DAE / IIA - 2019

MATH - 212 APPLIED MATHEMATICS - II

PART - A (OBJECTIVE)

Time : 30 Minutes Marks : 20

Q.1: *Encircle the correct answer.*

1. A function $f(x) = x^2 + 2x + 3$ is:

[a] Odd [b] Even
[c] Implicit [d] Explicit

2. Given $f(x) = 2(x-1) + 3$ then $f(2) = ?$

[a] 0 [b] 1
[c] 3 [d] 5

3. $\frac{d}{dx}(2x+3)^4 = ?$

[a] $8(2x+3)^3$ [b] $4(2x+3)^3$
[c] $(2x+3)^3$ [d] $4(2x+3)^2$

4. $\frac{d}{dx}\left(\frac{1}{x}\right) = ?$

[a] $\frac{1}{x^2}$ [b] $-\frac{1}{x^2}$
[c] $-\frac{1}{x^3}$ [d] $\frac{2}{x}$

5. $\frac{d}{dx}(\sin x^3) = ?$

[a] $\cos x^3$ [b] $-\cos x^3$
[c] $3x\cos x^3$ [d] $3x^2\cos x^3$

6. $\frac{d}{dx}\left[\cos\left(\frac{1}{x}\right)\right] = ?$

[a] $-\sin\left(\frac{1}{x}\right)$ [b] $\sin\left(\frac{1}{x}\right)$
[c] $\frac{1}{x^2}\sin\left(\frac{1}{x}\right)$ [d] $-\frac{1}{x^2}\sin\left(\frac{1}{x}\right)$

7. If $\frac{dy}{dx}$ changes sign from $+ve$ to $-ve$ then it is a point of:

[a] Maxima [b] Minima
[c] Inflection [d] None of these

8. If $\frac{dy}{dx}$ changes sign from $-ve$ to $+ve$ then it is a point of:

[a] Maxima [b] Minima
[c] Inflection [d] None of these

9. $\int\left(\frac{\cos x}{\sin x}\right)dx =$

[a] $\ln\cos x$ [b] $\ln\sin x$
[c] $\ln\cot x$ [d] $\frac{\cos^2 x}{2}$

10. $\int(\tan x\sec^2 x)\,dx = ?$

[a] $\ln\tan x$ [b] $\frac{\tan^2 x}{2}$
[c] $\frac{\sec^2 x}{3}$ [d] $\sec x\tan x$

11. ~~$\int\left(\frac{1}{\sqrt{1-x^2}}\right)dx = ?$~~

[a] $\sin^{-1}x$ [b] $\cos^{-1}x$
[c] $\sec^{-1}x$ [d] $\tan^{-1}x$

12. ~~$\int\left(\frac{-1}{\sqrt{1-x^2}}\right)dx = ?$~~

[a] $\sin^{-1}x$ [b] $\cos^{-1}x$
[c] $\sec^{-1}x$ [d] $\sqrt{1-x^2}$

13. $\int_0^1 (1)\,dx = ?$

[a] -1 [b] 0
[c] 1 [d] 2

14. $\int_0^{\pi/4}(\sec^2 x)\,dx = ?$

[a] 1 [b] 2
[c] 0 [d] 3

15. Slope of the line $\frac{x}{a}+\frac{y}{b}=1$ is:

[a] $\frac{a}{b}$ [b] $\frac{b}{a}$

[c] $-\frac{b}{a}$ [d] $-\frac{a}{b}$

16. $y=2$ is a line parallel to:

[a] $x-$axis [b] $y-$axis

[c] $y=x$ [d] $x=3$

17. $x=2$ is a line parallel to:

[a] $x-$axis [b] $y-$axis

[c] $y=x$ [d] $x=3$

18. Equation of line in slope intercept form is:

[a] $\frac{x}{a}+\frac{y}{b}=1$

[b] $y=mx+c$

[c] $y-y_1=m(x-x_1)$

[d] $y+y_1=m(x+x_1)$

19. Radius of the circle $x^2+y^2-2x-4y=8$ is:

[a] 8 [b] $\sqrt{8}$

[c] $\sqrt{12}$ [d] $\sqrt{13}$

20. Radius of the circle $(x-1)^2+(y-2)^2=16$ is:

[a] 2 [b] 1

[c] 4 [d] 16

Answer Key

1	d	2	d	3	a	4	b	5	d
6	d	7	a	8	b	9	b	10	b
11	a	12	b	13	c	14	a	15	c
16	a	17	b	18	b	19	d	20	c

