DAE / IIA - 2019

MATH - 233 APPLIED MATHEMATICS - II

PAPER 'B' PART - A (OBJECTIVE)

Time: 30 Minutes Marks: 15

Q.1: *Encircle the correct answer.*

1. $\int\left(\frac{\cos x}{\sin x}\right)dx = ?$

[a] $\ell n \cos x$ [b] $\ell n \sin x$

[c] $\ell n \cot x$ [d] $\frac{\cos^2 x}{2}$

2. $\int(\tan x \sec^2 x)dx = ?$

[a] $\ell n \tan x$ [b] $\frac{\tan^2 x}{2}$

[c] $\frac{\sec^2 x}{3}$ [d] $\sec x \tan x$

3. $\int(x^{n+1})dx = ?$

[a] $\frac{x^{n+1}}{n+1}$ [b] $(n+1)x^n$ [c] $\frac{x^{n+2}}{n+2}$ [d] $\frac{x^2}{2}$

4. $\int\left(\frac{\sec x \tan x}{3+\sec x}\right)dx = ?$

[a] $\sec x + \tan x$ [b] $3+\sec x$

[c] $\ell n(3+\sec x)$ [d] $\frac{x^3}{n}$

5. $\int(x \sin x)dx = ?$

[a] $-x\cos x + \sin x$ [b] $\sin x$

[c] $x + \sin x$ [d] $\frac{x^2}{2}\cos x$

6. $\int(\sin^4 x \cos x)dx = ?$

[a] $\frac{\sin^5 x}{5}$ [b] $\frac{\sin^5 x \cos x}{5}$

[c] $\frac{\cos^2 x}{2}$ [d] $-\sin x \cos x$

7. $\int_{\pi/3}^{\pi/4}\left(\frac{\csc^2 x}{\cot x}\right)dx = ?$

[a] 2 [b] $\sqrt{3}$ [c] $\frac{1}{\sqrt{3}}$ [d] $\ell n \frac{1}{\sqrt{3}}$

8. $\int_0^{\pi/2}(\cot x)dx = ?$

[a] -1 [b] 1 [c] 0 [d] $\pi/2$

9. Order of differential equation $\left(\frac{d^3y}{dx^3}\right)^2 + \frac{dy}{dx} + y = 0$ is:

[a] 2 [b] 1 [c] 0 [d] 3

10. Degree of differential equation $x\left(\frac{d^3y}{d^3}\right) = 1$ is:

[a] 0 [b] 1 [c] 2 [d] 3

11. If a function $f(-x) = f(x)$ then function is:

[a] Even [b] Odd

[c] Linear [d] Constant

12. ~~If an odd function, then Fourier coefficient~~ 'a_n' ~~is;~~

[a] 0 [b] 1 [c] -1 [d] 2

13. ~~$L^{-1}\left(\frac{S}{S^2+1}\right) = ?$~~

[a] $\sin t$ [b] $\cos t$ [c] $\sin\left(\frac{1}{t}\right)$ [d] $\cos\left(\frac{1}{t}\right)$

14. ~~$L^{-1}\left(\frac{1}{S-1}\right) = ?$~~

[a] e^{-t} [b] e^{2t} [c] $1/t$ [d] e^t

15. ~~$L^{-1}\left(\frac{1}{S+1}\right) = ?$~~

[a] e^t [b] e^{2t} [c] $1/t$ [d] e^{-t}

Answer Key

1	b	2	b	3	b	4	c	5	a
6	a	7	c	8	c	9	b	10	c
11	a	12	a	13	b	14	d	15	d

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DAE / IIA - 2019

MATH - 233 APPLIED MATHEMATICS - II

PAPER 'B' PART - B (SUBJECTIVE)

Time : 2 : 30Hrs Marks : 60

Section - I

Q.1. Write short answers to any Eighteen (18) questions.

1. Find $\int(\tan^2 x \operatorname{cosec}^2 x)dx$

Sol. $\int(\tan^2 x \operatorname{cosec}^2 x)dx$

$$= \int\left(\frac{\sin^2 x}{\cos^2 x}\cdot\frac{1}{\sin^2 x}\right)dx$$

$$= \int\left(\frac{1}{\cos^2 x}\right)dx$$

$$= \int \sec^2 x dx = \boxed{\tan x + c}$$

2. Find $\int(\sec 3x \tan 3x)dx$

Sol. $\int(\sec 3x \tan 3x)dx$

$$= \boxed{\frac{\sec 3x}{3} + c}$$

3. Evaluate $\int \cos^2 x dx$

Sol. $\int \cos^2 x dx = \int \frac{1+\cos 2x}{2} dx$

$$= \frac{1}{2}\int(1+\cos 2x)\,dx$$

$$= \boxed{\frac{1}{2}\left[x + \frac{\sin 2x}{2}\right] + c}$$

4. Find $\int(\tan^2 x)\,dx$

Sol. $\int(\tan^2 x)\,dx$

$$= \int(\sec^2 x - 1)\,dx$$

$$= \boxed{\tan x - x + c}$$

5. Evaluate $\int(\sin x - \cos x)^2\,dx$

Sol. $\int(\sin x - \cos x)^2\,dx$

$$= \int(\sin^2 x + \cos^2 x - 2\sin x\cos x)\,dx$$

$$= \int(1 - \sin 2x)\,dx \because \begin{Bmatrix}\sin^2 x+\cos^2 x=1\\ \sin 2x = 2\sin x \cos x\end{Bmatrix}$$

