EDUGATE Up to Date Solved Papers 35 Applied Mathematics-II (MATH-233) Paper A

DAE/IIA - 2018

MATH-233 APPLIED MATHEMATICS-II

PAPER 'A' PART - A (OBJECTIVE)

Time: 30 Minutes

Marks:15

Q.1: Encircle the correct answer.

- A function $f(x) = x^2 + 2x + 3$ is:
 - [a] Odd
- [b] Even
- [c] Implicit
- [d] Explicit
- If f(x) = 2(x-1)+3, then f(2) = ?2.
 - [a] 0 [b] 1 [c] 3 [d] 5

- $\frac{\mathrm{d}}{\mathrm{d}x}(2x+3)^4=?$ 3.
 - [c] $(2x+3)^3$ [d] $4(2x+3)^2$ [a] $8(2x+3)^3$ [b] $4(2x+3)^3$
- $\frac{d}{dx}\left(\frac{u}{c}\right) = ?$

 - [a] $\frac{du}{dx}$ [b] $\frac{1}{c} \frac{du}{dx}$
 - [c] $c \frac{d}{dx} \left(\frac{u}{c} \right)$ [d] 0
- $\frac{\mathbf{d}}{\mathbf{d}\mathbf{v}}\left(\frac{1}{\mathbf{v}}\right) = ?$ 5.
 - [a] $\frac{1}{v^2}$ [b] $-\frac{1}{v^2}$ [c] $-\frac{1}{v^3}$ [d] $\frac{2}{v}$
- $\frac{d}{dx}(\sin x^3) = ?$ 6.

- [a] $\cos x^3$ [b] $-\cos x^3$ [c] $3x \cos x^3$ [d] $3x^2 \cos x^3$
- $\frac{d}{dx} \left| \cos \left(\frac{1}{x} \right) \right| = ?$ 7.
 - [a] $-\sin\left(\frac{1}{x}\right)$ [b] $\sin\left(\frac{1}{x}\right)$
 - [c] $\frac{1}{\mathbf{x}^2} \sin\left(\frac{1}{\mathbf{x}}\right)$ [d] $-\frac{1}{\mathbf{x}^2} \sin\left(\frac{1}{\mathbf{x}}\right)$
- $\frac{d}{dx}(\sin ax + \cos ax) = ?$ 8.

- [a] $a\cos x + a\sin x$
- [b] $a\cos x a\sin x$
- [c] $\cos ax + \sin ax$
- [d] a cos ax sin ax
- $\frac{d}{dx}(\cos ec3x) =$ 5.
 - [a] $-\cos ec3x \cot 3x$
 - [b] $-3\cos ec3x \cot 3x$
 - [c] cot 3x
- [d] $\cos ec3x$
- For an increasing function $\frac{dy}{dz}$ is: 10.
 - [a] + ve [b] ve [c] zero [d] None
- For a decreasing function $\frac{dy}{dz}$ is: 11.
 - [a] +ve [b] -ve [c] zero [d] None
- A set of data is called:
 - [a] Continuous data
 - [b] Discontinuous
 - [c] Population [d] Sample
- 13. The difference between the lower and upper class boundaries is called:
 - [a] Common difference
 - [b] size or length of interval
 - [c] Difference operator
 - [d] Size of the table
- 14. A process of obtaining an observation is called:
 - [a] An experiment [b] Trail
 - [c] Out come [d] An event
- 15. The result obtained from an experiment or a trial is called:
 - [a] Sample space [b] An event
 - [c] Out come [d] Population

Answer Key

1	d	2	d	3	a	4	d	5	b
6	d	7	c	8	d	9	b	10	a
11	b	12	d	13	b	14	а	15	c

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DAE/IIA-2018

MATH-233 APPLIED MATHEMATICS-II

PAPER 'B' PART - B (SUBJECTIVE)

Time:2:30Hrs

Marks: 60

Section - I

- Q.1. Write short answers to any Eighteen (18) questions.
- 1. If $f(x)=3x^2-7x+4$, then find $f\left(\frac{1}{x}\right)$.
- **Sol.** As, $f(x) = 3x^2 7x + 4$

