DAE / IIA - 2017

MATH - 233 APPLIED MATHEMATICS - II

PAPER 'B' PART - A (OBJECTIVE)

Time : 30 Minutes Marks : 15

Q.1: *Encircle the correct answer.*

1. $\int(\sec x)\,dx = ?$

[a] $\ell n(\sec x \tan x)$ [b] $\tan x$

[c] $\ell n(\sec x + \tan x)$ [d] $\frac{\sec^2 x}{2}$

2. $\int(x^{n+1})\,dx = ?$

[a] $\frac{x^{n+1}}{n+1}$ [b] $(n+1)x^n$ [c] $\frac{x^{n+2}}{n+2}$ [d] $\frac{x^2}{2}$

3. $\int\left(\frac{\cos x}{\sin x}\right)dx = ?$

[a] $\ell n \cos x$ [b] $\ell n \sin x$

[c] $\ell n \cot x$ [d] $\frac{\cos^2 x}{2}$

4. $\int(x \sin x)\,dx = ?$

[a] $-x\cos x + \sin x$ [b] $\sin x$

[c] $x + \sin x$ [d] $\frac{x^2}{2}\cos x$

5. $\int(xe^x)\,dx = ?$

[a] $xe^x + e^x$ [b] $xe^x - e^x$ [c] e^x [d] $\frac{x^2}{2}e^x$

6. $\int(ax+b)^3\,dx = ?$

[a] $3(ax+b)^2$ [b] $3a(ax+b)^2$

[c] $\frac{(ax+b)^3}{4a}$ [d] $\frac{(ax+b)^4}{4a}$

7. ~~$\int_0^1\left(\frac{1}{x^2+1}\right)dx = ?$~~

[a] $-\frac{\pi}{4}$ [b] 1 [c] 0 [d] $\frac{\pi}{4}$

8. $\int_0^{\pi/2}(\cot x)\,dx = ?$

[a] -1 [b] 1 [c] 0 [d] $\pi/2$

9. An equation involving one or more derivative of a function is called:

[a] Quadratic [b] Linear

[c] Differential [d] Cubic

10. Degree of differential equation $\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^3 = 0$ is:

[a] 3 [b] 2 [c] 0 [d] 1

11. Solution of differential equation $xdy + ydx = 0$ is;

[a] $y = cx$ [b] $y = x$

[c] $xy = c$ [d] $x^2 + c = y^2$

12. ~~If an even function, the Fourier coefficient~~ 'b_n' ~~is:~~

[a] 0 [b] 1 [c] -1 [d] 2

13. Laplace transform of the function $f(t) = e^t$ is:

[a] $\frac{1}{s-1}$ [b] $\frac{1}{s}$ [c] $s-1$ [d] s

14. Laplace transform of the function $f(t) = \sin wt$ is:

[a] $\frac{1}{S^2+w^2}$ [b] $\frac{w}{S^2+w^2}$

[c] $\frac{S}{S^2+w^2}$ [d] $\frac{1}{S^2-w^2}$

15. ~~The Inverse Laplace transform~~ $L^{-1}\left(\frac{1}{S}\right)$ ~~is:~~

[a] 1 [b] 2 [c] 3 [d] 4

Answer Key

1	c	2	c	3	a	4	a	5	a
6	d	7	d	8	a	9	a	10	b
11	c	12	d	13	a	14	b	15	d

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DAE / IIA - 2017

MATH - 233 APPLIED MATHEMATICS - II

PAPER 'B' PART - B (SUBJECTIVE)

Time : 2 : 30Hrs Marks : 60

Section - I

Q.1. Write short answers to any Eighteen (18) questions.

1. Evaluate $\int\left(\frac{ax+bx^{-3}+cx^{-7}}{x^{-2}}\right)dx$

Sol. $\int\left(\frac{ax+bx^{-3}+cx^{-7}}{x^{-2}}\right)dx$

$$=\int x^2\left(ax+bx^{-3}+cx^{-7}\right)dx$$

$$=\int\left(ax^3+bx^{-1}+cx^{-5}\right)dx$$

$$=a\frac{x^4}{4}+b\,\ell nx+\frac{cx^{-4}}{-4}+c_1$$

$$=\boxed{\frac{1}{4}\left[ax^4+4b\ell nx-cx^{-4}\right]+c_1}$$

2. Evaluate $\int\frac{1}{2}\left(e^{\frac{1}{2}x}-e^{-\frac{1}{2}x}\right)dx$

Sol. $\int\frac{1}{2}\left(e^{\frac{1}{2}x}-e^{-\frac{1}{2}x}\right)dx$

$$=\frac{1}{2}\left(\frac{e^{\frac{1}{2}x}}{\frac{1}{2}}-\frac{e^{-\frac{1}{2}x}}{-\frac{1}{2}}\right)+c=\boxed{e^{\frac{1}{2}x}+e^{-\frac{1}{2}x}+c}$$

