DAE / IIA - 2018

MATH - 212 APPLIED MATHEMATICS - II

PART - A (OBJECTIVE)

Time : 30 Minutes Marks : 20

Q.1: *Encircle the correct answer.*

1. Given $f(x)=\frac{1}{x}-1$ then $f(2)=?$

[a] 1 [b] 2 [c] $-\frac{1}{2}$ [d] 3

2. If $f(x)=3^x-1$, then $f(3)=?$

[a] 27 [b] 8 [c] 26 [d] 16

3. mx^{m-1} is the differential w.r.t. x of:

[a] $m(m-1)x^{m-2}$

[b] $(m-1)x^{m-2}$

[c] x^m [d] mx^m

4. $\frac{d}{dx}(ax+b)^2=?$

[a] $2(ax+b)$ [b] $2a(ax+b)$

[c] $\frac{(ax+b)^3}{3}$ [d] $2(ax+b)b$

5. $\frac{d}{dx}(\tan x^2)=?$

[a] $2x\sec^2 x^2$ [b] $\sec^2 x^2$

[c] $\sec x^2$ [d] $\sec^2 x$

6. $\frac{d}{dx}(\sin\sqrt{x})=?$

[a] $\cos(\sqrt{x})$ [b] $-\cos(\sqrt{x})$

[c] $\frac{1}{2\sqrt{x}}\cos(\sqrt{x})$

[d] $-\frac{1}{2\sqrt{x}}\cos(\sqrt{x})$

7. $\frac{d}{dx}(\sec 3x)=?$

[a] $3\sec 3x\tan 3x$

[b] $\sec 3x\tan 3x$

[c] $3\sec 3x\cot 3x$

[d] $-3\sec 3x\tan 3x$

8. ~~$\frac{d}{dx}\left[\ell n(\sin^{-1}x)\right]=?$~~

[a] $\frac{1}{\sqrt{x^2-1}}$ [b] $\frac{1}{\sin^{-1}x\sqrt{1-x^2}}$

[c] $\frac{\sin^{-1}x}{\sqrt{1-x^2}}$ [d] $\frac{\cos^{-1}x}{\sqrt{1-x^2}}$

9. If $\frac{dy}{dx}$ changes sign from $+ve$ to $-ve$ then it is a point of:

[a] Maxima [b] Minima

[c] Inflection [d] None of these

10. If $\frac{dy}{dx}$ changes sign from $-ve$ to $+ve$ then it is a point of:

[a] Maxima [b] Minima

[c] Inflection [d] None of these

11. $\int\left(\frac{\cos x}{\sin x}\right)dx=$

[a] $\ell n\cos x$ [b] $\ell n\sin x$

[c] $\ell n\cot x$ [d] $\frac{\cos^2 x}{2}$

12. $\int(\tan x\sec^2 x)dx=?$

[a] $\ell n\tan x$ [b] $\frac{\tan^2 x}{2}$

[c] $\frac{\sec^2 x}{3}$ [d] $\sec x\tan x$

13. ~~$\int\left(\frac{e^x}{1+e^x}\right)dx=?$~~

[a] $1+e^x$ [b] $\ell n(1+e^x)$

[c] e^x [d] $\frac{(1+e^x)^2}{2}$

14. $\int\left(\frac{\sec x\tan x}{3+\sec x}\right)dx=?$

[a] $\sec x+\tan x$

[b] $3+\sec x$

[c] $\ell n(3+\sec x)$

[d] $\ell n(\sec x+\tan x)$

15. If lower and upper limits of $\int f(x)dx$ are replaced by each other then we get:

[a] $\int_a^b f(x)dx$ [b] $-\int_a^b f(x)dx$

[c] $+\int_a^b f(x)dx$ [d] $\int_a^b f(x)dx$

16. $\int_0^3 \left(\frac{2x}{x^2+1}\right)dx = ?$

[a] $\ell n2$ [b] $\ell n10 - \ell n6$