Replace 'x' by ' $\frac{1}{x}$ ', we have:

$$f\left(\frac{1}{x}\right) = 3\left(\frac{1}{x}\right)^{2} - 7\left(\frac{1}{x}\right) + 4$$

$$= \frac{3}{x^{2}} - \frac{7}{x} + 4 = \boxed{\frac{3 - 7x + 4x^{2}}{x^{2}}}$$

- 2. If $f(x)=2x\sqrt{1-x^2}$, then find $f(\sin \theta)$
- **Sol.** As, $f(x) = 2x\sqrt{1-x^2}$

Put $x = \sin \theta$, we have :

$$f(\sin \theta) = 2\sin \theta \sqrt{1 - \sin^2 \theta}$$
$$= 2\sin \theta \sqrt{\cos^2 \theta} = 2\sin \theta \cos \theta = \sin 2\theta$$

- 3. If $f(x) = \frac{2x}{1+x^2}$, then find $f(\tan A)$.
- Sol. As, $f(x) = \frac{2x}{1+x^2}$ Put $x = \tan A$, we have: $f(\tan A) = \frac{2 \tan A}{1+\tan^2 A}$

$$= \frac{2 \tan A}{\sec^2 A} = 2 \frac{\sin A}{\cos A} \cdot \cos^2 A$$

$$= 2\sin A\cos A = \sin 2A$$

- 4. If $f(x) = \frac{1}{1-x}$, then find f[f(5)]
- **Sol.** As, $f(x) = \frac{1}{1-x} \to (i)$

Put x = 5 in eq.(i)

$$f(5) = \frac{1}{1-5} = \frac{1}{-4} = -\frac{1}{4}$$

Put x = f(5) in eq.(i)

$$\mathbf{f}\left[\mathbf{f}\left(5\right)\right] = \frac{1}{1 - \left(-\frac{1}{4}\right)}$$

$$=\frac{1}{\frac{4+1}{4}} = \frac{1}{\frac{5}{4}} = \boxed{\frac{4}{5}}$$

- 5. Differentiate with respect to x by ab-initio x².
- **Sol.** Let, $y = x^2 \rightarrow (i)$

Step-I: then $y + \delta y = (x + \delta x)^2 \rightarrow (ii)$

Step-II: Subtracting eq.(i) from eq.(ii), we have:

$$y + \delta y - y = (x + \delta x)^2 - x^2$$

$$\delta y = x^2 + 2x\delta x + \delta x^2 - x^2$$

$$\delta y = 2x\delta x + \delta x^2$$

$$\delta y = \delta x (2x + \delta x)$$

Step-III: Dividing both sides by ' δx ':

$$\frac{\delta y}{\delta x} = 2x + \delta x$$

Step-IV: Taking limit $\delta x \rightarrow 0$ on both sides:

$$\underset{\delta x \to 0}{\text{Lim}} \frac{\delta y}{\delta x} = \underset{\delta x \to 0}{\text{Lim}} \left(2x + \delta x\right)$$

$$\frac{\mathrm{dy}}{\mathrm{dx}} = 2x + 0 = \boxed{2x}$$

6. Differentiate with respect to x by ab-initio x³.

Sol. Let, $y = x^3 \rightarrow (i)$

Step-I: then
$$y + \delta y = (x + \delta x)^3 \rightarrow (ii)$$

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Step-II: Subtracting eq.(i) from eq.(ii), we have:

$$y + \delta y - y = (x + \delta x)^3 - x^3$$
$$\delta y = x^3 + 3x^2 \delta x + 3x \delta x^2 + \delta x^3 - x^3$$
$$\delta y = 3x^2 \delta x + 3x \delta x^2 + \delta x^3$$
$$\delta y = \delta x (3x^2 + 3x \delta x + \delta x^2)$$

