DAE / IIA - 2018

MATH - 233 APPLIED MATHEMATICS - II

PAPER 'B' PART - A (OBJECTIVE)

Time : 30 Minutes Marks : 15

Q.1: *Encircle the correct answer.*

1. $\int x^3 dx = ?$

[a] $\frac{x^4}{4}$ [b] $\frac{x^4}{3}$ [c] $3x^2$ [d] $4x^4$

2. $\int (e^{2x}) dx = ?$

[a] $\frac{e^{2x}}{2}$ [b] $\frac{e^{x2}}{2}$ [c] $2e^{2x}$ [d] $\frac{e^{2x+1}}{2}$

3. $\int \left(\frac{\cos x}{\sin x}\right) dx = ?$

[a] $\ell n \cos x$ [b] $\ell n \sin x$

[c] $\ell n \cot x$ [d] $\frac{\cos^2 x}{2}$

4. $\int (x \sec^2 x) dx = ?$

[a] $x \tan x$ [b] $x \tan x + \ell n \sec x$

[c] $\tan x$ [d] $x \tan x - \ell n \sec x$

5. ~~$\int (xe^x) dx = ?$~~

[a] $xe^x + e^x$ [b] $xe^x - e^x$

[c] e^x [d] $\frac{x^2}{2} e^x$

6. $\int (ax+b)^3 dx = ?$

[a] $3(ax+b)^2$ [b] $3a(ax+b)^2$

[c] $\frac{(ax+b)^3}{4a}$ [d] $\frac{(ax+b)^4}{4a}$

7. $\int_0^1 (1) dx = ?$ [a] -1 [b] 0 [c] 1 [d] 2

8. $\int_0^{\pi/4} (\sec^2 x) dx = ?$

[a] 1 [b] 2 [c] 0 [d] 3

9. If $xdy + ydx = 0$ is the differentiation, then its variables separable form is:

[a] $ydy + xdx = 0$ [b] $\frac{1}{y}dy = \frac{1}{x}dx$

[c] $\frac{1}{2}dy = -\frac{1}{x}$ [d] $xdy = -ydx$

10. $y = \ell n x + c$ is the solution of differential equation:

[a] $xdy = dx$ [b] $xdx = dy$

[c] $dy = \frac{1}{x}$ [d] $dy = dx$

11. The series $\frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx)$ is:

[a] Binomial [b] Fourier

[c] Arithmetic [d] Geometric

12. If a function $f(-x) = -f(x)$ then function is:

[a] Even [b] Odd

[c] Linear [d] Constant

13. $L\{f(t)\}$ (the Laplace Transform of $f(t)$) is:

[a] $\int_0^\infty f(t)e^{-st}.dt$ [b] $\int_0^\infty f(t)e^{st}.dt$

[c] $\int_0^\infty f(t)e^{t}.dt$ [d] $\int_0^\infty f(t)e^{s}.dt$

14. $L\{1\} = ?$

[a] $\frac{1}{s^3}$ [b] $\frac{1}{s^2}$ [c] $\frac{1}{s}$ [d] $-\frac{1}{s}$

15. Laplace transform of the function $f(t) = t$ is:

[a] $\frac{1}{s}$ [b] $\frac{1}{s^2}$ [c] $\frac{1}{s^3}$ [d] $-\frac{1}{s}$

Answer Key

1	a	2	a	3	b	4	d	5	b
6	a	7	c	8	a	9	c	10	a
11	b	12	b	13	a	14	c	15	b

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DAE / IIA - 2018

MATH - 233 APPLIED MATHEMATICS - II

PAPER 'B' PART - B (SUBJECTIVE)

Time : 2 : 30Hrs Marks : 60

Section - I

Q.1. Write short answers to any Eighteen (18) questions.

1. Find $\int (2x+9)^{-5/2} dx$

Sol. $\int (2x+9)^{-5/2} dx$

$= \frac{1}{2}\int (2x+9)^{-5/2}(2)dx$

$= \frac{1}{2}\left[\frac{(2x+9)^{-3/2}}{-3/2}\right] + c \quad \{\text{using Rule-I}\}$

$= \boxed{-\frac{1}{3}(2x+9)^{-3/2} + c}$

2. Find $\int \frac{1}{\sqrt[3]{(3x+4)^2}} dx$

Sol. $\int \frac{1}{\sqrt[3]{(3x+4)^2}} dx$

$= \frac{1}{3}\int (3x+4)^{-2/3}(3)dx$

$= \frac{1}{3}\left[\frac{(3x+4)^{1/3}}{1/3}\right] + c \quad \{\text{using Rule-I}\}$