$$= x - \left(\frac{-\cos 2x}{2}\right) + c = \boxed{x + \frac{1}{2}\cos 2x + c}$$

6. Find $\int(\tan x + \cot x)^2\,dx$

Sol. $\int(\tan x + \cot x)^2\,dx$

$$= \int(\tan^2 x + \cot^2 x + 2\tan x.\cot x)dx$$

$$= \int\left[\sec^2 x - 1 + \operatorname{cosec}^2 x - 1 + 2\right]dx$$

$$= \int\left[\sec^2 x + \operatorname{cosec}^2 x\right]dx$$

$$= \boxed{\tan x - \cot x + c}$$

7. Find $6\int x^2 e^{x^3}\,dx$

Sol. $6\int x^2 e^{x^3}\,dx$

$$= 6\int e^{x^3}(x^2)dx$$

$$= 6\int e^t \frac{dt}{3}$$

$$= \frac{6}{3}\int e^t dt$$

$$= 2e^t + c$$

$$= \boxed{2e^{x^3} + c}$$

Put $x^3 = t$

$$\frac{d}{dx}(x^3) = \frac{d}{dx}(t)$$

$$3x^2 = \frac{dt}{dx}$$

8. Find $\int \frac{\sin^{-1} x}{\sqrt{1-x^2}}dx$

Sol. $\int \frac{\sin^{-1} x}{\sqrt{1-x^2}}dx$

$$= \int \sin^{-1} x\left(\frac{1}{\sqrt{1-x^2}}\right)dx$$

$$= \boxed{\frac{(\sin^{-1} x)^2}{2} + c}$$ {using Rule-I}

9. Integrate $\int \frac{\cos(\ell nx)}{x}dx$

Sol. $\int \frac{\cos(\ell nx)}{x}dx$

$= \int \cos(\ell nx).\left(\frac{1}{x}\right)dx$

Put $\ell nx = t$

$\frac{d}{dx}(\ell nx) = \frac{d}{dx}(t)$

$\frac{1}{x} = \frac{dt}{dx}$

$\left(\frac{1}{x}\right)dx = dt$

$= \int \cos t\, dt$

$= \sin t + c = \boxed{\sin(\ell nx) + c}$

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10. Find $\int\left(\frac{x-1}{x^2-2x+3}\right)dx$

Sol. $\int\left(\frac{x-1}{x^2-2x+3}\right)dx$

$= \frac{1}{2}\int \frac{2x-2}{(x^2-2x+3)}dx \because$
$\frac{d}{dx}(x^2-2x+3) = 2x-2(1)+0 = 2x-2$

$= \boxed{\frac{1}{2}\ell n(x^2-2x+3)+c}$ {using Rule-II}

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11. Find $\int(\cot^2 x)\, dx$

Sol. $\int(\cot^2 x)\, dx$

$= \int(\cos ec^2 x - 1)\, dx$

$= -\cot x - x + c = \boxed{-\cot x - x + c}$

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12. Find $\int 2\sec 2x\, dx$

Sol. $\int 2\sec 2x\, dx$

$= 2\int \sec 2x\, dx$

$= \not{2}\frac{\ell n(\sec 2x + \tan 2x)}{\not{2}} + c$

$= \boxed{\ell n(\sec 2x + \tan 2x) + c}$

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13. Find $\int \sin^9 x \cos x\, dx$

Sol. $\int \sin^9 x \cos x\, dx$

$\because \frac{d}{dx}(\sin x) = \cos x$

$= \boxed{\frac{\sin^{10} x}{10} + c}$ {using Rule-I}

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14. Evaluate $\int_0^{\pi/4}(\sec x \tan x)\, dx$

Sol. $\int_0^{\pi/4}(\sec x \tan x)\, dx$

$= [\sec x]_0^{\pi/4}$

$= \sec\left(\frac{\pi}{4}\right) - \sec(0)$

$= \sec 45° - \sec 0° = \boxed{\sqrt{2}-1}$

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15. Evaluate $\int_0^b (x^3 \cos x^4)\, dx$