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DAE / IIA - 2019

MATH - 212 APPLIED MATHEMATICS - II

PART - B (SUBJECTIVE)

Time: 2:30 Hrs Marks: 60

Section - I

Q.1: Write short answers to any Twenty Five (25) of the follwing questions. **25×2 = 50**

1. If $f(x)=2x^2+4x+9$, find the value of $\frac{f(3)-f(1)}{f(-1)+f(0)}$.

Sol. $f(x)=2x^2+4x+9 \rightarrow (i)$

Put $x=3$ in eq. (i)

$f(3)=2(3)^2+4(3)+9$

$f(3)=18+12+9=39$

Put $x=1$ in eq. (i)

$f(1)=2(1)^2+4(1)+9$

$f(1)=2+4+9=15$

Put $x=-1$ in eq. (i)

$f(-1)=2(-1)^2+4(-1)+9$

$f(-1)=2-4+9=7$

Put $x=0$ in eq. (i)

$f(0)=2(0)^2+4(0)+9=9$

$$\frac{f(3)-f(1)}{f(-1)+f(0)}=\frac{39-15}{7+9}=\frac{24}{16}=\boxed{\frac{3}{2}}$$

2. If $f(x)=\sin x+\cos x$, show that: $f(x+\pi)=-f(x)$

Sol. As, $f(x)=\sin x+\cos x$

L.H.S. $=f(x+\pi)$

$=\sin(x+\pi)+\cos(x+\pi)$

$=-\sin x-\cos x$

$=-(\sin x+\cos x)$

$=-f(x)=$ R.H.S. **Proved.**

3. **Show that the function $f(x) = x^4 - 7x^2 + 7$ is an even function of 'x'.**

Sol. $f(x) = x^4 - 7x^2 + 7$

Replace 'x' by '−x', we have:

$f(-x) = (-x)^4 - 7(-x)^2 + 7$

$f(-x) = x^4 - 7x^2 + 7$

$f(-x) = f(x)$

Hence $f(x)$ is an $\boxed{\text{even}}$ function.

4. **If $f(x) = 3x^3 + 2x^2 - x + 4$, prove that: $2f(3) = 25f(1)$**

Sol. As, $f(x) = 3(x)^3 + 2(x)^2 - x + 4 \rightarrow (i)$

Put $x = 3$, in eq. (i):

$f(3) = 3(3)^3 + 2(3)^2 - 3 + 4$

$f(3) = 81 + 18 - 3 + 4 = 100$

Put $x = 1$, in eq. (i):

$f(1) = 3(1)^3 + 2(1)^2 - 1 + 4$

$f(1) = 3 + 2 - 1 + 4 = 8$

$2f(3) = 25f(1)$

$2(100) = 25(8)$

$200 = 200$

L.H.S. = R.H.S. **Proved.**

5. **If $y = \sqrt{x} - \frac{1}{\sqrt{x}}$, then show that: $2x\frac{dy}{dx} + y = 2\sqrt{x}$**

Sol. $y = \sqrt{x} - \frac{1}{\sqrt{x}}$

Differentiate both sides w.r.t. 'x':

$\frac{d}{dx}(y) = \frac{d}{dx}\left(\sqrt{x} - \frac{1}{\sqrt{x}}\right)$

$\frac{dy}{dx} = \frac{d}{dx}\left(x^{1/2} - x^{-1/2}\right)$

$\frac{dy}{dx} = \frac{1}{2}x^{-1/2} - \left(-\frac{1}{2}\right)x^{-3/2}$

$\frac{dy}{dx} = \frac{1}{2}\left(x^{-1/2} + x^{-3/2}\right)$

Now take: L.H.S. $= 2x\frac{dy}{dx} + y$

$= 2x\left[\frac{1}{2}\left(x^{-1/2} + x^{-3/2}\right)\right] + \sqrt{x} - \frac{1}{\sqrt{x}}$