[c] $\ell n(x^2+1)$ [d] $\ell n10 + \ell n6$

17. Equation of line in slope intercept form is:

[a] $\frac{x}{a}+\frac{y}{b}=1$ [b] $y = mx + c$

[c] $y - y_1 = m(x - x_1)$

[d] $y + y_1 = m(x + x_1)$

18. Distance between $(4, 3)$ and $(7, 5)$ is:

[a] 25 [b] $\sqrt{13}$

[c] 5 [d] 7

19. Point $(-4, -5)$ lies in the quadrant:

[a] 1^{st} [b] 2^{nd}

[c] 3^{rd} [d] 4^{th}

20. Center of the circle $x^2 + y^2 - 2x - 4y = 8$ is:

[a] $(1, 2)$ [b] $(2, 4)$

[c] $(-1, -2)$ [d] $(-2, -4)$

Answer Key

1	c	2	c	3	c	4	c	5	a
6	c	7	a	8	b	9	a	10	b
11	b	12	b	13	b	14	c	15	b
16	a	17	b	18	b	19	c	20	a

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DAE / IIA - 2018

MATH - 212 APPLIED MATHEMATICS - II

PART - B (SUBJECTIVE)

Time : 2:30 Hrs Marks : 60

Section - I

Q.1: Write short answers to any Twenty Five (25) of the follwing questions. **25 × 2 = 50**

1. If $f(x) = 2x^2 + 4x + 9$, find the value of $\frac{f(3)-f(1)}{f(-1)+f(0)}$.

Sol. $f(x) = 2x^2 + 4x + 9 \rightarrow (i)$

Put $x = 3$ in eq. (i)

$f(3) = 2(3)^2 + 4(3) + 9$

$f(3) = 18 + 12 + 9 = 39$

Put $x = 1$ in eq. (i)

$f(1) = 2(1)^2 + 4(1) + 9$

$f(1) = 2 + 4 + 9 = 15$

Put $x = -1$ in eq. (i)

$f(-1) = 2(-1)^2 + 4(-1) + 9$

$f(-1) = 2 - 4 + 9 = 7$

Put $x = 0$ in eq. (i)

$f(0) = 2(0)^2 + 4(0) + 9 = 9$

$$\frac{f(3)-f(1)}{f(-1)+f(0)} = \frac{39-15}{7+9} = \frac{24}{16} = \boxed{\frac{3}{2}}$$

2. If $f(x) = \sin x + \cos x$, show that: $f(x+\pi) = -f(x)$

Sol. As, $f(x) = \sin x + \cos x$

$L.H.S. = f(x+\pi)$

$= \sin(x+\pi) + \cos(x+\pi)$

$= -\sin x - \cos x$

$= -(\sin x + \cos x)$

$= -f(x) = R.H.S.$ **Proved.**

3. Show that the function $f(x)=x^4-7x^2+7$ is an even function of x.

Sol. $f(x)=x^4-7x^2+7$

Replace 'x' by '−x', we have:

$$f(-x)=(-x)^4-7(-x)^2+7$$
$$f(-x)=x^4-7x^2+7$$
$$f(-x)=f(x)$$

Hence $f(x)$ is an $\boxed{\text{even}}$ function.

4. If $f(x)=3x^3+2x^2-x+4$, prove that: $2f(3)=25f(1)$

Sol. As, $f(x)=3(x)^3+2(x)^2-x+4 \rightarrow (i)$

Put $x=3$, in eq.(i):

$$f(3)=3(3)^3+2(3)^2-3+4$$
$$f(3)=81+18-3+4=100$$

Put $x=1$, in eq.(i):

$$f(1)=3(1)^3+2(1)^2-1+4$$
$$f(1)=3+2-1+4=8$$
$$2f(3)=25f(1)$$
$$2(100)=25(8)$$
$$200=200$$

