\right)^{\frac{x}{2}}\right)^2=\boxed{e^2}$$

5. **If** $y=5x^3-7x^2+9-\frac{8}{x}+\frac{7}{x^4}$**, find** $\frac{dy}{dx}$

Sol. $y=5x^3-7x^2+9-\frac{8}{x}+\frac{7}{x^4}$

Differentiate both sides w.r.t. 'x':

$$\frac{d}{dx}(y)=\frac{d}{dx}\left(5x^3-7x^2+9-\frac{8}{x}+\frac{7}{x^4}\right)$$

$$\frac{dy}{dx}=\frac{d}{dx}\left(5x^3-7x^2+9-8x^{-1}+7x^{-4}\right)$$

$$\frac{dy}{dx}=5\left(3x^2\right)-7\left(2x\right)+0-8\left(-1\right)x^{-2}+7\left(-4\right)x^{-5}$$

$$\boxed{\frac{dy}{dx}=15x^2-14x+\frac{8}{x^2}-\frac{28}{x^5}}$$

6. **If** $ax^2+by^2+2hxy=0$**, find** $\frac{dy}{dx}$

Sol. As, $ax^2+by^2+2hxy=0$

Differentiate both sides w.r.t. 'x':

$$\frac{d}{dx}\left(ax^2+by^2+2hxy\right)=\frac{d}{dx}(0)$$

$$a(2x)+b(2y)\frac{dy}{dx}+2h\left[\left(\frac{d}{dx}(x)\right)y+x\left(\frac{d}{dx}(y)\right)\right]=0$$

$$2ax+2by\frac{dy}{dx}+2h\left(1.y+x\frac{dy}{dx}\right)=0$$

$$2ax+2by\frac{dy}{dx}+2hy+2hx\frac{dy}{dx}=0$$

$$2by\frac{dy}{dx}+2hx\frac{dy}{dx}=-2ax-2hy$$

$$2\frac{dy}{dx}(by+2h)=-2(ax+hy)$$

$$\frac{dy}{dx}=\frac{-2(ax+hy)}{2(by+2h)}$$

$$\boxed{\frac{dy}{dx}=-\frac{(ax+hy)}{(by+hx)}}$$

7. **If** $y=\frac{1+x}{1-x}$**, find** $\frac{dy}{dx}$

Sol. $y=\frac{1+x}{1-x}$

Differentiate both sides w.r.t. 'x':

$$\frac{d}{dx}(y)=\frac{d}{dx}\left(\frac{1+x}{1-x}\right)\left\{\begin{array}{c}\text{By using}\\\text{Quotient Rule}\end{array}\right\}$$

$$\frac{dy}{dx}=\frac{(1-x)\left(\frac{d}{dx}(1+x)\right)-(1+x)\left(\frac{d}{dx}(1-x)\right)}{(1-x)^2}$$

$$\frac{dy}{dx}=\frac{(1-x)(0+1)-(1+x)(0-1)}{(1-x)^2}$$

$$\frac{dy}{dx}=\frac{1-x+1+x}{(1-x)^2}\Rightarrow\boxed{\frac{dy}{dx}=\frac{2}{(1-x)^2}}$$

8. **Differentiate**

$2x^2+x+1$ **w.r.t.** x^2-x-1

Sol. Let, $y=2x^2+x+1$ & $t=x^2-x-1$

Differentiate both sides w.r.t. 'x':

$$\frac{d}{dx}(y)=\frac{d}{dx}\left(2x^2+x+1\right) \quad\Big|\quad \frac{d}{dx}(t)=\frac{d}{dx}\left(x^2-x-1\right)$$