$= \boxed{(3x+4)^{1/3} + c}$

3. Find $\int \left(x+\frac{1}{x}\right)^2 dx$

Sol. $\int \left(x+\frac{1}{x}\right)^2 dx$

$= \int \left(x^2+\frac{1}{x^2}+2\right) dx$

$= \int \left(x^2+x^{-2}+2\right) dx$

$= \frac{x^3}{3}+\frac{x^{-1}}{-1}+2x+c$

$= \boxed{\frac{x^3}{3}-\frac{1}{x}+2x+c}$

4. Find $\int \left(\frac{1}{t^3}+\frac{1}{t^2}-2\right) dt$

Sol. $\int \left(\frac{1}{t^3}+\frac{1}{t^2}-2\right) dt$

$= \int \left(t^{-3}+t^{-2}-2\right) dt$

$= \frac{t^{-2}}{-2}+\frac{t^{-1}}{-1}-2t+c$

$= \boxed{\frac{-1}{2t^2}-\frac{1}{t}-2t+c}$

5. Find $\int \left(\frac{-2x}{\sqrt{4-x^2}}\right) dx$

Sol. $\int \left(\frac{-2x}{\sqrt{4-x^2}}\right) dx$

$= \int (4-x^2)^{-1/2}(-2x)\, dx$

$= \frac{(4-x^2)^{1/2}}{1/2} + c \quad \{\text{using Rule-I}\}$

$\frac{d}{dx}(4-x^2) = 0-2x = -2x$

$= \boxed{2\sqrt{4-x^2}+c}$

6. Find $\int \left(\frac{x^2+1}{x+1}\right) dx$

Sol. $\int \left(\frac{x^2+1}{x+1}\right) dx$

$$\begin{array}{r} x-1 \\ x+1\overline{)\,x^2+1} \\ \underline{\pm x^2 \pm x} \\ -x+1 \\ \underline{\mp x \mp 1} \\ 2 \end{array}$$

$$= \int\left(x - 1 + \frac{2}{x+1}\right)dx$$

$$= \boxed{\frac{x^2}{2} - x + 2\,\ell n(x+1) + c}$$

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7. Find $\int\left(1+\frac{3}{x^2}\right)^2 dx$

Sol. $\int\left(1+\frac{3}{x^2}\right)^2 dx$

$$= \int\left[(1)^2 + 2(1)\left(\frac{3}{x^2}\right) + \left(\frac{3}{x^2}\right)^2\right]dx$$

$$= \int\left(1 + \frac{6}{x^2} + \frac{9}{x^4}\right)dx$$

$$= \int\left(1 + 6x^{-2} + 9x^{-4}\right)dx$$

$$= x + \frac{6x^{-1}}{-1} + \frac{9x^{-3}}{-3} + c$$

$$= \boxed{x - \frac{6}{x} - \frac{3}{x^3} + c}$$

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8. Evaluate $\int \frac{dx}{x(1+\ell nx)}$

Sol. $\int \frac{dx}{x(1+\ell nx)}$

$$= \int \frac{1}{(1+\ell nx)} \cdot \frac{1}{x} . dx$$

$$= \boxed{\ell n(1+\ell nx) + c}$$

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9. Evaluate $\int(x\cos x)dx$

Sol. $\int(x\cos x)dx$

Integrating by parts:

taking $u = x$ & $v = \cos x$

$$= x\int \cos x dx - \int\left[\frac{d}{dx}(x)\int \cos x dx\right]dx$$

$$= x(\sin x) - \int 1.(\sin x)dx$$

$$= x\sin x - \int \sin x dx$$

$$= x\sin x - (-\cos x) + c$$

$$= \boxed{x\sin x + \cos x + c}$$

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10. Evaluate $\int(x\sec^2 x)dx$

Sol. $\int(x\sec^2 x)dx$

Integrating by parts:

taking $u = x$ & $v = \sec^2 x$

$$= x\int \sec^2 x dx - \int\left[\frac{d}{dx}(x)\int \sec^2 x dx\right]dx$$

$$= x\tan x - \int(1 \cdot \tan x)dx$$

$$= x\tan x - \int(\tan x)dx$$

$$= \boxed{x\tan x - \ell n\sec x + c}$$

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11. Evaluate $\int(\ell n\, x)dx$

