DAE / IIA - 2019

MATH - 233 APPLIED MATHEMATICS - II

PAPER 'A' PART - A (OBJECTIVE)

Time: 30 Minutes Marks: 15

Q.1: *Encircle the correct answer.*

1. $\lim_{x \to 0} (1+x)^{1/x} = ?$

[a] 0 [b] 1 [c] e [d] e^2

2. $\lim_{\theta \to \frac{\pi}{2}} \frac{\sin\theta}{\theta} = ?$ [a] 1 [b] $\frac{\pi}{2}$ [c] $\frac{2}{\pi}$ [d] $\frac{1}{2}$

3. If $y = u^2$ and $u = x$ then $\frac{dy}{dx} = ?$

[a] $2x$ [b] u^2 [c] x [d] $2x^2$

4. If $y = \frac{x+1}{x}$, then $\frac{dy}{dx} = ?$

[a] $-\frac{1}{x^2}$ [b] $\frac{x+1}{x^2}$ [c] $\frac{2}{x^2}$ [d] $\frac{x^2-1}{x^2}$

5. If $y = x^{2/3}$ then at $x = 8$, $\frac{dy}{dx} =$

[a] $\frac{1}{2}$ [b] $\frac{1}{3}$ [c] $\frac{2}{3}$ [d] $\frac{3}{2}$

6. $\frac{d}{dx}\left(\sec^{-1} 2x\right) = ?$

[a] $\frac{1}{x\sqrt{4x^2-1}}$ [b] $\frac{1}{x\sqrt{4x^2-1}}$

[c] $\frac{1}{x\sqrt{x^2-1}}$ [d] $\frac{1}{2x\sqrt{x^2-1}}$

7. $\frac{d}{dx}\left(\tan^{-1} x^2\right) = ?$

[a] $\frac{1}{1+x^2}$ [b] $\frac{1}{1-x^2}$

[c] $\frac{2x}{1+x^4}$ [d] $\frac{2x}{1-x^4}$

8. $\frac{d}{dx}\left(a^x\right) = ?$

[a] $a^x \ln a$ [b] xa^{x-1} [c] a^{x-1} [d] a^x

9. $\frac{d}{dx}\left(\ln \sin x\right) = ?$

[a] $\cot x$ [b] $\frac{1}{\sin x}\ln \sin x$

[c] $\ln \cos x$ [d] $\tan x$

10. If 2^{nd} derivative is $-ve$ at a point, then function is:

[a] Maximum [b] Minimum

[c] Point of inflection [d] None

11. A function is maximum at a point if its 2^{nd} derivative is:

[a] $+ve$ [b] $-ve$ [c] zero [d] None

12. A value which occurs maximum number of times in the data is called;

[a] Mean [b] Geometric Mean

[c] Harmonic Mean [d] Mode

13. Mode can be calculated by the formula:

[a] Mode = 3Media − 4Mean

[b] Mode = 4Media − 3Mean

[c] Mode = 3Media − 2Mean

[d] Mode = 2Media − 3Mean

14. Toss of a fair coin is an example of:

[a] Equally likely events

[b] Independent events

[c] Simple events

[d] Dependent events

15. A perfect coin is tossed, what is the probability that it shows head:

[a] $\frac{1}{2}$ [b] zero [c] 1 [d] $\frac{3}{4}$

Answer Key

1	c	2	c	3	a	4	a	5	b
6	a	7	c	8	a	9	a	10	a
11	b	12	d	13	c	14	a	15	a

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DAE / IIA - 2019

MATH - 233 APPLIED MATHEMATICS - II

PAPER 'B' PART - B (SUBJECTIVE)

Time : 2 : 30Hrs   Marks : 60

Section - I

Q.1. Write short answers to any Eighteen (18) questions.