Sol. $\int_0^b (x^3 \cos x^4)\, dx$

$= \frac{1}{4}\int_0^b \cos x^4 (4x^3)\, dx$

$= \frac{1}{4}\left[\sin x^4\right]_0^b$

$= \frac{1}{4}\left[\sin b^4 - \sin(0)^4\right]$

$= \frac{1}{4}\left[\sin b^4 - 0\right] = \boxed{\frac{1}{4}\sin b^4}$

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16. Evaluate $\int_0^{\pi/2} \frac{\cos x}{3+4\sin x}\, dx$

Sol. $\int_0^{\pi/2} \frac{\cos x}{3+4\sin x}\, dx$

$$=\frac{1}{4}\int_0^{\pi/2}\frac{4\cos x}{3+4\sin x}dx$$

$$=\frac{1}{4}\left[\ell n(3+4\sin x)\right]_0^{\pi/2} \quad \left\{\begin{array}{c}\text{using}\\ \text{Rule-II}\end{array}\right\}$$

$$=\frac{1}{4}\left[\ell n\left(3+4\sin\left(\pi/2\right)\right)-\ell n(3+4\sin(0))\right]$$

$$=\frac{1}{4}\left[\ell n(3+4\sin 90^\circ)-\ell n(3+4\sin 0^\circ)\right]$$

$$=\frac{1}{4}\left[\ell n(3+4(1))-\ell n(3+4(0))\right]$$

$$=\frac{1}{4}\left[\ell n(3+4)-\ell n(3+0)\right]$$

$$=\frac{1}{4}\left[\ell n(7)-\ell n(3)\right]$$

$$=\boxed{\frac{1}{4}\ell n\left(\frac{7}{3}\right)}\left\{\begin{array}{c}\text{By using logrithm law}\\ \ell n\left(\frac{m}{n}\right)=\ell n(m)-\ell n(n)\end{array}\right\}$$

17. Evaluate $\int_0^{\pi/6}(2\sin 2x)\,dx$

Sol. $\int_0^{\pi/6}(2\sin 2x)dx$

$$=2\left[-\frac{\cos 2x}{2}\right]_0^{\pi/6}=-\left[\cos 2x\right]_0^{\pi/6}$$

$$=-\left(\cos 2\left(\frac{\pi}{6}\right)-\cos 2(0)\right)$$

$$=-\left(\cos 2(30^\circ)-\cos 2(0^\circ)\right)$$

$$=-\left(\cos 60^\circ-\cos 0^\circ\right)$$

$$=-\left(\frac{1}{2}-1\right)\left\{\begin{array}{c}\text{using calculator}\\ \cos 60^\circ=\frac{1}{2}\ \&\ \cos 0^\circ=1\end{array}\right\}$$

$$=-\left(\frac{1-2}{2}\right)=-\left(-\frac{1}{2}\right)=\boxed{\frac{1}{2}}$$

18. Find the solution of $\frac{dy}{dx}=-4xy^2$

Sol. $\frac{dy}{dx}=-4xy^2$

$$\frac{dy}{y^2}=-4xdx$$

Integrating both sides, we have:

$$\int\frac{dy}{y^2}dy=\int -4xdx$$

$$\int y^{-2}dy=\int -4xdx$$

$$\frac{y^{-2+1}}{-1}=-\frac{4x^{1+1}}{2}+c$$

$$-\frac{1}{y}=-2x^2+c$$

$$\boxed{y=\frac{-1}{-2x^2+c}}$$

19. Find the solution of $\frac{dy}{dx}=\frac{x}{y^2}$

Sol. $\frac{dy}{dx}=\frac{x}{y^2}$

Integrating both sides, we have:

$$\int y^2dy=\int xdx$$

$$\boxed{\frac{y^3}{3}=\frac{x^2}{2}+c}$$

20. Find the solution of

$$\frac{dy}{dx}=\frac{y}{4+x^2}$$

Sol. $\frac{dy}{dx}=\frac{y}{4+x^2}$

$$\frac{dy}{y}=\frac{dx}{4+x^2}$$

Integrating both sides, we have:

$$\int\left(\frac{1}{y}\right)dy=\int\left(\frac{1}{(2)^2+(x)^2}\right)dx$$

$$\boxed{\ell n\,y=\frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right)+c}$$