Step-III: Dividing both sides by ' δx ':

$$\frac{\delta y}{\delta x} = 3x^2 + 3x\delta x + \delta x^2$$

Step-IV: Taking limit $\delta x \rightarrow 0$ on both sides:

$$\lim_{\delta x \to 0} \frac{\delta y}{\delta x} = \lim_{\delta x \to 0} \left(3x^2 + 3x\delta x + \delta x^2 \right)$$

$$\frac{dy}{dx} = 3x^2 + 3x(0) + (0)^2$$

$$\frac{dy}{dx} = 3x^2 + 0 + 0 = \boxed{3x^2}$$
Sol. $y = \sqrt{x} - \frac{1}{\sqrt{x}}$

Differentiate 1 w.r.t. 'x' by 1st 7.

Sol. Let,
$$y = \frac{1}{x^2} \rightarrow (i)$$

Step-I: then
$$y + \delta y = \frac{1}{(x + \delta x)^2} \rightarrow (ii)$$

Step-II: Subtracting eq.(i) from eq.(ii), we have:

$$y+\delta y-y=\frac{1}{\left(x+\delta x\right)^{2}}-\frac{1}{x^{2}}$$

$$\delta y = \frac{x^2 - (x + \delta x)^2}{(x + \delta x)^2 x^2} \left\{ \text{taking L.C.M.} \right\}$$

$$\delta y = \frac{x^2 - (x^2 + 2x\delta x + \delta x^2)}{x^2 (x + \delta x)^2}$$

$$\delta y = \frac{x^{2} - x^{2} - 2x\delta x - \delta x^{2}}{x^{2} (x + \delta x)^{2}}$$

$$\delta y = \frac{\delta x \left(-2x - \delta x\right)}{x^2 \left(x + \delta x\right)^2}$$

Step-III: Dividing both sides by ' δx ':

$$\frac{\delta y}{\delta x} = \frac{-2x - \delta x}{x^2 (x + \delta x)^2}$$

Step-IV: Taking limit $\delta x \rightarrow 0$ on both sides:

$$\lim_{\delta x \to 0} \frac{\delta y}{\delta x} = \lim_{\delta x \to 0} \left(\frac{-2x - \delta x}{x^2 (x + \delta x)^2} \right)$$

$$\frac{dy}{dx} = \frac{-2x - 0}{x^2 (x + 0)^2} = \frac{-2x}{x^4} = \boxed{-\frac{2}{x^3}}$$

8. If
$$y = \sqrt{x} - \frac{1}{\sqrt{x}}$$
, then show

that:
$$2x \frac{dy}{dx} + y = 2\sqrt{x}$$

Sol.
$$y = \sqrt{x} - \frac{1}{\sqrt{x}}$$

Differentiate both sides w.r.t. 'x':

$$\frac{\mathrm{d}}{\mathrm{d}x} \Big(y \Big) = \frac{\mathrm{d}}{\mathrm{d}x} \Bigg(\sqrt{x} - \frac{1}{\sqrt{x}} \Bigg)$$

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{d}}{\mathrm{d}x} \left(x^{\frac{1}{2}} - x^{-\frac{1}{2}} \right)$$

$$\frac{dy}{dx} = \frac{1}{2}x^{-\frac{1}{2}} - \left(-\frac{1}{2}\right)x^{-\frac{3}{2}}$$

$$\frac{dy}{dx} = \frac{1}{2} \left(x^{-1/2} + x^{-3/2} \right)$$

Now take: L.H.S. =
$$2x \frac{dy}{dx} + y$$

$$=2x\left[\frac{1}{2}\left(x^{-\frac{1}{2}}+x^{-\frac{3}{2}}\right)\right]+\sqrt{x}-\frac{1}{\sqrt{x}}$$

$$= x^{\frac{1}{2}} + x^{-\frac{1}{2}} + x^{\frac{1}{2}} - x^{-\frac{1}{2}}$$

$$=2x^{1/2}=\mathrm{R.H.S.}$$
 Proved.

$$= 2x^{\frac{1}{2}} = \text{R.H.S.} \qquad \text{Proved.}$$
9. If $y = x^2 + \frac{1}{x^2}$, then find $\frac{dy}{dx}$.