$= x^{1/2} + x^{-1/2} + x^{1/2} - x^{-1/2}$

$= 2x^{1/2} = \text{R.H.S.}$ **Proved.**

6. **Find: $\frac{d}{dx}\left(\frac{1}{(ax+b)^m}\right)$**

Sol. $\frac{d}{dx}\left(\frac{1}{(ax+b)^m}\right)$

$= \frac{d}{dx}(ax+b)^{-m}$

$= -m(ax+b)^{-m-1}\frac{d}{dx}(ax+b)$

$= -m\frac{1}{(ax+b)^{m+1}}\cdot(a(1)+0)$

$= \boxed{-\frac{am}{(ax+b)^{m+1}}}$

7. **If $y = x - \sqrt{x^2+1}$ then show that $(y-x)\frac{dy}{dx} = y$**

Sol. $y = x - \sqrt{x^2+1}$

Differentiate both sides w.r.t. 'x':

$\frac{d}{dx}(y) = \frac{d}{dx}\left(x - \sqrt{x^2+1}\right)$

$\frac{dy}{dx} = 1 - \frac{1}{2}(x^2+1)^{-1/2}\left(\frac{d}{dx}(x^2+1)\right)$

$\frac{dy}{dx} = 1 - \frac{1}{2\sqrt{x^2+1}}(2x+0)$

$\frac{dy}{dx} = 1 - \frac{x}{\sqrt{x^2+1}} = \frac{\sqrt{x^2+1} - x}{\sqrt{x^2+1}}$

$$\text{L.H.S.} = (y-x)\frac{dy}{dx}$$

$$= \left(x - \sqrt{x^2+1} - x\right)\left(\frac{\sqrt{x^2+1}-x}{\sqrt{x^2+1}}\right)$$

$$= \left(-\sqrt{x^2+1}\right)\left(\frac{\sqrt{x^2+1}-x}{\sqrt{x^2+1}}\right)$$

$$= -\left(\sqrt{x^2+1}-x\right)$$

$$= -\sqrt{x^2+1}+x$$

$$= x-\sqrt{x^2+1}$$

$= y = \text{R.H.S.}$ **Proved.**

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8. If $y = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \ldots,$ then show that $\frac{dy}{dx} = y$

Sol. $y = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \ldots$

Differentiate both sides w.r.t. 'x' :

$$\frac{d}{dx}(y) = \frac{d}{dx}\left(1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \frac{x^4}{24} + \ldots\right)$$

$$\frac{dy}{dx} = 0 + 1 + \frac{2x}{2} + \frac{3x^2}{6} + \frac{4x^3}{24} + \ldots$$

$$\frac{dy}{dx} = 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \ldots$$

$$\frac{dy}{dx} = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots$$

$$\boxed{\frac{dy}{dx} = y}$$ **Proved.**

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9. ~~Find the derivative of $\sin^{-1}\left(\frac{x}{a}\right)$~~

Sol. $\frac{d}{dx}\left(\sin^{-1}\left(\frac{x}{a}\right)\right)$

$$= \frac{1}{\sqrt{1-\left(\frac{x}{a}\right)^2}} \cdot \frac{d}{dx}\left(\frac{x}{a}\right)$$

$$= \frac{1}{\sqrt{1-\frac{x^2}{a^2}}} \times \left(\frac{1}{a}\right) = \frac{1}{\sqrt{\frac{a^2-x^2}{a^2}}} \cdot \frac{1}{a}$$

$$= \frac{1}{\frac{\sqrt{a^2-x^2}}{a}} \cdot \frac{1}{a} = \boxed{\frac{1}{\sqrt{a^2-x^2}}}$$

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10. ~~Find the value of $\frac{d}{dx}\left(\sin^{-1}x + \cos^{-1}x\right)$~~

Sol. $\frac{d}{dx}\left(\sin^{-1}x + \cos^{-1}x\right)$

$$= \frac{1}{\sqrt{1-x^2}} - \frac{1}{\sqrt{1-x^2}} = \boxed{0}$$

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11. ~~Find the value of $\frac{d}{dx}\left(\sec^{-1}\left(\sqrt{x}\right)\right)$~~