L.H.S. = R.H.S. **Proved.**

5. Find $\frac{dy}{dx}$ if $\sqrt{x}+\sqrt{y}=5$

Sol. $\sqrt{x}+\sqrt{y}=5$

Differentiate both sides w.r.t. 'x':

$$\frac{d}{dx}\left(\sqrt{x}+\sqrt{y}\right)=\frac{d}{dx}(5)$$
$$\frac{1}{2}x^{-1/2}+\frac{1}{2}y^{-1/2}\frac{d}{dx}(y)=0$$
$$\frac{1}{2\sqrt{x}}+\frac{1}{2\sqrt{y}}\frac{dy}{dx}=0$$
$$\frac{1}{2\sqrt{y}}\frac{dy}{dx}=-\frac{1}{2\sqrt{x}}$$
$$\frac{dy}{dx}=-\left(\frac{1}{2\sqrt{x}}\right)\left(2\sqrt{y}\right)$$
$$\frac{dy}{dx}=-\frac{\sqrt{y}}{\sqrt{x}} \Rightarrow \boxed{\frac{dy}{dx}=-\sqrt{\frac{y}{x}}}$$

6. Find $\frac{dy}{dx}$ if $y=x^3+x^2+2x+3$

Sol. Differentiate both sides w.r.t. 'x':

$$\frac{d}{dx}(y)=\frac{d}{dx}\left(x^3+x^2+2x+3\right)$$
$$\frac{dy}{dx}=3x^2+2x+2(1)+0$$
$$\boxed{\frac{dy}{dx}=3x^2+2x+2}$$

7. If $y=\left(3x^2+2x+9\right)^7$, find $\frac{dy}{dx}$

Sol. $y=\left(3x^2+2x+9\right)^7$

Differentiate both sides w.r.t. 'x':

$$\frac{d}{dx}(y)=\frac{d}{dx}\left(3x^2+2x+9\right)^7$$
$$\frac{dy}{dx}=7\left(3x^2+2x+9\right)^6\left[\frac{d}{dx}\left(3x^2+2x+9\right)\right]$$
$$\frac{dy}{dx}=7\left(3x^2+2x+9\right)^6\left[3(2x)+2(1)+0\right]$$
$$\frac{dy}{dx}=7\left(3x^2+2x+9\right)^6(6x+2)$$
$$\boxed{\frac{dy}{dx}=7(6x+2)\left(3x^2+2x+9\right)^6}$$

8. If $y=5x^3-7x^2+9-\frac{8}{x}+\frac{7}{x^4}$, find $\frac{dy}{dx}$

Sol. $y=5x^3-7x^2+9-\frac{8}{x}+\frac{7}{x^4}$

Differentiate both sides w.r.t. 'x':

$$\frac{d}{dx}(y)=\frac{d}{dx}\left(5x^3-7x^2+9-\frac{8}{x}+\frac{7}{x^4}\right)$$
$$\frac{dy}{dx}=\frac{d}{dx}\left(5x^3-7x^2+9-8x^{-1}+7x^{-4}\right)$$
$$\frac{dy}{dx}=5\left(3x^2\right)-7(2x)+0-8(-1)x^{-2}+7(-4)x^{-5}$$
$$\boxed{\frac{dy}{dx}=15x^2-14x+\frac{8}{x^2}-\frac{28}{x^5}}$$

9. ~~Find the derivative of~~ $\sin^{-1}\left(\frac{x}{a}\right)$

Sol. $\frac{d}{dx}\left(\sin^{-1}\left(\frac{x}{a}\right)\right)$

$$= \frac{1}{\sqrt{1-\left(\frac{x}{a}\right)^2}} \cdot \frac{d}{dx}\left(\frac{x}{a}\right)$$

$$= \frac{1}{\sqrt{1-\frac{x^2}{a^2}}} \times \left(\frac{1}{a}\right) = \frac{1}{\sqrt{\frac{a^2-x^2}{a^2}}} \cdot \frac{1}{a}$$

$$= \frac{1}{\frac{\sqrt{a^2-x^2}}{a}} \cdot \frac{1}{a} = \boxed{\frac{1}{\sqrt{a^2-x^2}}}$$

10. ~~Find the value of~~ $\frac{d}{dx}\left(\sin^{-1}x + \cos^{-1}x\right)$

Sol. $\frac{d}{dx}\left(\sin^{-1}x + \cos^{-1}x\right)$

$$= \frac{1}{\sqrt{1-x^2}} - \frac{1}{\sqrt{1-x^2}} = \boxed{0}$$

11. ~~Find the value of~~ $\frac{d}{dx}\left(\sec^{-1}\left(\sqrt{x}\right)\right)$