Sol.
$$y = x^2 + \frac{1}{x^2}$$

Differentiate both sides w.r.t. 'x':

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$$\frac{d}{dx}(y) = \frac{d}{dx}\left(x^2 + \frac{1}{x^2}\right)$$

$$\frac{dy}{dx} = \frac{d}{dx}\left(x^2 + x^{-2}\right)$$

$$\frac{dy}{dx} = 2x + (-2)x^{-3}$$

$$\frac{dy}{dx} = 2x - \frac{2}{x^3} \Rightarrow \boxed{\frac{dy}{dx} = \frac{2x^4 - 2}{x^3}}$$

Differentiate $(x + x^{-1})^2$ w.r.t. 'x'. 10.

Sol.
$$\frac{d}{dx} \left(x + x^{-1} \right)^{2}$$

$$= 2 \left(x + x^{-1} \right)^{2-1} \left(\frac{d}{dx} \left(x + x^{-1} \right) \right)$$

$$= 2 \left(x + x^{-1} \right) \left(1 + \left(-1 \right) x^{-2} \right)$$

$$= 2 \left(x + \frac{1}{x} \right) \left(1 - \frac{1}{x^{2}} \right)$$

$$= 2 \left(\frac{x^{2} + 1}{x} \right) \left(\frac{x^{2} - 1}{x^{2}} \right)$$

$$= 2 \left(\frac{\left(x^{2} \right)^{2} - \left(1 \right)^{2}}{x^{3}} \right) = 2 \left(\frac{x^{4} - 1}{x^{3}} \right)$$

Find the value of $\frac{d}{dx} \left(\frac{1 - \cos x}{\sin x} \right)$ 11.

Sol.
$$\frac{d}{dx} \left(\frac{1 - \cos x}{\sin x} \right) \left\{ \begin{array}{l} \underset{\text{product Rule}}{\text{sin } x} \right\}$$

$$= \frac{\sin x \left(\frac{d}{dx} (1 - \cos x) \right) - (1 - \cos x) \left(\frac{d}{dx} (\sin x) \right)}{\sin^2 x}$$

$$= \frac{\sin x \left(0 - (-\sin x) \right) - (1 - \cos x) (\cos x)}{\sin^2 x}$$

$$= \frac{\sin^2 x - \cos x + \cos^2 x}{\sin^2 x}$$

$$= \frac{1 - \cos x}{\sin^2 x} \quad \because \left\{ \sin^2 x + \cos^2 x = 1 \right\}$$

$$\begin{split} &= \frac{2\sin^{2}\frac{x}{2}}{\left(2\sin\frac{x}{2}\cos\frac{x}{2}\right)^{2}} \cdot \cdot \left\{\frac{\sin x}{\sin x}\right\} \\ &= \frac{2\sin^{2}\frac{x}{2}}{4\sin^{2}\frac{x}{2}\cos^{2}\frac{x}{2}} = \frac{1}{2\cos^{2}\frac{x}{2}} = \frac{1}{2\cos^{2}\frac{x}{2}} \end{split}$$

Differentiate cos² x w.r.t. sin² x.

Sol. Let, $y = \cos^2 x \& t = \sin^2 x$ Differentiate both equations

both sides w.r.t. 'x':

$$=2\left(x+x^{-1}\right)^{2-1}\left(\frac{d}{dx}\left(x+x^{-1}\right)\right)$$

$$=2\left(x+x^{-1}\right)\left(1+\left(-1\right)x^{-2}\right)$$

$$=2\left(x+\frac{1}{x}\right)\left(1-\frac{1}{x^{2}}\right)$$

$$=2\left(\frac{x^{2}+1}{x}\right)\left(\frac{x^{2}+1}{x^{2}}\right)$$

$$=2\left(\frac{\left(x^{2}\right)^{2}-\left(1\right)^{2}}{x^{3}}\right)$$

$$=2\left(\frac{\left(x^{2}\right)^{2}-\left(1\right)^{2}-\left(1\right)^{2}}{x^{3}}\right)$$