Sol. $\frac{d}{dx}\left(\sec^{-1}\left(\sqrt{x}\right)\right)$

$$= \frac{1}{\sqrt{x}\sqrt{\left(\sqrt{x}\right)^2-1}} \cdot \frac{d}{dx}\left(\sqrt{x}\right)$$

$$= \frac{1}{\sqrt{x}\sqrt{x-1}} \cdot \frac{1}{2}(x)^{-1/2}$$

$$= \frac{1}{\sqrt{x}\sqrt{x-1}}\ \frac{1}{2\sqrt{x}}$$

$$= \frac{1}{2\left(\sqrt{x}\right)^2\sqrt{x-1}} = \boxed{\frac{1}{2x\sqrt{x-1}}}$$

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12. ~~Find the value of $\frac{d}{dx}\left(\cos^{-1}\left(1-2x^2\right)\right)$~~

Sol. $\frac{d}{dx}\left(\cos^{-1}\left(1-2x^2\right)\right)$

$$= \frac{-1}{\sqrt{1-\left(1-2x^2\right)^2}} \cdot \left(\frac{d}{dx}\left(1-2x^2\right)\right)$$

$$= \frac{-1}{\sqrt{1-\left(1-4x^2+4x^4\right)}}.\left(0-2(2x)\right)$$

$$= \frac{-1}{\sqrt{1-1+4x^2-4x^4}}(-4x)$$

$$= \frac{4x}{\sqrt{4x^2-4x^4}} = \frac{4x}{\sqrt{4x^2(1-x^2)}}$$

$$= \frac{4x}{2x\sqrt{1-x^2}} = \boxed{\frac{2}{\sqrt{1-x^2}}}$$

13. If $y = x^4 - 3x^2 + 4x - 1$, find $\frac{d^2y}{dx^2}$

Sol. $y = x^4 - 3x^2 + 4x - 1$

Differentiate both sides w.r.t. 'x':

$$\frac{d}{dx}(y) = \frac{d}{dx}(x^4 - 3x^2 + 4x - 1)$$

$$\frac{dy}{dx} = 4x^3 - 3(2x) + 4(1) - 0$$

$$\frac{dy}{dx} = 4x^3 - 6x + 4$$

Differentiate both sides w.r.t. 'x':

$$\frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dx}(4x^3 - 6x + 4)$$

$$\frac{d^2y}{dx^2} = 4(3x^2) - 6(1) + 0$$

$$\boxed{\frac{d^2y}{dx^2} = 12x^2 - 6}$$

14. If $y = \ell n x$, find y_2

Sol. $y = \ell n x$

Differentiate both sides w.r.t. 'x':

$$\frac{d}{dx}(y) = \frac{d}{dx}(\ell n x)$$

$$y_1 = \frac{1}{x}$$

Differentiate both sides w.r.t. 'x':

$$\frac{d}{dx}(y_1) = \frac{d}{dx}\left(\frac{1}{x}\right)$$

$$y_2 = \frac{d}{dx}(x^{-1}) = -1(x)^{-2} = \boxed{\frac{-1}{x^2}}$$

15. If $y = \cos 3x + \sin 3x$, show that $y_2 + 9y = 0$

Sol. $y = \cos 3x + \sin 3x$

Differentiate both sides w.r.t. 'x':

$$\frac{d}{dx}(y) = \frac{d}{dx}(\cos 3x + \sin 3x)$$

$$y_1 = -\sin 3x(3) + \cos 3x(3)$$

$$y_1 = -3\sin 3x + 3\cos 3x$$

Differentiate both sides w.r.t. 'x':

$$\frac{d}{dx}(y_1) = \frac{d}{dx}(-3\sin 3x + 3\cos 3x)$$

$$y_2 = -3\cos 3x(3) + 3(-\sin 3x)(3)$$

$$y_2 = -9\cos 3x - 9\sin 3x$$

$$y_2 = -9(\cos 3x + \sin 3x)$$

$$y_2 = -9y \Rightarrow \boxed{y_2 + 9y = 0}$$ **Proved.**

16. If $y = Ae^{mx} + Be^{-mx}$, show that $y_2 - m^2y = 0$

Sol. $y = Ae^{mx} + Be^{-mx}$

Differentiate both sides w.r.t. 'x':