Sol. $\frac{d}{dx}\left(\sec^{-1}\left(\sqrt{x}\right)\right)$

$$= \frac{1}{\sqrt{x}\sqrt{\left(\sqrt{x}\right)^2-1}} \cdot \frac{d}{dx}\left(\sqrt{x}\right)$$

$$= \frac{1}{\sqrt{x}\sqrt{x-1}} \cdot \frac{1}{2}(x)^{-1/2}$$

$$= \frac{1}{\sqrt{x}\sqrt{x-1}} \frac{1}{2\sqrt{x}}$$

$$= \frac{1}{2\left(\sqrt{x}\right)^2\sqrt{x-1}} = \boxed{\frac{1}{2x\sqrt{x-1}}}$$

12. ~~Find the value of~~ $\frac{d}{dx}\left(\cos^{-1}\left(1-2x^2\right)\right)$

Sol. $\frac{d}{dx}\left(\cos^{-1}\left(1-2x^2\right)\right)$

$$= \frac{-1}{\sqrt{1-\left(1-2x^2\right)^2}} \cdot \left(\frac{d}{dx}\left(1-2x^2\right)\right)$$

$$= \frac{-1}{\sqrt{1-\left(1-4x^2+4x^4\right)}} \cdot \left(0-2(2x)\right)$$

$$= \frac{-1}{\sqrt{1-1+4x^2-4x^4}}(-4x)$$

$$= \frac{4x}{\sqrt{4x^2-4x^4}} = \frac{4x}{\sqrt{4x^2\left(1-x^2\right)}}$$

$$= \frac{4x}{2x\sqrt{1-x^2}} = \boxed{\frac{2}{\sqrt{1-x^2}}}$$

14. **Find the slope of the tangent to the curve $y = \sin 2x$ at $x = \frac{\pi}{6}$.**

Sol. $y = \sin 2x$

Differentiate both sides w.r.t. 'x':

$$\frac{d}{dx}(y) = \frac{d}{dx}(\sin 2x)$$

$$\frac{dy}{dx} = \cos 2x(2)$$

$$\frac{dy}{dx} = 2\cos 2x$$

At $x = \pi/6$

$$\left.\frac{dy}{dx}\right|_{x=\pi/6} = 2\cos 2\left(\pi/6\right)$$

$$\left.\frac{dy}{dx}\right|_{x=\pi/6} = 2\cos\left(\pi/3\right)$$

$$\left.\frac{dy}{dx}\right|_{x=\pi/6} = 2\left(\frac{1}{2}\right) = \boxed{1}$$

15. **Find the slope of tangent to the curve $y = \cos^2 x$ at $x = \frac{\pi}{4}$.**

Sol. $y = \cos^2 x$

Differentiate both sides w.r.t. 'x':

$$\frac{d}{dx}(y)=\frac{d}{dx}\left(\cos^2 x\right)$$

$$\frac{dy}{dx}=2\cos x(-\sin x)$$

$$\frac{dy}{dx}=-2\sin x\cos x$$

$$\frac{dy}{dx}=-\sin 2x$$

At $x=\pi/4$

$$\left.\frac{dy}{dx}\right|_{x=\pi/4}=-\sin 2\left(\pi/4\right)$$

$$\left.\frac{dy}{dx}\right|_{x=\pi/4}=-\sin\left(\pi/2\right)$$

$$\left.\frac{dy}{dx}\right|_{x=\pi/4}=\boxed{-1}$$

16. Find the slope of tangent to the curve $y=x^3-3x+2$ at $(0, 2)$.

Sol. $y=x^3-3x+2$

Differentiate both sides w.r.t. 'x':