$$\frac{dy}{dx}=2(2x)+1+0 \quad\Big|\quad \frac{dt}{dx}=2x-1-0$$

$$\frac{dy}{dx}=4x+1 \quad\Big|\quad \frac{dt}{dx}=2x-1$$

$$\frac{dx}{dt}=\frac{1}{2x-1}$$

using chain rule: $\frac{dy}{dt}=\frac{dy}{dx}\times\frac{dx}{dt}$

$$\frac{dy}{dt}=(4x+1)\left(\frac{1}{2x-1}\right)=\boxed{\frac{4x+1}{2x-1}}$$

9. **Find the value of** $\frac{d}{dx}\left(\frac{1-\cos x}{\sin x}\right)$

Sol. $\frac{d}{dx}\left(\frac{1-\cos x}{\sin x}\right)$ {By using Product Rule}

$$=\frac{\sin x\left(\frac{d}{dx}(1-\cos x)\right)-(1-\cos x)\left(\frac{d}{dx}(\sin x)\right)}{\sin^2 x}$$

$$=\frac{\sin x(0-(-\sin x))-(1-\cos x)(\cos x)}{\sin^2 x}$$

$$=\frac{\sin^2 x-\cos x+\cos^2 x}{\sin^2 x}$$

$$=\frac{1-\cos x}{\sin^2 x} \quad \because \{\sin^2 x+\cos^2 x=1\}$$

$$=\frac{2\sin^2\frac{x}{2}}{\left(2\sin\frac{x}{2}\cos\frac{x}{2}\right)^2} \because \left\{\sin x = 2\sin\frac{x}{2}\cos\frac{x}{2}\right\}$$

$$=\frac{2\sin^2\frac{x}{2}}{4\sin^2\frac{x}{2}\cos^2\frac{x}{2}}=\frac{1}{2\cos^2\frac{x}{2}}=\boxed{\frac{1}{2}\sec^2\frac{x}{2}}$$

10. Find the derivative of $\left(\sec^{-1}x\right)^3$

Sol. $\frac{d}{dx}\left(\sec^{-1}x\right)^3$

$$=3\left(\sec^{-1}x\right)^2\left(\frac{d}{dx}\left(\sec^{-1}x\right)\right)$$

$$=3\left(\sec^{-1}x\right)^2\frac{1}{x\sqrt{x^2-1}}$$

$$=\boxed{\frac{3\left(\sec^{-1}x\right)^2}{x\sqrt{x^2-1}}}$$

11. ~~If $x=\sin 2t,\ y=2\cos t,$ then find $\frac{dy}{dx}$.~~

Sol. As, $x=\sin 2t$ & $y=2\cos t$

Differentiate both equations both sides w.r.t. 't':

$$\frac{d}{dt}(t)=\frac{d}{dt}(\sin 2t)$$

$$\frac{dt}{dx}=\cos 2t\frac{d}{dt}(2t)$$

$$\frac{dt}{dx}=\cos 2t(2)$$

$$\frac{dx}{dt}=\frac{1}{2\cos 2t}$$

$$\frac{d}{dt}(y)=\frac{d}{dt}(2\cos t)$$

$$\frac{dy}{dt}=2(-\sin t)$$

$$\frac{dy}{dt}=-2\sin t$$

By using Chain's Rule: $\frac{dy}{dx}=\frac{dy}{dt}\times\frac{dt}{dx}$

$$\frac{dy}{dx}=(-2\sin t)\left(\frac{1}{2\cos 2t}\right)=\boxed{-\frac{\sin t}{\cos 2t}}$$

12. ~~Find $\frac{d}{dx}\left(a^{x^2}\right)$~~

Sol. $\frac{d}{dx}\left(a^{x^2}\right)$

$$=a^{x^2}(\ell n\, a)\left(\frac{d}{dx}\left(x^2\right)\right)$$

$$=a^{x^2}(\ell n\, a)(2x)=\boxed{2x(\ell n\, a)a^{x^2}}$$

13. If $y=\ell n x$, find y_2

Sol. $y=\ell n x$

Differentiate both sides w.r.t. 'x':