$$\frac{d}{dx}(y) = \frac{d}{dx}(Ae^{mx} + Be^{-mx})$$

$$y_1 = Ae^{mx}(m) + Be^{-mx}(-m)$$

$$y_1 = Ame^{mx} - Bme^{-mx}$$

Differentiate both sides w.r.t. 'x':

$$\frac{d}{dx}(y_1) = \frac{d}{dx}(Ame^{mx} - Bme^{-mx})$$

$$y_2 = Ame^{mx}(m) - Bme^{-mx}(-m)$$

$$y_2 = Am^2e^{mx} + Bm^2e^{-mx}$$

$$y_2 = m^2(Ae^{mx} + Be^{-mx})$$

$$y_2 = m^2y \Rightarrow \boxed{y_2 - m^2y = 0}$$ **Proved.**

17. Find $\int (2x+9)^{-5/2}\,dx$

Sol. $\int (2x+9)^{-5/2}\,dx$

$$= \frac{1}{2}\int (2x+9)^{-5/2}(2)\,dx$$

$$= \frac{1}{2}\left[\frac{(2x+9)^{-3/2}}{-3/2}\right] + c \quad \left\{\begin{matrix}\text{using}\\ \text{Rule-I}\end{matrix}\right\}$$

$$= \boxed{-\frac{1}{3}(2x+9)^{-3/2} + c}$$

18. Find $\int \frac{1}{\sqrt[3]{(3x+4)^2}}dx$

Sol. $\int \frac{1}{\sqrt[3]{(3x+4)^2}}dx$

$$= \frac{1}{3}\int (3x+4)^{-2/3}(3)dx$$

$$= \frac{1}{3}\left[\frac{(3x+4)^{1/3}}{1/3}\right] + c \quad \left\{\begin{matrix}\text{using}\\ \text{Rule-I}\end{matrix}\right\}$$

$$= \boxed{(3x+4)^{1/3} + c}$$

19. Find $\int \left(x + \frac{1}{x}\right)^2 dx$

Sol. $\int \left(x + \frac{1}{x}\right)^2 dx$

$$= \int \left(x^2 + \frac{1}{x^2} + 2\right) dx$$

$$= \int \left(x^2 + x^{-2} + 2\right) dx$$

$$= \frac{x^3}{3} + \frac{x^{-1}}{-1} + 2x + c = \boxed{\frac{x^3}{3} - \frac{1}{x} + 2x + c}$$

20. Find $\int \left(\frac{1}{t^3} + \frac{1}{t^2} - 2\right) dt$

Sol. $\int \left(\frac{1}{t^3} + \frac{1}{t^2} - 2\right) dt$

$$= \int \left(t^{-3} + t^{-2} - 2\right) dt$$

$$= \frac{t^{-2}}{-2} + \frac{t^{-1}}{-1} - 2t + c$$

$$= \boxed{\frac{-1}{2t^2} - \frac{1}{t} - 2t + c}$$

21. Find $6\int x^2 e^{x^3} dx$

Sol. $6\int x^2 e^{x^3} dx$

$$= 6\int e^{x^3}(x^2)dx$$

$$= 6\int e^t \frac{dt}{3}$$

Put $x^3 = t$

$\frac{d}{dx}(x^3) = \frac{d}{dx}(t)$

$3x^2 = \frac{dt}{dx}$

$$= \frac{6}{3}\int e^t dt$$

$$= 2e^t + c$$

$$= \boxed{2e^{x^3} + c}$$

22. Find $\int \frac{\sin^{-1}x}{\sqrt{1-x^2}}dx$

Sol. $\int \frac{\sin^{-1}x}{\sqrt{1-x^2}}dx$

$$= \int \sin^{-1}x\left(\frac{1}{\sqrt{1-x^2}}\right)dx$$

$$= \boxed{\frac{(\sin^{-1}x)^2}{2} + c} \quad \left\{\begin{matrix}\text{using}\\ \text{Rule-I}\end{matrix}\right\}$$