$$\frac{d}{dx}(y)=\frac{d}{dx}\left(x^3-3x+2\right)$$

$$\frac{dy}{dx}=3x^2-3(1)+0$$

$$\frac{dy}{dx}=3x^2-3$$

At $x=0$

$$\left.\frac{dy}{dx}\right|_{x=0}=3(0)^2-3$$

$$\left.\frac{dy}{dx}\right|_{x=0}=0-3=\boxed{-3}$$

17. Find $\int\left(\frac{-2x}{\sqrt{4-x^2}}\right)dx$

Sol. $\int\left(\frac{-2x}{\sqrt{4-x^2}}\right)dx$

$$=\int\left(4-x^2\right)^{-1/2}(-2x)\,dx$$

$$=\frac{\left(4-x^2\right)^{1/2}}{1/2}+c \quad \left\{\begin{matrix}\text{using}\\ \text{Rule-I}\end{matrix}\right\}$$

$$\boxed{\begin{aligned}&\frac{d}{dx}\left(4-x^2\right)\\ &=0-2x\\ &=-2x\end{aligned}}$$

$$=\boxed{2\sqrt{4-x^2}+c}$$

18. Find $\int\left(\frac{x^2+1}{x+1}\right)dx$

Sol. $\int\left(\frac{x^2+1}{x+1}\right)dx$

$$\begin{array}{r|l} & x-1 \\ \hline x+1 & x^2+1 \\ & \pm x^2 \pm x \\ \hline & -x+1 \\ & \mp x \mp 1 \\ \hline & 2 \end{array}$$

$$=\int\left(x-1+\frac{2}{x+1}\right)dx$$

$$=\boxed{\frac{x^2}{2}-x+2\,\ell n(x+1)+c}$$

19. Find $\int\left(1+\frac{3}{x^2}\right)^2 dx$

Sol. $\int\left(1+\frac{3}{x^2}\right)^2 dx$

$$=\int\left[(1)^2+2(1)\left(\frac{3}{x^2}\right)+\left(\frac{3}{x^2}\right)^2\right]dx$$

$$=\int\left(1+\frac{6}{x^2}+\frac{9}{x^4}\right)dx$$

$$=\int\left(1+6x^{-2}+9x^{-4}\right)dx$$

$$=x+\frac{6x^{-1}}{-1}+\frac{9x^{-3}}{-3}+c$$

$$=\boxed{x-\frac{6}{x}-\frac{3}{x^3}+c}$$

20. Evaluate $\int\left(\frac{x}{a+x}\right)dx$

Sol. $\int\left(\frac{x}{a+x}\right)dx$ {Improper Fraction}

$$=\int\left(1-\frac{a}{a+x}\right)dx \quad \begin{array}{r|l} x+a & x \\ & \underline{\pm x \pm a} \\ & -a \end{array} \quad \text{quotient } 1$$

$$=\int\left(1-a(a+x)^{-1}\right)dx$$

$$=\boxed{x-a\,\ell n(a+x)+c} \quad \{\text{using Rule-II}\}$$

21. Integrate $\int\frac{\cos(\ell nx)}{x}dx$

Sol. $\int\frac{\cos(\ell nx)}{x}dx$

$$=\int\cos(\ell nx).\left(\frac{1}{x}\right)dx$$

Put $\ell nx=t$

$$\frac{d}{dx}(\ell nx)=\frac{d}{dx}(t)$$

$$\frac{1}{x}=\frac{dt}{dx}$$

$$\left(\frac{1}{x}\right)dx=dt$$

$$=\int\cos t\,dt$$

$$=\sin t+c=\boxed{\sin(\ell nx)+c}$$

22. Find $\int\left(\frac{x-1}{x^2-2x+3}\right)dx$

Sol. $\int\left(\frac{x-1}{x^2-2x+3}\right)dx$

$$=\frac{1}{2}\int\frac{2x-2}{(x^2-2x+3)}dx \quad \because \frac{d}{dx}(x^2-2x+3)=2x-2(1)+0=2x-2$$