$$\frac{d}{dx}(y)=\frac{d}{dx}(\ell n x)$$

$$y_1=\frac{1}{x}$$

Differentiate both sides w.r.t. 'x':

$$\frac{d}{dx}(y_1)=\frac{d}{dx}\left(\frac{1}{x}\right)$$

$$y_2=\frac{d}{dx}\left(x^{-1}\right)=-1(x)^{-2}=\boxed{\frac{-1}{x^2}}$$

14. $\sin[\sin(\cos x)]$

Sol. $\frac{d}{dx}[\sin[\sin(\cos x)]]$

$$=\cos[\sin(\cos x)]\frac{d}{dx}[\sin(\cos x)]$$

$$=\cos\left[\sin(\cos x)\right].\left[\cos(\cos x)\right].\frac{d}{dx}(\cos x)$$

$$=\cos\left[\sin(\cos x)\right]\left[\cos(\cos x)\right].(-\sin x)$$

$$=\boxed{-\cos\left[\sin(\cos x)\right]\left[\cos(\cos x)\right]\left[\sin x\right]}$$

15. **If $s=\log(t)$,, find the velocity and acceleration at $t=3\,\text{sec}$.**

Sol. $s=\log(t)$

Differentiate both sides w.r.t. 't':

$$\frac{d}{dt}(s)=\frac{d}{dt}(\log t)$$

$$v=\frac{1}{t}\rightarrow(i)$$

Diff. again both sides w.r.t. 't':

$$\frac{d}{dt}(v)=dt\left(\frac{1}{t}\right)$$

$$a=-\frac{1}{t^2}\rightarrow(ii)$$

Put $t=3$ in eq.(i) & eq.(ii), we get:

$$\boxed{v\big|_{t=3}=\frac{1}{3}\ m/s}\quad \&$$

$$a\big|_{t=3}=-\frac{1}{(3)^2}=\boxed{\frac{-1}{9}\,m/sec^2}$$

16. **Find the turning points of the curve $y=2x^3-15x^2+36x+10$**

Sol. $y=2x^3-15x^2+36x+10$

Differentiate both sides w.r.t. 'x':

$$\frac{d}{dx}(y)=\frac{d}{dx}\left(2x^3-15x^2+36x+10\right)$$

$$\frac{dy}{dx}=2\left(3x^2\right)-15(2x)+36(1)+0$$

$$\frac{dy}{dx}=6x^2-30x+36$$

For turning point, put $\frac{dy}{dx}=0$

$$6x^2-30x+36=0$$

Dividing each term on '6', we get:

$$x^2-5x+6=0$$

$$x^2-3x-2x+6=0$$

$$x(x-3)-2(x-3)=0$$

$$(x-3)(x-2)=0$$

Either	**OR**
$x-3=0$	$x-2=0$
$\boxed{x=3}$	$\boxed{x=2}$

17. **Find $\int\left(\frac{1}{t^3}+\frac{1}{t^2}-2\right)dt$**

Sol. $\int\left(\frac{1}{t^3}+\frac{1}{t^2}-2\right)dt$

$$=\int\left(t^{-3}+t^{-2}-2\right)dt$$

$$=\frac{t^{-2}}{-2}+\frac{t^{-1}}{-1}-2t+c$$

$$=\boxed{\frac{-1}{2t^2}-\frac{1}{t}-2t+c}$$

18. **Find $\int\left(\frac{x^2+1}{x+1}\right)dx$**

Sol. $\int\left(\frac{x^2+1}{x+1}\right)dx$

```
          x - 1
x + 1 ) x^2 + 1
        ±x^2 ± x
          - x + 1
          ∓ x ∓ 1
              2
```

$$=\int\left(x-1+\frac{2}{x+1}\right)dx$$

$$=\boxed{\frac{x^2}{2}-x+2\,\ell n(x+1)+c}$$

19. **Find $\int\left(\frac{\sin 2x}{\cos^2 2x}\right)dx$**

Sol. $\int\frac{\sin 2x}{\cos^2 2x}\,dx$

$$= \int \left(\frac{\sin 2x}{\cos 2x . \cos 2x} \right) dx$$

$$= \int \left(\frac{1}{\cos 2x} . \frac{\sin 2x}{\cos 2x} \right) dx$$

$$= \int (\sec 2x \tan 2x) dx$$

$$= \boxed{\frac{\sec 2x}{2} + c} \quad \left\{ \begin{array}{l} \text{Using formula \#15} \\ \text{from page \# 226} \end{array} \right\}$$