1. If $f(x)=\ln(x)$, then prove that:

$$f(pq)=f(p)+f(q)$$

Sol. As, $f(x)=\ln(x)$

$$\text{L.H.S.}=f(pq)$$
$$=\ln(pq) \qquad \because f(x)=\ln(x)$$
$$=\ln(p)+\ln(q)$$
$$=f(p)+f(q)=\text{R.H.S.}$$ **Proved.**

2. If $f(x)=\ln(x)$, then prove that:

$$f\left(\frac{p}{q}\right)=f(p)-f(q)$$

Sol. As, $f(x)=\ln(x)$

$$\text{L.H.S.}=f\left(\frac{p}{q}\right)$$
$$=\ln\left(\frac{p}{q}\right) \qquad \because f(x)=\ln(x)$$
$$=\ln(p)-\ln(q)$$
$$=f(p)-f(q)=\text{R.H.S.}$$ **Proved.**

3. If $f(x)=\sin x+\cos x$, show that:

$$f(x+\pi)=-f(x)$$

Sol. As, $f(x)=\sin x+\cos x$

$$\text{L.H.S.}=f(x+\pi)$$
$$=\sin(x+\pi)+\cos(x+\pi)$$
$$=-\sin x-\cos x$$
$$=-(\sin x+\cos x)$$
$$=-f(x)=\text{R.H.S.}$$ **Proved.**

4. Show that the function $f(x)=x^4-7x^2+7$ is an even function of x.

Sol. $f(x)=x^4-7x^2+7$

Replace x by $-x$, we have:

$$f(-x)=(-x)^4-7(-x)^2+7$$
$$f(-x)=x^4-7x^2+7$$
$$f(-x)=f(x)$$

Hence f(x) is an **even** function. **Proved.**

5. Find the derivative of $(a+x)\sqrt{a-x}$ w.r.t. 'x'.

Sol. $\frac{d}{dx}\left[(a+x)\sqrt{a-x}\right]$ {using Product Rule}

$$=\left(\frac{d}{dx}(a+x)\right)\left(\sqrt{a-x}\right)+(a+x)\left(\frac{d}{dx}\sqrt{a-x}\right)$$
$$=(0+1)\sqrt{a-x}+(a+x)\frac{1}{2}(a-x)^{-1/2}\left(\frac{d}{dx}(a-x)\right)$$
$$=\sqrt{a-x}+\frac{(a+x)(0-1)}{2\sqrt{a-x}}$$
$$=\frac{2\left(\sqrt{a-x}\right)^2+(a+x)(-1)}{2\sqrt{a-x}}$$
$$=\frac{2a-2x-a-x}{2\sqrt{a-x}}=\boxed{\frac{a-3x}{2\sqrt{a-x}}}$$

6. If $y=\frac{1}{(x-3)(x+2)}$, find $\frac{dy}{dx}$

Sol. $y=\frac{1}{(x-3)(x+2)}$

Differentiate both sides w.r.t. 'x':

$$\frac{d}{dx}(y)=\frac{d}{dx}\left(\frac{1}{(x-3)(x+2)}\right)$$ {using Quotient Rule}

$$\frac{dy}{dx}=\frac{(x-3)(x+2)\left(\frac{d}{dx}(1)\right)-(1)\left(\frac{d}{dx}((x-3)(x+2))\right)}{((x-3)(x+2))^2}$$

$$\frac{dy}{dx}=\frac{(x-3)(x+2)(0)-\left[\left(\frac{d}{dx}(x-3)\right)(x+2)+(x-3)\left(\frac{d}{dx}(x+2)\right)\right]}{\left[(x-3)(x+2)\right]^2}$$

$$\frac{dy}{dx}=\frac{0-\left[(1-0)(x+2)+(x-3)(1+0)\right]}{\left[(x-3)(x+2)\right]^2}$$

$$\frac{dy}{dx}=\frac{-(x+2+x-3)}{\left[(x-3)(x+2)\right]^2}$$

$$\frac{dy}{dx}=\frac{-(2x-1)}{\left[(x-3)(x+2)\right]^2}$$

$$\boxed{\frac{dy}{dx}=\frac{1-2x}{\left[(x-3)(x+2)\right]^2}}$$

7. Differentiate $\frac{x}{x^2+1}$ w.r.t. 'x'

Sol. Differentiate w.r.t. 'x':

$$\frac{d}{dx}\left(\frac{x}{x^2+1}\right) \quad \{\text{using Quotient Rule}\}$$

$$=\frac{(x^2+1)\left(\frac{d}{dx}(x)\right)-x\left(\frac{d}{dx}(x^2+1)\right)}{(x^2+1)^2}$$