$$=\boxed{\frac{1}{2}\ell n(x^2-2x+3)+c} \quad \{\text{using Rule-II}\}$$

23. Find $\int\left(\cot^2 x\right)dx$

Sol. $\int\left(\cot^2 x\right)dx$

$$=\int\left(\operatorname{cosec}^2 x-1\right)dx$$

$$=-\cot x-x+c=\boxed{-\cot x-x+c}$$

24. Find $\int 2\sec 2x\,dx$

Sol. $\int 2\sec 2x\,dx$

$$=2\int\sec 2x\,dx$$

$$=\cancel{2}\,\frac{\ell n(\sec 2x+\tan 2x)}{\cancel{2}}+c$$

$$=\boxed{\ell n(\sec 2x+\tan 2x)+c}$$

25. Evaluate $\int_0^{\pi/4}(\sec x\tan x)\,dx$

Sol. $\int_0^{\pi/4}(\sec x\tan x)\,dx$

$$=\left[\sec x\right]_0^{\pi/4}$$

$$=\sec\left(\frac{\pi}{4}\right)-\sec(0)$$

$$=\sec 45^\circ-\sec 0^\circ=\boxed{\sqrt{2}-1}$$

26. Evaluate $\int_0^b\left(x^3\cos x^4\right)dx$

Sol. $\int_0^b\left(x^3\cos x^4\right)dx$

$$=\frac{1}{4}\int_0^b\cos x^4\left(4x^3\right)dx$$

$$=\frac{1}{4}\left[\sin x^4\right]_0^b$$

$$=\frac{1}{4}\left[\sin b^4-\sin(0)^4\right]$$

$$=\frac{1}{4}\left[\sin b^4-0\right]=\boxed{\frac{1}{4}\sin b^4}$$

27. Evaluate $\int_0^{\pi/2}\frac{\cos x}{3+4\sin x}dx$

Sol. $\int_0^{\pi/2}\frac{\cos x}{3+4\sin x}dx$

$$=\frac{1}{4}\int_0^{\pi/2}\frac{4\cos x}{3+4\sin x}dx$$

$$=\frac{1}{4}\left[\ell n(3+4\sin x)\right]_0^{\pi/2} \quad \left\{\begin{matrix}\text{using}\\ \text{Rule-II}\end{matrix}\right\}$$

$$=\frac{1}{4}\left[\ell n\left(3+4\sin\left(\pi/2\right)\right)-\ell n\left(3+4\sin(0)\right)\right]$$

$$=\frac{1}{4}\left[\ell n\left(3+4\sin 90^\circ\right)-\ell n\left(3+4\sin 0^\circ\right)\right]$$

$$=\frac{1}{4}\left[\ell n(3+4(1))-\ell n(3+4(0))\right]$$

$$=\frac{1}{4}\left[\ell n(3+4)-\ell n(3+0)\right]$$

$$=\frac{1}{4}\left[\ell n(7)-\ell n(3)\right]=\boxed{\frac{1}{4}\ \ell n\left(\frac{7}{3}\right)}$$

28. **Evaluate** $\int_0^{\pi/6}(2\sin 2x)\,dx$

Sol. $\int_0^{\pi/6}(2\sin 2x)\,dx$

$$=2\left[-\frac{\cos 2x}{2}\right]_0^{\pi/6}=-\left[\cos 2x\right]_0^{\pi/6}$$

$$=-\left(\cos 2\left(\frac{\pi}{6}\right)-\cos 2(0)\right)$$

$$=-\left(\cos 2(30^\circ)-\cos 2(0^\circ)\right)$$

$$=-\left(\cos 60^\circ-\cos 0^\circ\right)$$

$$=-\left(\frac{1}{2}-1\right)\left\{\begin{matrix}\text{using calculator}\\ \cos 60^\circ=\frac{1}{2}\ \&\ \cos 0^\circ=1\end{matrix}\right\}$$

$$=-\left(\frac{1-2}{2}\right)=-\left(-\frac{1}{2}\right)=\boxed{\frac{1}{2}}$$

29. **Find the triangle whose vertices are $A(0,1)$, $B(7,2)$ and $C(3,8)$. Find the length of the median from C to AB.**

Sol. Let D be midpoint of $\overline{AB}$:

$$\text{So,}\quad D=\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$$

$$D=\left(\frac{0+7}{2},\frac{2+1}{2}\right)=\left(\frac{7}{2},\frac{3}{2}\right)$$