Sol. $\int(\ell n\, x)dx$

$$= \int(\ell n x \cdot 1)dx$$

Integrating by parts:

taking $u = \ell n x$ & $v = 1$

$$= \ell n x\int(1)dx - \int\left[\frac{d}{dx}(\ell n x)\int(1)dx\right]dx$$

$$= \ell n x(x) - \int \frac{1}{x}.(x)dx$$

$$= x\ell n x - \int(1)dx$$

$$= x\ell n x - (x) + c$$

$$= x\ell n x - x + c = \boxed{x(\ell n x - 1) + c}$$

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12. Evaluate $\int(x\ell nx)\,dx$

Sol. $\int(x\ell nx)\,dx$

$$= \int \ell nx. x\,dx$$

Integrating by parts:

taking $u = \ell nx$ & $v = x$

$$= \ell nx\int x\,dx - \int\left\{\frac{d}{dx}(\ell nx)\int x\ dx\right\}dx$$

$$= \ell nx.\frac{x^2}{2} - \int\frac{1}{x}.\frac{x^2}{2}dx$$

$$= \frac{x^2}{2}\ell nx - \frac{1}{2}\int x\,dx$$

$$= \frac{x^2}{2}\ell nx - \frac{1}{2}.\frac{x^2}{2} + c$$

$$= \boxed{\frac{x^2}{2}\ell nx - \frac{1}{4}x^2 + c}$$

13. Find $\int(x.\sin x)\,dx$

Sol. $\int(x.\sin x)\,dx$

Integrating by parts:

taking $u = x$ & $v = \sin x$

$$= x\int \sin x dx - \int\left\{\frac{d}{dx}(x)\int \sin x dx\right\}dx$$

$$= x(-\cos x) - \int 1.(-\cos x)dx$$

$$= -x\cos x + \int \cos x dx$$

$$= \boxed{-x\cos x + \sin x + c}$$

14. Evaluate $\int_0^3 \sqrt[3]{(3x-1)^2}\,dx$

Sol. $\int_0^3 \sqrt[3]{(3x-1)^2}\,dx$

$$= \int_0^3 (3x-1)^{2/3}\ dx$$

$$= \frac{1}{3}\int_0^3 (3x-1)^{2/3}(3)\ dx$$

$$= \frac{1}{3}\left[\frac{(3x-1)^{5/3}}{5/3}\right]_0^3 \quad \left\{\begin{matrix}\text{using}\\ \text{Rule-I}\end{matrix}\right\}$$

$$= \frac{1}{3}\times\frac{3}{5}\left[(3x-1)^{5/3}\right]_0^3$$

$$= \frac{1}{5}\left[(3(3)-1)^{5/3} - (3(0)-1)^{5/3}\right]$$

$$= \frac{1}{5}\left[(8)^{5/3} - (-1)^{5/3}\right]$$

$$= \frac{1}{5}\left[32 - (-1)\right] = \boxed{\frac{33}{5}}$$

15. Evaluate $\int_1^3 \frac{1}{x+1}dx$

Sol. $\int_1^3 \frac{1}{x+1}dx$

$$= \left[\ell n(x+1)\right]_1^3 \quad \left\{\begin{matrix}\text{using}\\ \text{Rule-II}\end{matrix}\right\}$$

$$= \ell n(3+1) - \ell n(1+1)$$

$$= \ell n(4) - \ell n(2) = \ell n\left(\frac{4}{2}\right) = \boxed{\ell n2}$$

16. Evaluate $\int_{\pi/6}^{\pi/3}(\sin 2x)dx$

Sol. $\int_{\pi/6}^{\pi/3}(\sin 2x)dx$

$$= \left[\frac{-\cos 2x}{2}\right]_{\pi/6}^{\pi/3} = -\frac{1}{2}\left[\cos 2x\right]_{\pi/6}^{\pi/3}$$

$$= -\frac{1}{2}\left[\cos 2\left(\frac{\pi}{3}\right) - \cos 2\left(\frac{\pi}{6}\right)\right]$$

$$= -\frac{1}{2}\left[\cos 120^\circ - \cos 60^\circ\right]$$

$$= -\frac{1}{2}\left[-\frac{1}{2} - \frac{1}{2}\right] = -\frac{1}{2}[-1] = \boxed{1/2}$$

17. Evaluate $\int_{\pi/6}^{\pi/3}\left(\operatorname{cosec}^2 x\right)dx$

Sol. $\int_{\pi/6}^{\pi/3}\left(\operatorname{cosec}^2 x\right)dx$

$$= -\left[\cot\ x\right]_{\pi/6}^{\pi/3}$$

$$= -\left[\cot\left(\frac{\pi}{3}\right) - \cot\left(\frac{\pi}{6}\right)\right]$$

$$= -\left[\cot 60° - \cot 30°\right]$$

$$= -\left[\frac{1}{\sqrt{3}} - \sqrt{3}\right] = -\left(\frac{1-\left(\sqrt{3}\right)^2}{\sqrt{3}}\right)$$

$$= -\left(\frac{1-3}{\sqrt{3}}\right) = -\left(\frac{-2}{\sqrt{3}}\right) = \boxed{2/\sqrt{3}}$$

18. Define differential equation and give example.

Sol. An equation involving derivatives or differentials is called a differential equation.