$$=2\left(\frac{\left(x^{2}\right)^{2}-\left(1\right)^{2}-\left(1\right)^{2}-\left(1\right)^{2}}{x^{3}}\right)$$

$$=2\left(\frac{$$

13. Find the derivative of $x \cot x$ w.r.t. 'x'

Sol.
$$\frac{d}{dx}(x \cot x)$$

$$= \left(\frac{d}{dx}(x)\right) \cot x + x \left(\frac{d}{dx}(\cot x)\right)$$

$$= 1 \cdot \cot x + x \left(-\csc^2 x\right)$$

$$= \cot x - x \cos ec^2 x$$

14. Find $\frac{dy}{dx}$ if $x = a \sec \theta$, $y = b \tan \theta$

Sol. $x = a \sec \theta, y = b \tan \theta$ Differentiate both equations both sides w.r.t. ' θ ':

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$$\begin{vmatrix} \frac{d}{d\theta}(x) = \frac{d}{d\theta}(a\sec\theta) \\ \frac{dx}{d\theta} = a\sec\theta\tan\theta \\ \frac{d\theta}{dx} = \frac{1}{a\sec\theta\tan\theta} \end{vmatrix} \begin{vmatrix} \frac{d}{d\theta}(y) = \frac{d}{d\theta}(b\tan\theta) \\ \frac{dy}{d\theta} = b\sec^2\theta \end{vmatrix}$$

By using Chain's Rule :
$$\frac{dy}{dx} = \frac{dy}{d\theta} \times \frac{d\theta}{dx}$$

$$\frac{dy}{dx} = b \sec^2 \theta \left(\frac{1}{a \sec \theta \tan \theta} \right) = \frac{b}{a} \frac{\sec \theta}{\tan \theta}$$

$$\frac{dy}{dx} = \frac{b}{a} \cot \theta \cdot \sec \theta$$

$$\frac{dy}{dx} = \frac{b}{a} \frac{\cos \theta}{\sin \theta} \times \frac{1}{\cos \theta} = \boxed{\frac{b}{a} cos ec\theta}$$

15. Find the derivative of sin 1

Sol.
$$\frac{d}{dx} \left(\sin^{-1} \left(\frac{x}{a} \right) \right)$$

$$= \frac{1}{\sqrt{1 - \left(\frac{x}{a} \right)^2}} \cdot \frac{d}{dx} \left(\frac{x}{a} \right)$$

$$= \frac{1}{\sqrt{1 - \frac{x^2}{a^2}}} \times \left(\frac{1}{a} \right) = \frac{1}{\sqrt{\frac{a^2 - x^2}{a^2}}} \cdot \frac{1}{a}$$

$$= \frac{1}{\sqrt{a^2 - x^2}} \cdot \frac{1}{a} = \frac{1}{\sqrt{a^2 - x^2}}$$

$$\frac{d}{dx} \left(\sin^{-1} x + \cos^{-1} x \right)$$
Sol.
$$\frac{d}{dx} \left(\sin^{-1} x + \cos^{-1} x \right)$$

$$\frac{dx}{x^{1}-x^{2}} - \frac{1}{x^{1}-x^{2}} = 0$$

17. Find the value of
$$\frac{d}{dx}$$
 sec \sqrt{x}

Sol.
$$\frac{d}{dx} \left(\sec^{-1} \left(\sqrt{x} \right) \right)$$

$$= \frac{1}{\sqrt{x}} \sqrt{\left(\sqrt{x} \right)^2 - 1} \cdot \frac{d}{dx} \left(\sqrt{x} \right)$$