$\left|\overline{CD}\right|$ = Required length of median

$$=\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}$$

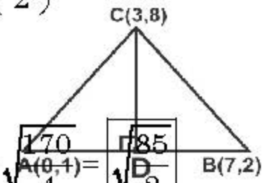
$$=\sqrt{\left(3-\frac{7}{2}\right)^2+\left(8-\frac{3}{2}\right)^2}$$

$$=\sqrt{\left(\frac{6-7}{2}\right)^2+\left(\frac{16-3}{2}\right)^2}$$

$$=\sqrt{\left(-\frac{1}{2}\right)^2+\left(\frac{13}{2}\right)^2}$$

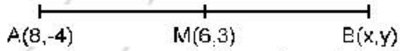
$$=\sqrt{\frac{1}{4}+\frac{169}{4}}$$

$$=\sqrt{\frac{1+169}{4}}=\sqrt{\frac{170}{4}}=\boxed{\frac{\sqrt{85}}{2}}$$



30. **If the mid-point of a segment is (6, 3) and one end point is (8, – 4), what are the coordinates of the other end point.**

Sol. Let B(x, y) be require end point.



$$\text{As, Mid-point}=(6,3)$$

$$\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)=(6,3)$$

$$\left(\frac{8+x}{2},\frac{-4+y}{2}\right)=(6,3)$$

Comparing both order pairs, we have :

$$\frac{8+x}{2}=6\ \text{and}\ \frac{-4+y}{2}=3$$

$8+x=12$	$-4+y=6$
$x=12-8$	$y=6+4$
$x=4$	$y=10$

Hence other end point $=\boxed{(4,10)}$

31. ~~Find the angle between the lines having slopes -3 and 2.~~

Sol. Let, $m_1 = -3$ and $m_2 = 2$

$$\theta = \tan^{-1}\left(\frac{m_2 - m_1}{1 + m_2 m_1}\right)$$

$$\theta = \tan^{-1}\left(\frac{2-(-3)}{1+(2)(-3)}\right)$$

$$\theta = \tan^{-1}\left(\frac{2+3}{1-6}\right) = \tan^{-1}\left(\frac{5}{-5}\right)$$

$$\theta = \tan^{-1}(-1) = \boxed{135^\circ}$$

32. **Find the slope of a line which is perpendicular to the line joining $P_1(2, 4)$, $P_2(-2, 1)$.**

Sol. Slope of line joining given point:

$$= m_1 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1-4}{-2-2} = \frac{-3}{-4} = \frac{3}{4}$$

Slope of require line $= m_2 = ?$

As, both lines are perpendicular,

So, $m_1 m_2 = -1 \Rightarrow \left(\frac{3}{4}\right) m_2 = -1$

$$\Rightarrow m_2 = -1 \times \frac{4}{3} \Rightarrow \boxed{m_2 = -\frac{4}{3}}$$

33. **Find the equation of a line through the point $(3, -2)$ with slope $m = \frac{3}{4}$.**

Sol. Equation of line in point - slope form:

$$y - y_1 = m(x - x_1)$$

$$y - (-2) = \frac{3}{4}(x - 3)$$

$$4(y + 2) = 3(x - 3)$$

$$4y + 8 = 3x - 9$$

$$4y + 8 - 3x + 9 = 0$$

$$-3x + 4y + 17 = 0$$

$$\boxed{3x - 4y - 17 = 0}$$

34. **Find the equation of circle with center on origin and radius is $\frac{1}{2}$.**

Sol. Standard form of equation of circle:

$$(x-h)^2 + (y-k)^2 = r^2$$

Put $h = 0$, $k = 0$ & $r = \frac{1}{2}$

$$(x-0)^2 + (y-0)^2 = \left(\frac{1}{2}\right)^2$$

$$\boxed{x^2 + y^2 - \frac{1}{4} = 0}$$

35. **Find center and radius of the circle $x^2 + y^2 + 9x - 7y - 33 = 0$**

Sol. Comparing with general equation of circle.

$$x^2 + y^2 + 9x - 7y - 33 = 0$$

$2g = 9$	$2f = -7$	
$g = \frac{9}{2}$	$f = -\frac{7}{2}$	$c = -33$

Center $= (-g, -f) =$

Center $= \left(-\frac{9}{2}, -\left(-\frac{7}{2}\right)\right) = \boxed{\left(-\frac{9}{2}, \frac{7}{2}\right)}$