$$=\frac{(x^2+1)(1)-x(2x+0)}{(x^2+1)^2}$$

$$=\frac{x^2+1-2x^2}{(x^2+1)^2}=\boxed{\frac{1-x^2}{(x^2+1)^2}}$$

8. If $y=\frac{1+x}{1-x}$, find $\frac{dy}{dx}$

Sol. $y=\frac{1+x}{1-x}$

Differentiate both sides w.r.t. 'x':

$$\frac{d}{dx}(y)=\frac{d}{dx}\left(\frac{1+x}{1-x}\right) \quad \left\{\begin{matrix}\text{using}\\ \text{Quotient Rule}\end{matrix}\right\}$$

$$\frac{dy}{dx}=\frac{(1-x)\left(\frac{d}{dx}(1+x)\right)-(1+x)\left(\frac{d}{dx}(1-x)\right)}{(1-x)^2}$$

$$\frac{dy}{dx}=\frac{(1-x)(0+1)-(1+x)(0-1)}{(1-x)^2}$$

$$\frac{dy}{dx}=\frac{1-x+1+x}{(1-x)^2}\Rightarrow\boxed{\frac{dy}{dx}=\frac{2}{(1-x)^2}}$$

9. If $y=\frac{x^2+1}{x-1}$, find $\frac{dy}{dx}$ at $x=2$.

Sol. $y=\frac{x^2+1}{x-1}$

Differentiate both sides w.r.t. 'x':

$$\frac{d}{dx}(y)=\frac{d}{dx}\left(\frac{x^2+1}{x-1}\right)\left\{\begin{matrix}\text{using by}\\ \text{Quotient Rule}\end{matrix}\right\}$$

$$\frac{dy}{dx}=\frac{(x-1)\left(\frac{d}{dx}(x^2+1)\right)-(x^2+1)\left(\frac{d}{dx}(x-1)\right)}{(x-1)^2}$$

$$\frac{dy}{dx}=\frac{(x-1)(2x+0)-(x^2+1)(1-0)}{(x-1)^2}$$

$$\frac{dy}{dx}=\frac{2x^2-2x-x^2-1}{(x-1)^2}$$

$$\frac{dy}{dx}=\frac{x^2-2x-1}{(x-1)^2}$$

At x = 2

$$\frac{dy}{dx}=\frac{(2)^2-2(2)-1}{(2-1)^2}$$

$$\frac{dy}{dx}=\frac{4-4-1}{(1)^2}=\frac{-1}{1}=\boxed{-1}$$

10. If $y=u^n$ & $u=(3x^3-7x^2+x+1)$ find $\frac{dy}{dx}$.

Sol. As, $y=u^n$

$$\Rightarrow \quad y=(3x^3-7x^2+x+1)^n$$

Differentiate both sides w.r.t. 'x':

$$\frac{d}{dx}(y)=\frac{d}{dx}\left(3x^3-7x^2+x+1\right)^n$$

$$\frac{dy}{dx}=n\left(3x^3-7x^2+x+1\right)^{n-1}\frac{d}{dx}\left(3x^3-7x^2+x+1\right)$$

$$\frac{dy}{dx}=n\left(3x^3-7x^2+x+1\right)^{n-1}\left(3\left(3x^2\right)-7(2x)+1+0\right)$$

$$\boxed{\frac{dy}{dx}=n\left(3x^3-7x^2+x+1\right)^{n-1}\left(9x^2-14x+1\right)}$$

11. Find the derivative of $x^2\sec 4x$

Sol. $\frac{d}{dx}\left(x^2\sec 4x\right)$ {using Product Rule}

$$=\left(\frac{d}{dx}\left(x^2\right)\right)\sec 4x+x^2\left(\frac{d}{dx}(\sec 4x)\right)$$

$$=2x\sec 4x+x^2\sec 4x\tan 4x\frac{d}{dx}(4x)$$

$$=2x\sec 4x+x^2\sec 4x\tan 4x(4)$$

$$=2x\sec 4x+4x^2\sec 4x\tan 4x$$

$$=\boxed{2x\sec 4x(1+2x\tan 4x)}$$

12. ~~Differentiate $\sin^{-1}\sqrt{x}$ w.r.t. 'x'.~~

Sol. $\frac{d}{dx}\left(\sin^{-1}\sqrt{x}\right)$

$$=\frac{1}{\sqrt{1-\left(\sqrt{x}\right)^2}}\frac{d}{dx}\left(\sqrt{x}\right)$$

$$=\frac{1}{\sqrt{1-x}}\cdot\frac{1}{2}(x)^{\frac{1}{2}-1}\frac{d}{dx}(x)$$

$$=\frac{1}{\sqrt{1-x}}\cdot\frac{1}{2}(x)^{-\frac{1}{2}}(1)$$

$$=\frac{1}{\sqrt{1-x}}\cdot\frac{1}{2\sqrt{x}}=\boxed{\frac{1}{2\sqrt{x}\sqrt{1-x}}}$$