$$= \frac{1}{\sqrt{x}} \sqrt{x - 1} \cdot \frac{1}{2} (x)^{-\frac{1}{2}}$$

$$= \frac{1}{\sqrt{x}} \sqrt{x - 1}} \frac{1}{2\sqrt{x}}$$

$$= \frac{1}{2(\sqrt{x})^2 \sqrt{x - 1}} = \boxed{\frac{1}{2x\sqrt{x} - 1}}$$

18. If
$$y = x^4 - 3x^2 + 4x - 1$$
, find $\frac{d^2y}{dx^2}$

Sol.
$$y = x^4 - 3x^2 + 4x - 1$$

Differentiate both sides w.r.t. 'x':

$$\frac{d}{dx}(y) = \frac{d}{dx}(x^4 - 3x^2 + 4x - 1)$$
$$\frac{dy}{dx} = 4x^3 - 3(2x) + 4(1) - 0$$
$$\frac{dy}{dx} = 4x^3 - 6x + 4$$

Differentiate both sides w.r.t. 'x':

$$\frac{d}{dx} \left(\frac{dy}{dx} \right) = \frac{d}{dx} \left(4x^3 - 6x + 4 \right)$$

$$\frac{d^2y}{dx^2} = 4 \left(3x^2 \right) - 6 \left(1 \right) + 0$$

$$\frac{d^2y}{dx^2} = 12x^2 - 6$$

19. If
$$y = l n x$$
, find y_2

Sol.
$$y = \ell n x$$

Differentiate both sides w.r.t. 'x':

$$\frac{d}{dx}(y) = \frac{d}{dx}(\ell n x)$$
$$y_1 = \frac{1}{x}$$

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Differentiate both sides w.r.t. 'x':

$$\frac{d}{dx}(y_1) = \frac{d}{dx}\left(\frac{1}{x}\right)$$

$$y_{_{2}}=\frac{d}{dx}\Big(x^{_{-1}}\Big)\!=\!-1\Big(x\Big)^{^{_{-2}}}\!=\!\overline{\left[\frac{-1}{x^{_{^{2}}}}\right]}$$

- 20. If $y = \cos 3x + \sin 3x$, show that: $y_2 + 9y = 0$
- **Sol.** $y = \cos 3x + \sin 3x$ Differentiate both sides w.r.t. 'x':

$$\frac{d}{dx}(y) = \frac{d}{dx}(\cos 3x + \sin 3x)$$

$$y_1 = -\sin 3x(3) + \cos 3x(3)$$

$$y_1 = -3\sin 3x + 3\cos 3x$$

Differentiate both sides w.r.t. 'x':

$$\frac{d}{dx}(y_1) = \frac{d}{dx}(-3\sin 3x + 3\cos 3x)$$

$$y_2 = -3\cos 3x(3) + 3(-\sin 3x)(3)$$

$$y_2 = -9\cos 3x - 9\sin 3x$$

$$y_2 = -9(\cos 3x + \sin 3x)$$

$$\boldsymbol{y}_{2}=-9\boldsymbol{y} \Longrightarrow \boldsymbol{y}_{2}+9\boldsymbol{y}=0$$
 Proved.

- 21. If $y = Ae^{mx} + Be^{-mx}$, show that: $y_2 - m^2y = 0$
- **Sol.** $y = Ae^{mx} + Be^{-mx}$

Differentiate both sides w.r.t. 'x':

$$\frac{d}{dx}(y) = \frac{d}{dx} \left(Ae^{mx} + Be^{-mx} \right)$$

$$y_1 = Ae^{mx}(m) + Be^{-mx}(-m)$$

$$y_1 = Ame^{mx} - Bme^{-mx}$$

Differentiate both sides w.r.t. 'x':

$$\frac{d}{dx}(y_1) = \frac{d}{dx}(Ame^{mx} - Bme^{-mx})$$

$$\boldsymbol{y}_{2}=\boldsymbol{A}\boldsymbol{m}\boldsymbol{e}^{m\boldsymbol{x}}\left(\boldsymbol{m}\right)-\boldsymbol{B}\boldsymbol{m}\boldsymbol{e}^{-m\boldsymbol{x}}\left(-\boldsymbol{m}\right)$$

$$y_2 = Am^2e^{mx} + Bm^2e^{-mx}$$

$$y_2 = m^2 \left(A e^{mx} + B e^{-mx} \right)$$

$$y_9 = m^2 y$$

$$y_2 - m^2 y = 0$$
 Proved.