20. Integrate $\int \sin^2 x dx$

Sol. $\int \sin^2 x dx$

$$= \int \left(\frac{1 - \cos 2x}{2} \right) dx \quad \because \left\{ \begin{array}{l} \sin^2 x \\ \frac{1-\cos 2x}{2} \end{array} \right\}$$

$$= \frac{1}{2} \left[\int (1) dx - \int (\cos 2x) dx \right]$$

$$= \boxed{\frac{1}{2} \left[x - \frac{\sin 2x}{2} \right] + c}$$

21. Find $\int (\cot^2 x) dx$

Sol. $\int (\cot^2 x) dx$

$$= \int (\operatorname{cosec}^2 x - 1) dx$$

$$= -\cot x - x + c = \boxed{-\cot x - x + c}$$

22. Find $\int \left(\frac{1}{25 + x^2} \right) dx$

Sol. $\int \left(\frac{1}{25 + x^2} \right) dx$

$$= \int \frac{1}{(5)^2 + (x)^2} dx$$

$$= \boxed{\frac{1}{5} \tan^{-1} \left(\frac{x}{5} \right) + c} \quad \left\{ \begin{array}{l} \text{Using formula \#17} \\ \text{from page \# 226} \end{array} \right\}$$

23. Evaluate $\int (x \cos x) dx$

Sol. $\int (x \cos x) dx$

Integrating by parts :

taking $u = x$ & $v = \cos x$

$$= x \int \cos x dx - \int \left[\frac{d}{dx}(x) \int \cos x dx \right] dx$$

$$= x \sin x - \int [(1) \sin x] dx$$

$$= x \sin x - \int \sin x \, dx$$

$$= x \sin x - (-\cos x) + c$$

$$= \boxed{x \sin x + \cos x + c}$$

24. Evaluate $\int \left(x . e^{x^2} \right) dx$

Sol. $\int \left(x . e^{x^2} \right) dx$

$$= \int \left(e^{x^2} \right) x dx$$

$$= \int (e^t) \frac{dt}{2}$$

$$= \frac{1}{2} e^t + c$$

Put $x^2 = t$

$\frac{d}{dx}(x^2) = \frac{d}{dx}(t)$

$2x = \frac{dt}{dx}$

$(x) dx = \frac{dt}{2}$

$$= \boxed{\frac{1}{2} e^{x^2} + c}$$

25. Evaluate $\int_0^{\pi/6} (2 \sin 2x) dx$

Sol. $\int_0^{\pi/6} (2 \sin 2x) dx$

$$= 2 \left[-\frac{\cos 2x}{2} \right]_0^{\pi/6} = -[\cos 2x]_0^{\pi/6}$$

$$= -\left(\cos 2\left(\frac{\pi}{6} \right) - \cos 2(0) \right)$$

$$= -(\cos 2(30^\circ) - \cos 2(0^\circ))$$

$$= -(\cos 60^\circ - \cos 0^\circ)$$

$$= -\left(\frac{1}{2} - 1 \right) \left\{ \begin{array}{c} \text{using calculator} \\ \cos 60^\circ = \frac{1}{2} \ \& \ \cos 0^\circ = 1 \end{array} \right\}$$

$$= -\left(\frac{1-2}{2} \right) = -\left(-\frac{1}{2} \right) = \boxed{\frac{1}{2}}$$