23. Integrate $\int \frac{\cos(\ell nx)}{x}dx$

Sol. $\int \frac{\cos(\ell nx)}{x}dx$

$$= \int \cos(\ell nx).\left(\frac{1}{x}\right) dx$$

Put $\ell nx = t \Rightarrow \frac{d}{dx}(\ell nx) = \frac{d}{dx}(t)$

$\frac{1}{x} = \frac{dt}{dx} \Rightarrow \left(\frac{1}{x}\right)dx = dt$

$$= \int \cos t\, dt = \sin t + c = \boxed{\sin(\ell nx) + c}$$

24. Find $\int \left(\frac{x-1}{x^2-2x+3}\right)dx$

Sol. $\int \left(\frac{x-1}{x^2-2x+3}\right)dx$

$$=\frac{1}{2}\int\frac{2x-2}{(x^2-2x+3)}dx \quad \because \boxed{\frac{d}{dx}(x^2-2x+3)=2x-2}$$

$$=\boxed{\frac{1}{2}\ln(x^2-2x+3)+c} \quad \{\text{using Rule-II}\}$$

25. Evaluate $\int_0^{\pi/2}\frac{\cos x}{3+4\sin x}\,dx$

Sol. $\int_0^{\pi/2}\frac{\cos x}{3+4\sin x}\,dx$

$$=\frac{1}{4}\int_0^{\pi/2}\frac{4\cos x}{3+4\sin x}dx$$

$$=\frac{1}{4}\left[\ln(3+4\sin x)\right]_0^{\pi/2} \quad \{\text{using Rule-II}\}$$

$$=\frac{1}{4}\left[\ln\left(3+4\sin\left(\pi/2\right)\right)-\ln(3+4\sin(0))\right]$$

$$=\frac{1}{4}\left[\ln(3+4\sin 90^\circ)-\ln(3+4\sin 0^\circ)\right]$$

$$=\frac{1}{4}\left[\ln(3+4(1))-\ln(3+4(0))\right]$$

$$=\frac{1}{4}\left[\ln(3+4)-\ln(3+0)\right]$$

$$=\frac{1}{4}\left[\ln(7)-\ln(3)\right]=\boxed{\frac{1}{4}\ln\left(\frac{7}{3}\right)}$$

26. Evaluate $\int_0^{\pi/6}(2\sin 2x)\,dx$

Sol. $\int_0^{\pi/6}(2\sin 2x)dx$

$$=2\left[-\frac{\cos 2x}{2}\right]_0^{\pi/6}$$

$$=-\left[\cos 2x\right]_0^{\pi/6}$$

$$=-\left(\cos 2\left(\frac{\pi}{6}\right)-\cos 2(0)\right)$$

$$=-\left(\cos 2(30^\circ)-\cos 2(0^\circ)\right)$$

$$=-\left(\cos 60^\circ-\cos 0^\circ\right)$$

$$=-\left(\frac{1}{2}-1\right)=-\left(\frac{1-2}{2}\right)=-\left(-\frac{1}{2}\right)=\boxed{\frac{1}{2}}$$

27. Evaluate $\int_{-\pi/2}^{\pi/2}(\cos x)\,dx$

Sol. $\int_{-\pi/2}^{\pi/2}(\cos x)dx$

$$=\left[\sin x\right]_{-\pi/2}^{\pi/2}$$

$$=\sin\left(\frac{\pi}{2}\right)-\sin\left(-\frac{\pi}{2}\right)$$

$$=\sin(90^\circ)-\sin(-90^\circ)\left\{\begin{array}{l}\frac{\pi}{2}\times\frac{180}{\pi}=90^\circ\\ -\frac{\pi}{2}\times\frac{180}{\pi}=-90^\circ\end{array}\right\}$$

$$=1-(-1)=1+1=\boxed{2}$$

28. Evaluate $\int_0^{\pi/6}(\sec^2 x)\,dx$

Sol. $\int_0^{\pi/6}(\sec^2 x)dx$

$$=\left[\tan x\right]_0^{\pi/6}$$

$$=\tan\left(\frac{\pi}{6}\right)-\tan(0)$$

$$=\tan(30^\circ)-\tan(0^\circ)$$

$$=\frac{1}{\sqrt{3}}-0=\boxed{\frac{1}{\sqrt{3}}}$$

29. Write distance formula between two points and give one example.

Sol. Let $A(x_1, y_1)$ & $B(x_2, y_2)$ be two different points, then,

$$\text{Distance}=|AB|=\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}$$

Example:

Let A(0, 0), B(1, 1) be two points, then

$$|\overline{AB}|=\sqrt{(0-1)^2+(0-1)^2}$$

$$|\overline{AB}|=\sqrt{1+1}=\sqrt{2}$$

30. Find the distance between the points $(-3\ ,1)$ and $(3\ ,-2)$.

Sol. Distance between $(-3\ ,1)$ & $(3\ ,-2)$.

$$= \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

$$= \sqrt{(-3-3)^2 + (1-(-2))^2}$$

$$= \sqrt{(-6)^2 + (3)^2} = \sqrt{36+9}$$

$$= \sqrt{45} = \sqrt{9\times 5} = \boxed{3\sqrt{5}}$$

31. **Show that the points $A(-1, -1)$, $B(4, 1)$ and $C(12, 4)$ lies on a straight line.**

Sol. $\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = \begin{vmatrix} -1 & -1 & 1 \\ 4 & 1 & 1 \\ 12 & 4 & 1 \end{vmatrix}$

$$= -1\begin{vmatrix} 1 & 1 \\ 4 & 1 \end{vmatrix} - (-1)\begin{vmatrix} 4 & 1 \\ 12 & 1 \end{vmatrix} + 1\begin{vmatrix} 4 & 1 \\ 12 & 4 \end{vmatrix}$$

$$= -1(1-4) + 1(4-12) + 1(16-12)$$

$$= -1(-3) + 1(-8) + 1(4)$$

$$= 3 - 8 + 4 = \boxed{-1} \neq 0$$

Hence given points do not lie on a straight line.

32. **Find the coordinates of the mid-point of the segment $P_1(3, 7)$ and $P_2(-2, 3)$.**

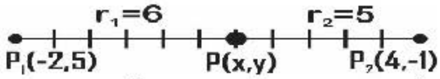
Sol. $\text{Mid-point} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

$$= \left(\frac{3+(-2)}{2}, \frac{7+3}{2}\right) = \left(\frac{3-2}{2}, \frac{10}{2}\right) = \boxed{\left(\frac{1}{2}, 5\right)}$$

33. ~~**Find the coordinates of the point $P(x, y)$ which divide internally the segment through $P_1(-2, 5)$ and $P_2(4, -1)$ of the ratio of $\frac{r_1}{r_2} = \frac{6}{5}$.**~~

Sol. Here: $r_1 = 6,\ r_2 = 5,\ (x_1, y_1) = (-2, 5)$

& $(x_2, y_2) = (4, -1)$



$$P(x, y) = \left(\frac{r_1 x_2 + r_2 x_1}{r_1 + r_2}, \frac{r_1 y_2 + r_2 y_1}{r_1 + r_2}\right)$$

$$= \left(\frac{6(4)+5(-2)}{6+5}, \frac{6(-1)+5(5)}{6+5}\right)$$

$$= \left(\frac{24-10}{11}, \frac{-6+25}{11}\right) = \boxed{\left(\frac{14}{11}, \frac{19}{11}\right)}$$

34. **Find the equation of circle with center on origin and radius is $\frac{1}{2}$.**

Sol. Standard form of equation of circle:

$$(x-h)^2 + (y-k)^2 = r^2$$

Put $h = 0,\ k = 0\ \&\ r = \frac{1}{2}$

$$(x-0)^2 + (y-0)^2 = \left(\frac{1}{2}\right)^2$$

$$\boxed{x^2 + y^2 - \frac{1}{4} = 0}$$

35. **Find center and radius of the circle $x^2 + y^2 + 9x - 7y - 33 = 0$**

Sol. Comparing with general equation of circle.