- 22. Define statistics.
- **Sol.** Statistics is the science of estimation and probabilities.
- 23. What is primary data?
- **Sol.** It is most origin data and has not undergone any statistical treatment.
- 24. What is primary data?
- **Sol.** It is the data which has gone through statistical treatment, at least once.
- 25. If a die is rolled once, what is the probability of getting an even number?
- **Sol.** $S = \{1, 2, 3, 4, 5, 6\}$, n(S) = 6Let A be event that even number

appear.

$$A = \{2, 4, 6\}, n(A) = 3$$

$$P(A) = \frac{n(A)}{n(S)} = \frac{3}{6} = \boxed{\frac{1}{2}}$$

- 26. A card is drawn at random from a deck of 52 cards. Find the probability of getting a diamond.
- Sol. n(S) = 52

Let A be event that it is a diamond card. n(A) = 13

$$\therefore P(A) = \frac{n(A)}{n(S)} = \frac{13}{52} = \boxed{\frac{1}{4}}$$

- 27. A fair coin is tossed twice what is the probability that we get at least on head.
- **Sol.** $S = \{HH, HT, TH, TT\}, n(S) = 4$

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Let A be event that at least one head appears.

$$A = \{HH, HT, TH\} \Rightarrow n(A) = 4$$

$$\therefore P(A) = \frac{n(A)}{n(S)} = \boxed{\frac{3}{4}}$$

Section - II

Note: Attemp any three (3) questions $3 \times 8 = 24$

Q.2.(a) Prove that:
$$f[f(x)] = x$$
, for the function $f(x) = \frac{x+1}{x-1}$

Sol. See Q.10 of Ex # 1.1 (Page # 9)

(b) Evaluate
$$\lim_{x\to 1} x^n - a^m$$

Sol. See Q.3(iii) of Ex # 1.3 (Page # 30)

Q.3.(a) Find
$$\frac{dy}{dx}$$
 of $x^6 + y^6 = 5a^2x^2y^2$

Sol. See Q.2(iii) of Ex # 2.3 (Page # 71)

(b) Find the derivative of
$$\frac{\sqrt{x^2 + 1} - \sqrt{x^2 - 1}}{\sqrt{x^2 + 1} + \sqrt{x^2 - 1}}$$

Sol. See Q.2 of Ex # 2.4 (Page # 90)

Q.4.(a) If
$$xy = cos(x+y)$$
, show that
$$\frac{dy}{dx} + \frac{y + sin(x+y)}{x + sin(x+y)} = 0$$

Sol. See Q.5[a] of Ex # 3.1 (Page # 120)

(b) If
$$y = \tan(p \tan^{-1}x)$$
, show that
$$(1+x^2) \frac{dy}{dx} = p(1+y^2)$$

Sol. See Q.3 of Ex # 3.2 (Page # 132)

Q.5.(a) Use differentials to find the approximate value of $\sqrt[9]{124}$

Sol. See Q.6(ii) of Ex # 4.1 (Page # 177)

(b) Find the maximum and minimum (extreme) values of the function $(x-4)^2(x-2)$

Sol. See Q.2(vii) of Ex # 4.2 (Page # 193)

Q.6.(a) Calculate the median from the following table.

1	Class	Frequency
0	65 – 84	7
	85-104	6
	105-124	8
)	125 – 144	2
	145 – 164	2
	165 – 184	2
	185 – 204	3

Sol. See example # 7 of Chapter 05.

(b) Calculate S.D. from the following

Group	Frequency	
20 - 24	1	
25 – 29	4	
30-34	8	
35 – 39	11	
40 – 44	15	
45 – 49	9	
50-54	2	

Sol. See Q.7 of Ex# 5.2 (Page # 243)