26. Evaluate $\int_{\pi/6}^{\pi/3}\left(\operatorname{cosec}^2 x\right)dx$

Sol. $\int_{\pi/6}^{\pi/3}\left(\operatorname{cosec}^2 x\right)dx = -\left[\cot x\right]_{\pi/6}^{\pi/3}$

$= -\left[\cot\left(\frac{\pi}{3}\right) - \cot\left(\frac{\pi}{6}\right)\right]$

$= -\left[\cot 60^\circ - \cot 30^\circ\right]$

$= -\left[\frac{1}{\sqrt{3}} - \sqrt{3}\right] = -\left(\frac{1-\left(\sqrt{3}\right)^2}{\sqrt{3}}\right)$

$= -\left(\frac{1-3}{\sqrt{3}}\right) = -\left(\frac{-2}{\sqrt{3}}\right) = \boxed{2/\sqrt{3}}$

27. Evaluate $\int_1^3 \frac{1}{x+1}dx$

Sol. $\int_1^3 \frac{1}{x+1}dx$

$= \left[\ell n(x+1)\right]_1^3$ {using Rule-II}

$= \ell n(3+1) - \ell n(1+1)$

$= \ell n(4) - \ell n(2)$

$= \ell n\left(\frac{4}{2}\right) = \boxed{\ell n 2}$

28. Evaluate $\int \frac{dx}{x(1+\ell n x)}$

Sol. $\int \frac{dx}{x(1+\ell n x)}$

$= \int \frac{1}{(1+\ell n x)}.\frac{1}{x}dx$

$= \boxed{\ell n(1+\ell n x)+c}$

29. Write distance formula between two points.

Sol. Let $A(x_1, y_1)$ & $B(x_2, y_2)$ be two different points, then,

$\text{Distance} = |AB| = \sqrt{(x_1-x_2)^2+(y_1-y_2)^2}$

30. ~~Find an equation on the line with the following intercepts: a = 2, b = −5~~

Sol. Equation of line in intercept form:

$\frac{x}{a}+\frac{y}{b}=1 \Rightarrow \frac{x}{2}+\frac{y}{-5}=1$

$\frac{5x-2y}{10}=1 \Rightarrow 5x-2y=10$

$\Rightarrow \boxed{5x-2y-10=0}$

31. Show that the lines passing through points $(0,-7), (8,-5)$ and $(5, 7), (8,-5)$ are perpendicular.

Sol. $\ell_1 : (0,-7)$ & $(8,-5)$ | $\ell_2 : (5, 7)$ & $(8,-5)$

Slope of ℓ_1 =	Slope of ℓ_2 =
$m_1 = \frac{y_2-y_1}{x_2-x_1}$	$m_2 = \frac{y_2-y_1}{x_2-x_1}$
$m_1 = \frac{-5+7}{8} = \frac{2}{8} = \frac{1}{4}$	$m_2 = \frac{-5-7}{8-5} = \frac{-12}{3} = -4$

As, $m_1 m_2 = \left(\frac{1}{4}\right)(-4) = -1$

Hence both lines ℓ_1 & ℓ_2 are **perpendicular**. **Proved.**

32. Find the distance to the line $3x-2y+12=0$ from the point $(-1, 7)$.

Sol. $D = \frac{|ax_1+by_1+c|}{\sqrt{a^2+b^2}}$

$D = \frac{|3(-1)-2(7)+12|}{\sqrt{(3)^2+(-2)^2}}$

$D = \frac{|-3-14+12|}{\sqrt{9+4}} = \boxed{\frac{5}{\sqrt{13}}}$

33. Find the midpoint of the points $A(6,-2)$ and $B(2, 1)$.

Sol. $\text{Mid-Point} = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

$= \left(\frac{6+2}{2}, \frac{-2+1}{2}\right) = \left(\frac{8}{2}, \frac{-1}{2}\right) = \boxed{\left(4, -\frac{1}{2}\right)}$

34. **Define real circle.**

Sol. A circle is called imaginary circle if $r > 0$.