$$x^2 + y^2 + 9x - 7y - 33 = 0$$

$2g = 9$	$2f = -7$	$c = -33$
$g = \frac{9}{2}$	$f = -\frac{7}{2}$	

Center $= (-g, -f) =$

$$\text{Center} = \left(-\frac{9}{2}, -\left(-\frac{7}{2}\right)\right) = \boxed{\left(-\frac{9}{2}, \frac{7}{2}\right)}$$

$$\text{Radius} = r = \sqrt{g^2 + f^2 - c}$$

$$r = \sqrt{\left(\frac{9}{2}\right)^2 + \left(-\frac{7}{2}\right)^2 - (-33)}$$

$$r = \sqrt{\frac{81}{4} + \frac{49}{4} + 33} = \sqrt{\frac{81+49+132}{4}}$$

$$r = \sqrt{\frac{262}{4}} = \boxed{\sqrt{\frac{131}{2}}}$$

36. **Find the center and radius of the circle $6x^2+6y^2-18y=0$**

Sol. $6x^2+6y^2-18y=0$

Dividing each term by 6, we get:

$x^2+y^2-3y=0$

Comparing with general form:

$x^2+y^2+2gx+2fy+c=0$

$2g=0$	$2f=-3$	$c=0$
$g=0$	$f=-3/2$	

Center $=(-g,-f)$

Center $=\left(0,-\left(-\frac{3}{2}\right)\right)=\boxed{\left(0,3/2\right)}$

Radius $=r=\sqrt{g^2+f^2-c}$

$r=\sqrt{(0)^2+\left(-\frac{3}{2}\right)^2-0}=\sqrt{\frac{9}{2}}=\boxed{\frac{3}{2}}$

37. **What type of circle is represented by $x^2+y^2-2x+4y+8=0$**

Sol. $x^2+y^2-2x+4y+8=0$

Comparing this equation with general form of equation of circle:

$x^2+y^2+2gx+2fy+c=0$

$2g=-2$	$2f=4$	
$g=-\frac{2}{2}=-1$	$f=\frac{4}{2}=2$	$c=8$

Radius $=r=\sqrt{g^2+f^2-c}$

$r=\sqrt{(-1)^2+(2)^2-8}$

$r=\sqrt{1+4-8}=\sqrt{-3}=\boxed{\sqrt{3}i}$

So, it is an **Imaginary circle**.

Section - II

Note: Attemp any three (3) questions $\boxed{3\times10=30}$

Q.2.[a] If $f(x)=\frac{x-1}{x+1}$, show that;

$$\frac{f(x)-f(y)}{1+f(x)f(y)}=\frac{x-y}{1+xy}$$

Sol. See Q.9 of Ex # 1.1 (Page # 7)

[b] ~~**Differentiate $x^{2/3}$ by ab-initio method.**~~

Sol. See Q.1(iv) of Ex # 2.1 (Page # 38)

Q.3.[a] **Differentiate $\cos 2x$ from first principle method.**

Sol. See Q.1(iii) of Ex # 3.1 (Page # 109)

[b] **Use differentials to find the approximate value of $\sqrt{65}$.**

Sol. See Q.6(i) of Ex # 4.1 (Page # 174)

Q.4.[a] **Integrate $\int\left(\frac{1}{\sqrt{1+x}-\sqrt{x}}\right)dx$**

Sol. See Q.16 of Ex # 5.1 (Page # 232)

[b] ~~**Evaluate $\int\frac{dx}{\sqrt{a^2-x^2}}$**~~

Sol. See example # 9 of Chapter 06.

Q.5.[a] **Calculate the integral**

$$\int_0^3 \sqrt[3]{(3x-1)^2}\,dx$$

Sol. See Q.1(iv) of Ex # 7.1 (Page # 319)

[b] **Find which of the two circles $x^2+y^2-3x+4y=0$ and $x^2+y^2-6x-8y=0$ is greater.**

Sol. See Q.7 of Ex # 9 (Page # 446)

Q.6.[a] **If a line ℓ_1 contains P(2, 6) and Q(0, y). Find 'y' if ℓ_1 is parallel to ℓ_2 and that the slope of $\ell_2=\frac{3}{4}$.**

Sol. See Q.2 of Ex # 8.3 (Page # 374)

[b] ~~**Find an equation of the line which is perpendicular to the line 4x + 7y = 5 and passes through (-1, 2).**~~

Sol. See Q.10 of Ex # 8.4 (Page # 389)

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