21. Find the general solution of

$$xdy = 3ydx$$

Sol. $xdy = 3ydx$

$$\frac{1}{y}dy = \frac{3}{x}dx$$

Integrating both sides, we have:

$$\int \frac{1}{y}dy = \int \frac{3}{x}dx$$

$$\ell n y = 3\ell n x + \ell n c$$

$$\ell n y = \ell n x^3 + \ell n c$$

$$\ell n y = \ell n\left(cx^3\right) \Rightarrow \boxed{y = cx^3}$$

22. Find the solution of

$$3x^2\left(1+y^2\right)dx = dy$$

Sol. $3x^2\left(1+y^2\right)dx = dy$

$$3x^2dx = \frac{1}{1+y^2}dy$$

Integrating both sides, we have:

$$\int 3x^2dx = \int \frac{1}{1+y^2}dy$$

$$3\frac{x^3}{3} = \tan^{-1}y + c$$

$$x^3 - c = \tan^{-1}y$$

$$\tan\left(x^3 - c\right) = y$$

$$\boxed{y = \tan\left(x^3 - c\right)}$$

23. Prove that:

$$L\{u'(t)\} = sL\{u(t)\} - u(0)$$

Sol. $L.H.S. = L\{u'(t)\}$

$$= \int_0^\infty e^{-st}u'(t)dt$$

$$= e^{-st}\int_0^\infty u'(t)dt - \int_0^\infty\left(\frac{d}{dt}\left(e^{-st}\right)\int u'(t)dt\right)dt$$

$$= \left[e^{-st}u(t)\right]_0^\infty - \int_0^\infty\left(e^{-st}(-s)u(t)\right)dt$$

$$= \left[\frac{u(t)}{e^{st}}\right]_0^\infty + s\int_0^\infty e^{-st}u(t)dt$$

$$= \left(\frac{u(\infty)}{e^\infty} - \frac{u(0)}{e^0}\right) + sL\{u(t)\}$$

$$= 0 - u(0) + sL\{u(t)\}$$

$$= sL\{u(t)\} - u(0) = R.H.S.$$ **Proved.**

24. Write the formula for $L\{u''(t)\}$.

Sol. $L\{u''(t)\}$

$$= s^2\left(L\{u(t)\}\right) - su(0) - u'(0)$$

25. Write Laplace transformation of e^{at}.

Sol. $L\{e^{at}\} = \boxed{\frac{1}{s-a}}$

26. If $L\{t^n\} = \frac{n!}{s^{n+1}}$ then what will be $L\{t^7\}$.

Sol. As, $L\{t^n\} = \frac{n!}{s^{n+1}}$

Put $n = 7$, we have:

$$L\{t^7\} = \frac{7!}{s^{7+1}} = \boxed{\frac{5040}{s^8}}$$

27. Write Laplace transformation $t\,e^{at}$.

Sol. $L\{t\,e^{at}\} = (-1)\frac{d}{ds}L\{e^{at}\}$

$$= -\frac{d}{ds}\left(\frac{1}{s-a}\right) = -\frac{d}{ds}\left((s-a)^{-1}\right)$$

$$= -(-1)(s-a)^{-2}\left(\frac{d}{ds}(s-a)\right)$$

$$= \frac{1}{(s-a)^2}(1-0) = \boxed{\frac{1}{(s-a)^2}}$$

Section - II

Note: Attemp any three (3) questions $3 \times 8 = 24$

Q.2.[a] Evaluate

$$\int \left(\frac{a\sin^3 x + b\cos^3 x}{\sin^2 x \cos^2 x} \right) dx$$

Sol. See Q.8 of Ex # 7.2 (Page # 293)

[b] Evaluate $\int \frac{dx}{x^{1/3}\left(x^{2/3} - 1\right)}$

Sol. See Q.2(iv) of Ex # 7.3 (Page # 302)

Q.3.[a] Evaluate $\int \left(\sin^2 x \cos^3 x\right) dx$

Sol. See Q.1(iv) of Ex # 7.3 (Page # 397)

[b] ~~Evaluate $\int \left(\frac{1}{\sqrt{a^2 - x^2}} \right) dx$~~

Sol. See example # 09 of Chapter 8.

Q.4[a] Calculate the integral

$$\int_0^3 \sqrt[3]{(3x-1)^2}\, dx$$

Sol. See Q.1(iv) of Ex # 9.1 (Page # 375)

[b] ~~Find area between the curve~~

~~$y = 3x^2 - 3$ and x – axis.~~

Sol. See Q.3 of Ex # 9.2 (Page # 388)

Q.5.[a] Find the general solution of equation

$$(y + xy)\,dx + \left(x - xy^2\right)dy = 0$$

Sol. See Q.14 of Ex # 10 (Page # 418)

[b] Find the general solution of differential equation:

$$3x^2y^2dx + y^2dx + dy = 0,$$

Given y = 1 when x = 2

Sol. See Q.17 of Ex # 10 (Page # 420)

Q.6. Find the Laplace transformation of the following functions:

(i) $f(t) = e^{at}$ when $t \geq 0$, and a is constant.

Sol. See example # 03 of Chapter 12.

(ii) If $f(t) = 2\sin wt$. Find $L\{f(t)\}$.

Sol. See Q.5 of Ex # 12 (Page # 470)

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