Example: $\frac{dy}{dx} + 2x = 0$

19. Define the Order of differential equation with example.

Sol. The order of a differential equation is the order of the highest derivative which appears in the differential equation.

Examples: $\frac{dy}{dx} + 2x = 0$ is a differential equation with order 1.

$$x^2\frac{dy}{dx^2} + x\frac{dy}{dx} + \left(x^2 - 4\right)y = 0$$

is a differential equation with order 2.

20. Define the Degree of differential equation with example.

Sol. The "Degree" of a differential equation is the highest power of the highest order derivative in the equation, after making it free from radicals and fractions.

Example: $\left(\frac{d^2y}{dx^2}\right)^2 + \frac{x^2}{dy/dx} = x$ has degree 3.

21. What is general solution of differential equation?

Sol. A solution which contains the same number of arbitrary constants as the order of the differential equation is called General Solution.

22. What the particular solution of differential equation?

Sol. A solution obtained from the general solution by giving particular numerical values to arbitrary constants, by applying the initial and boundary conditions, is called particular solution.

23. What is Laplace transformation of $\sin 7t$?

Sol. $L\{\sin 7t\}$

$$= \frac{7}{s^2 + (7)^2} = \boxed{\frac{7}{s^2 + 49}}$$

24. If $L\left\{e^{at}\right\} = \frac{1}{s-a}$ then what will be the Laplace transformation of e^{-4t}.

Sol. As, $L\left\{e^{at}\right\} = \frac{1}{s-a}$

Put $a = -4$, we have:

$$L\left\{e^{-4t}\right\} = \frac{1}{s-(-4)} = \boxed{\frac{1}{s+4}}$$

25. What is Laplace transformation of $\cos 6t$?

Sol. $L\{\cos 6t\}$

$$= \frac{s}{(s)^2 + (6)^2} = \boxed{\frac{s}{s^2 + 36}}$$

26. ~~Define inverse Laplace transformation.~~

Sol. $f(t)$ is called inverse Laplace transformation of $F(S)$ and written as: $f(t)=L^{-1}\{F(S)\}$

27. ~~What is the most important method to find the inverse Laplace transformation of function?~~

Sol. The partial fractions method is most important method to find inverse Laplace transformation.

Section - II

Note : Attemp any three (3) questions $3 \times 8 = 24$

Q.2.[a] Evaluate $\int \frac{\sin\sqrt{x}}{\sqrt{x}\cos\sqrt{x}}dx$

Sol. See Q.2(x) of Ex # 7.3 (Page # 340)

[b] Evaluate $\int(\sin^3 x\cos x)dx$

Sol. See example # 21 of Chapter 07.

Q.3.[a] Evaluate

$$\int(\cot^3 2x\ \mathrm{cosec}^3\ 2x)dx$$

Sol. See Q.4(vi) of Ex # 8.1 (Page # 325)

[b] ~~Evaluate $\int(\sin^{-1}x)dx$.~~

Sol. See Q.2(iv) of Ex # 8.3 (Page # 343)

Q.4.[a] Evaluate $\int_0^a \frac{dx}{\sqrt{x+a}+\sqrt{x}}$

Sol. See Q.1(viii) of Ex # 9.1 (Page # 377)

[b] ~~Show that area of a circle of radius r is πr^2.~~

Sol. See example # 19 of Chapter 09.

Q.5.[a] Find the general solution of equation

$$ydx - xdy = x(dy - ydx)$$

Sol. See Q.4 of Ex # 10 (Page # 412)

[b] Find the particular solution of:

$dy = x(2ydx - xdy)$ subject to the conditions $x = 1,\ y = 4$

Sol. See example # 3 of Chapter 10.

Q.6. ~~Expand the Fourier Series;~~

$$f(x)=\begin{cases}1, & 0 \le x \le \pi \\ 2, & \pi \le x \le 2\pi\end{cases} \quad \text{Period } 2\pi$$

Sol. See Q.2(i) of Ex# 11 (Page # 441)

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