35. **Find the equation of circle with center $(-1, 2)$ and radius $r = \sqrt{2}$.**

Sol. Standard form of equation of circle:

$$(x-h)^2 + (y-k)^2 = r^2$$

Put $h = -1, \ k = 2 \ \& \ r = \sqrt{2}$

$$(x-(-1))^2 + (y-2)^2 = (\sqrt{2})^2$$

$$(x+1)^2 + (y-2)^2 = 2$$

$$x^2 + 2x + 1 + y^2 - 4y + 4 - 2 = 0$$

$$\boxed{x^2 + y^2 + 2x - 4y + 3 = 0}$$

36. **Reduce the equation $x^2 + y^2 - 4x + 6y - 12 = 0$ into standard form.**

Sol. As given equation:

$$x^2 + y^2 - 4x + 6y - 12 = 0$$

$$x^2 - 4x + y^2 + 6y = 12$$

Adding the square of one half of the coefficient of 'x' & 'y' on both sides:

$$x^2 - 4x + (2)^2 + y^2 + 6y + (3)^2 = 12 + (2)^2 + (3)^2$$

$$(x-2)^2 + (y+3)^2 = 12 + 4 + 9$$

$$(x-2)^2 + (y+3)^2 = 25$$

$$\boxed{(x-2)^2 + (y+3)^2 = (5)^2}$$

37. **Write the general form of the circle, also represent the center and radius in this form.**

Sol. $x^2 + y^2 + 2gx + 2fy + c = 0$

Center $= \boxed{(-g, -f)}$

& Radius $= \boxed{\sqrt{g^2 + f^2 - c}}$

Section - II

Note: Attemp any three (3) questions $\boxed{3 \times 10 = 30}$

Q.2.[a] **Prove that: $f[f(x)] = x$, for the function $f(x) = \dfrac{x+1}{x-1}$.**

Sol. See Q.10 of Ex # 1.1 (Page # 7)

[b] **If $x = \dfrac{1-t^2}{1+t^2}, \ y = \dfrac{2t}{1+t^2}$ then prove that: $y\dfrac{dy}{dx} + x = 0$**

Sol. See Q.4(iii) of Ex # 2.3 (Page # 78)

Q.3.[a] **Find the derivative of $\sin^m x \sin mx$ w.r.t. 'x'.**

Sol. See Q.3(iii) of Ex # 3.1 (Page # 112)

[b] **Find the maximum and minimum (extreme) values of the function $(x-2)^2(x-1)$.**

Sol. See Q.2(vi) of Ex # 4.2 (Page # 190)

Q.4.[a] **Find the anti-derivative of $\int (\sin x + \cos x)^2 (\cos^2 x - \sin^2 x) dx$**

Sol. See Q.1(xv) of Ex # 5.2 (Page # 245)

[b] **Evaluate $\int (x \cos^2 x) dx$**

Sol. See Q.1(iii) of Ex # 6.3 (Page # 282)

Q.5.[a] **Evaluate $\int (\sec^3 x) dx$**

Sol. See Q.1(v) of Ex # 6.3 (Page # 283)

[b] **Is the point $(0, 4)$ inside or outside the circle of radius 4 with center at $(-3, 1)$.**

Sol. See Q.3 of Ex # 8.1 (Page # 356)

Q.6.[a] **If a line ℓ_1 contains P(2, 6) and Q(0, y). Find 'y' if ℓ_1 is parallel to ℓ_2 and that the slope of $\ell_2 = \dfrac{3}{4}$.**

Sol. See Q.2 of Ex # 8.3 (Page # 374)

[b] **Find which of the two circles $x^2 + y^2 - 3x + 4y = 0$ and $x^2 + y^2 - 6x - 8y = 0$ is greater.**

Sol. See Q.7 of Ex # 9 (Page # 446)

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