Radius $= r = \sqrt{g^2 + f^2 - c}$

$$r = \sqrt{\left(\frac{9}{2}\right)^2 + \left(-\frac{7}{2}\right)^2 - (-33)}$$

$$r = \sqrt{\frac{81}{4} + \frac{49}{4} + 33}$$

$$r = \sqrt{\frac{81 + 49 + 132}{4}}$$

$$r = \sqrt{\frac{262}{4}} = \boxed{\sqrt{\frac{131}{2}}}$$

36. **Find the center and radius of the circle $6x^2 + 6y^2 - 18y = 0$**

Sol. $6x^2+6y^2-18y=0$

Dividing each term by 6, we get:

$x^2+y^2-3y=0$

Comparing with general form:

$x^2+y^2+2gx+2fy+c=0$

$2g=0$	$2f=-3$	$c=0$
$g=0$	$f=-3/2$	

Center $=(-g,-f)$

Center $=\left(0,-\left(-\frac{3}{2}\right)\right)=\boxed{\left(0,3/2\right)}$

Radius $=r=\sqrt{g^2+f^2-c}$

$r=\sqrt{(0)^2+\left(-\frac{3}{2}\right)^2-0}=\sqrt{\frac{9}{2}}=\boxed{\frac{3}{2}}$

37. **What type of circle is represented by $x^2+y^2-2x+4y+8=0$**

Sol. $x^2+y^2-2x+4y+8=0$

Comparing this equation with general form of equation of circle:

$x^2+y^2+2gx+2fy+c=0$

$2g=-2$	$2f=4$	
$g=-\frac{2}{2}$	$f=\frac{4}{2}$	$c=8$
$g=-1$	$f=2$	

$\text{Radius}=r=\sqrt{g^2+f^2-c}$

$r=\sqrt{(-1)^2+(2)^2-8}$

$r=\sqrt{1+4-8}=\sqrt{-3}=\boxed{\sqrt{3}i}$

So, it is an **Imaginary circle.**

Section - II

Note: Attemp any three (3) questions $\boxed{3\times10=30}$

Q.2.[a] **If $f(x)=\log\frac{1-x}{1+x}$, Prove that:**

$$f(x)+f(y)=f\left(\frac{x+y}{1+xy}\right)$$

Sol. See Q.13 of Ex # 1.1 (Page # 9)

[b] **Find $\frac{dy}{dx}$ when**

$$x=\frac{a(1-t^2)}{1+t^2} \quad \text{and} \quad y=\frac{2bt}{1+t^2}$$

Sol. See Q.3(iv) of Ex # 2.3 (Page # 72)

Q.3.[a] ~~**Differentiate**~~ $\cos 2x$ ~~**from the first principle method.**~~

Sol. See Q.1(iii) of Ex # 3.1 (Page # 109)

[b] **Find the maximum and minimum (extreme) values of the function $(x-2)^2(x-1)$.**

Sol. See Q.2(vi) of Ex # 4.2 (Page # 190)

Q.4.[a] **Integrate** $\int\left(\frac{1}{\sqrt{1+x}-\sqrt{x}}\right)dx$

Sol. See Q.16 of Ex # 5.1 (Page # 232)

[b] **Find the coordinates of the point that is equidistant from the points (2,3), (0,-1) and (4,5).**

Sol. See Q.9 of Ex # 8.1 (Page # 359)

Q.5.[a] **Calculate** $\int_0^{\pi/3}\frac{dx}{1-\sin x}$

Sol. See example # 7 of Chapter 07.

[b] ~~**Evaluate**~~ ~~$\int\frac{dx}{\sqrt{a^2-x^2}}$~~

Sol. See example # 9 of Chapter 06.

Q.6.[a] **Find the equation of the circle passing through the points (0, 1), (3, -3) and (3, -1).**

Sol. See Q.3[b] of Ex # 9 (Page # 435)

[b] **Differentiate the expansion:**

~~$\sec^{-1}\left(\frac{x^2+1}{x^2-1}\right)$~~ **w.r.t. 'x'.**

Sol. See Q.2(iv) of Ex # 3.2 (Page # 129)

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