13. ~~Differentiate $\sin^{-1}\left(\frac{x}{3}\right)$ w.r.t. 'x'.~~

Sol. $\frac{d}{dx}\left[\sin^{-1}\left(\frac{x}{3}\right)\right]$

$$=\frac{1}{\sqrt{1-\left(\frac{x}{3}\right)^2}}\frac{d}{dx}\left(\frac{x}{3}\right)$$

$$=\frac{1}{\sqrt{1-\frac{x^2}{9}}}\cdot\left(\frac{1}{3}\right)=\frac{1}{\sqrt{\frac{9-x^2}{9}}}\cdot\left(\frac{1}{3}\right)$$

$$=\frac{1}{\frac{\sqrt{9-x^2}}{3}}\cdot\left(\frac{1}{3}\right)=\boxed{\frac{1}{\sqrt{9-x^2}}}$$

14. ~~Differentiate $x^2\cot^{-1}x$ w.r.t. 'x'.~~

Sol. $\frac{d}{dx}\left(x^2\cot^{-1}x\right)$ {using Product Rule}

$$=\left(\frac{d}{dx}\left(x^2\right)\right)\cot^{-1}x+x^2\left(\frac{d}{dx}\left(\cot^{-1}x\right)\right)$$

$$=2x\cot^{-1}x+x^2\left(\frac{-1}{1+x^2}\right)$$

$$=\boxed{2x\cot^{-1}x-\frac{x^2}{1+x^2}}$$

15. Differentiate $\ell n\sqrt{x}$ w.r.t. 'x'.

Sol. $\frac{d}{dx}\left(\ell n\sqrt{x}\right)$

$$=\frac{1}{\sqrt{x}}\frac{d}{dx}\left(\sqrt{x}\right)$$

$$=\frac{1}{\sqrt{x}}\cdot\frac{1}{2}(x)^{\frac{1}{2}-1}\frac{d}{dx}(x)$$

$$=\frac{1}{\sqrt{x}}\cdot\frac{1}{2}(x)^{-\frac{1}{2}}(1)$$

$$=\frac{1}{\sqrt{x}}\cdot\frac{1}{2\sqrt{x}}$$

$$=\frac{1}{2\left(\sqrt{x}\right)^2}=\boxed{\frac{1}{2x}}$$

16. Differentiate $\sin(\ell n\tan x)$

Sol. $\frac{d}{dx}\left[\sin(\ell n\tan x)\right]$

$$= \cos(\ell n \tan x)\frac{d}{dx}(\ell n \tan x)$$

$$= \cos(\ell n \tan x)\frac{1}{\tan x}\frac{d}{dx}(\tan x)$$

$$= \cos(\ell n \tan x)\frac{1}{\tan x}\sec^2 x$$

$$= \boxed{\frac{\cos(\ell n \tan x)\sec^2 x}{\tan x}}$$

17. **Differentiate $x\,\ell n\, 3x$ w.r.t. 'x'.**

Sol. $\frac{d}{dx}(x\,\ell n 3x)$ {using Product Rule}

$$= \left(\frac{d}{dx}(x)\right)\ell n 3x + x\left(\frac{d}{dx}(\ell n 3x)\right)$$

$$= 1.\ell n 3x + x\left(\frac{1}{3x}.\frac{d}{dx}(3x)\right)$$

$$= \ell n 3x + x\left(\frac{1}{3x}(3(1))\right)$$

$$= \boxed{\ell n 3x + 1}$$

18. **Find slope of tangent to**

$x = a\cos\theta$, $y = b\sin\theta$ at $\theta = \frac{\pi}{4}$

Sol. As, $x = a\cos\theta,\ y = b\sin\theta$

Diff. both equations both sides w.r.t. 'x':

$$\frac{d}{d\theta}(x) = \frac{d}{d\theta}(a\cos\theta)$$

$$\frac{dx}{d\theta} = a(-\sin\theta)$$

$$\frac{d\theta}{dx} = -\frac{1}{a\sin\theta}$$

$$\frac{d}{d\theta}(y) = \frac{d}{d\theta}(b\sin\theta)$$

$$\frac{dy}{d\theta} = b\cos\theta$$

By using Chain's Rule: $\frac{dy}{dx} = \frac{dy}{d\theta}\times\frac{d\theta}{dx}$

$$\frac{dy}{dx} = (b\cos\theta)\left(-\frac{1}{a\sin\theta}\right) = -\frac{b\cos\theta}{a\sin\theta}$$

$$\frac{dy}{dx} = -\frac{b}{a}\cot\theta$$

At $x = \pi/4$

$$\frac{dy}{dx} = -\frac{b}{a}\cot\left(\frac{\pi}{4}\right) = -\frac{b}{a}(1) = \boxed{-\frac{b}{a}}$$

19. **Find the slope of tangent to the curve $y = x^3 - 3x + 2$ at $(0, 2)$.**

Sol. $y = x^3 - 3x + 2$

Differentiate both sides w.r.t. 'x':

$$\frac{d}{dx}(y) = \frac{d}{dx}(x^3 - 3x + 2)$$

$$\frac{dy}{dx} = 3x^2 - 3(1) + 0$$

$$\frac{dy}{dx} = 3x^2 - 3$$

At $x = 0$

$$\frac{dy}{dx} = 3(0)^2 - 3 = 0 - 3 = \boxed{-3}$$

20. **For the curve $x = t^2 - 1$, $y = t^2 - t$, the tangent is parallel to x-axis, find the value of t.**

Sol. $x = t^2 - 1,\ y = t^2 - t$

Differentiate both equations both sides w.r.t. 't':

$$\frac{d}{dt}(x) = \frac{d}{dt}(t^2 - 1)$$

$$\frac{dx}{dt} = 2t - 0$$

$$\frac{dt}{dx} = \frac{1}{2t}$$

$$\frac{d}{dt}(y) = \frac{d}{dt}(t^2 - t)$$

$$\frac{dy}{dt} = 2t - 1$$

Using Chain's Rule: $\frac{dy}{dx} = \frac{dy}{dt}.\frac{dt}{dx}$

$$\frac{dy}{dx}=(2t-1)\left(\frac{1}{2t}\right)=\frac{2t-1}{2t}$$

As tangent is parallel to x-axis,

so $\frac{dy}{dx}=0$

$$\frac{2t-1}{2t}=0 \Rightarrow 2t-1=0 \Rightarrow \boxed{t=\frac{1}{2}}$$

21. ~~If $x=a\cos\theta,\ y=a\sin\theta$, find $\frac{d^2y}{dx^2}$~~

Sol. Differentiate both equations both sides w.r.t. 'θ':

$x=a\cos\theta$	$y=a\sin\theta$
$\frac{d}{d\theta}(x)=\frac{d}{d\theta}(a\cos\theta)$	$\frac{d}{d\theta}(y)=\frac{d}{d\theta}(a\sin\theta)$
$\frac{dx}{d\theta}=a(-\sin\theta)$	$\frac{dy}{d\theta}=a\cos\theta$

$$\frac{d\theta}{dx}=-\frac{1}{a\sin\theta}$$

Using Chain's Rule: $\frac{dy}{dx}=\frac{dy}{d\theta}\cdot\frac{d\theta}{dx}$

$$\frac{dy}{dx}=(a\cos\theta)\left(\frac{-1}{a\sin\theta}\right)=-\cot\theta$$

Differentiate both sides w.r.t. 'x':

$$\frac{d}{dx}\left(\frac{dy}{dx}\right)=\frac{d}{d\theta}(-\cot\theta)\cdot\frac{d\theta}{dx}$$

$$\frac{d^2y}{dx^2}=(\operatorname{cosec}^2\theta)\cdot\left(\frac{-1}{a\sin\theta}\right)=\boxed{\frac{-1}{a\sin^3\theta}}$$

22. If mode = 15, Median = 12 find mean.

Sol. As, Mode = 15

$$3\text{Median}-2\text{Mean}=15$$
$$3(12)-2\text{Mean}=15$$
$$-2\text{Mean}=15-36$$
$$-2\text{Mean}=-21$$
$$\text{Mean}=\frac{-21}{-2}=\boxed{10.5}$$

23. Define arithmetic mean.

Sol. Let $x_1, x_2, x_3, \ldots, x_n$ be n values of a variable x, then their A.M. is defined as: $\bar{x}=\frac{\sum x_i}{n}$; $i=1, 2, \ldots, n$

24. The arithmetic mean of 7 values is 6 find the sum of values.

Sol. Here A.M = 6, n = 7, & $\sum x=?$

As, A.M. = 6

$$\frac{\sum x}{n}=6$$
$$\frac{\sum x}{7}=6$$
$$\sum x=6\times 7$$
$$\boxed{\sum x=42}$$

25. If two coins are tossed find the probability that only one head.

Sol. $S=\{HH, HT, TH, TT\}$, $n(S)=4$

Let A be event that only one head appears. $A=\{HT, TH\}$

$$n(A)=2 \therefore P(A)=\frac{n(A)}{n(S)}=\frac{2}{4}=\boxed{\frac{1}{2}}$$

26. Tickets numbered 1 to 20 are mixed up and then a ticket is drawn at random. What is the probability that ticket drawn has a number multiple of 3 or 5.

Sol. $S=\{T_1, T_2, T_3, \ldots, T_{20}\}$, $n(S)=20$

Let A be event that ticket number is multiple of 3 or 5.

$$A=\left\{\begin{matrix}T_3, & T_5, & T_6, & T_9, & T_{10},\\ T_{12}, & T_{15}, & T_{18}, & T_{20}\end{matrix}\right\}$$

$$n(A)=9$$

$$\therefore P(A)=\frac{n(A)}{n(S)}=\boxed{\frac{9}{20}}$$

27. **If a die is rolled once, what is the probability of getting a 4?**

Sol. $S=\{1, 2, 3, 4, 5, 6\}$, $n(S)=6$

Let A be event that getting number is 4. $A=\{4\}$

$n(A)=1 \therefore P(A)=\frac{n(A)}{n(S)}=\boxed{\frac{1}{6}}$

Section - II

Note : Attemp any three (3) questions $3\times 8=24$

Q.2.(a) Show that $x.\frac{a^x+1}{a^x-1}$ is an Even function of x.

Sol. See Q.12(ii) of Ex # 1.1 (Page # 10)

(b) ~~Evaluate: $\lim_{\theta\to 0}\frac{1-\cos p\theta}{1-\cos q\theta}$~~

Sol. See Q.1(ix) of Ex # 1.3 (Page # 27)

Q.3.(a) ~~Differentiate $x^{2/3}$ by ab-initio method (or first principle).~~

Sol. See Q.1(iv) of Ex # 2.1 (Page # 40)

(b) Differentiate $x\sqrt{\frac{a+x}{a-x}}$ w.r.t. 'x'.

Sol. See Q.4(vi) of Ex # 2.2 (Page # 58)

Q.4(a) If $\sin y = x\sin(a+y)$,

prove that : $\frac{dy}{dx}=\frac{\sin^2(a+y)}{\sin a}$

Sol. See Q.4 of Ex # 3.1 (Page # 119)

(b) Find the derivative of

$\ell n\left(\frac{x^2+x+1}{x^2-x+1}\right)$

Sol. See Q.1(vi) of Ex # 3.3 (Page # 140)

Q.5.(a) Use differentials to find the approximate value of $\sqrt{65}$.

Sol. See Q.6(i) of Ex # 4.1 (Page # 176)

(b) Find the maximum and minimum (extreme) values of the function

$\frac{x^3}{3}-\frac{3x^2}{2}+2x+5$

Sol. See Q.2(iv) of Ex # 4.2 (Page # 189)

Q.6.(a) Find the mean for the following distribution showing marks obtained by 50 students in English.

Marks	Frequency
20 – 24	1
25 – 29	4
30 – 34	8
35 – 39	11
40 – 44	15
45 – 49	9
50 – 54	2

Sol. See example # 02 of Chapter 05.

(b) ~~Calculate the S.D from the Mean for the following data, 2, 6, 9, 12, 8, 13, 5, 6, 23, 16.~~

Sol. See Q.1 of Ex # 5.2 (Page # 240)

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