3. Solve $\int\left(\frac{\cos^2x-\sin^2x}{\sin 2x}\right)dx$

Sol. $\int\left(\frac{\cos^2x-\sin^2x}{\sin 2x}\right)dx$

$$=\int\frac{\cos 2x}{\sin 2x}dx \quad \because \left\{\cos 2x = \cos^2x-\sin^2x\right\}$$

$$=\int\cot 2x\,dx$$

$$=\boxed{\frac{\ell n(\sin 2x)}{2}+c} \quad \left\{\text{Using formula \# 10 from page \# 282}\right\}$$

4. Evaluate $\int\left(\tan^4x+\tan^2x\right)dx$

Sol. $\int\left(\tan^4x+\tan^2x\right)dx$

$$=\int\tan^2x\left(\tan^2x+1\right)dx$$

$$=\int\tan^2x.\sec^2x\,dx \quad \because\left\{1+\tan^2x=\sec^2x\right\}$$

$$=\frac{\tan^3x}{3}+c=\boxed{\frac{1}{3}\tan^3x+c}$$

5. Evaluate $\int\left(\sin x-\cos x\right)^2dx$

Sol. $\int\left(\sin x-\cos x\right)^2dx$

$$=\int\left(\sin^2x+\cos^2x-2\sin x\cos x\right)dx$$

$$=\int\left(1-\sin 2x\right)dx \quad \because\left\{\begin{array}{l}\sin^2x+\cos^2x=1\\ \sin 2x=2\sin x\cos x\end{array}\right\}$$

$$=x-\left(\frac{-\cos 2x}{2}\right)+c=\boxed{x+\frac{1}{2}\cos 2x+c}$$

6. Integrate $\int\left(\frac{\sin 2x}{1+\sin^2x}\right)dx$

Sol. $\int\left(\frac{\sin 2x}{1+\sin^2x}\right)dx=\int\frac{2\sin x\cos x}{1+\sin^2x}dx$

Put $1+\sin^2x=t$

$$\frac{d}{dx}\left(1+\sin^2x\right)=\frac{d}{dx}(t)$$

$$0+2\sin x\cos x=\frac{dt}{dx}$$

$$\left(2\sin x\cos x\right)dx=dt$$

$$=\int\frac{dt}{t}=\int\frac{1}{t}dt$$

$$=\ell n(t)+c=\boxed{\ell n\left(1+\sin^2x\right)+c}$$

7. Integrate $\int \cos x \sin 3x dx$

Sol. $\int \cos x \sin 3x dx$

$= \frac{1}{2}\int (2\cos x \sin 3x)dx$ $\left\{\begin{matrix}\because 2\cos\alpha\sin\beta \\ =\sin(\alpha+\beta)-\sin(\alpha+\beta)\end{matrix}\right\}$

$= \frac{1}{2}\int [\sin(x+3x)-\sin(x-3x)]dx$

$= \frac{1}{2}\int (\sin 4x + \sin 2x)dx \quad \because \left\{\begin{matrix}\sin(-\theta) \\ =-\sin\theta\end{matrix}\right\}$

$= \frac{1}{2}\left[-\frac{\cos 4x}{4} - \frac{\cos 2x}{2}\right] + c$ {Using formula # 07 from page # 282}

$= \frac{1}{2}\left[\frac{-\cos 4x - 2\cos 2x}{4}\right] + c$

$= \boxed{-\frac{1}{8}[\cos 4x + 2\cos 2x] + c}$

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8. Evaluate $\int \frac{dx}{x(1+\ell nx)}$

Sol. $\int \frac{dx}{x(1+\ell nx)}$

$= \int \frac{1}{(1+\ell nx)} \cdot \frac{1}{x} . dx$

$= \boxed{\ell n(1+\ell nx)+c}$

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9. Evaluate $\int (\sin^3 x) dx$

Sol. $\int (\sin^3 x) dx$

$= \int \sin^2 x . \sin x \, dx$

$= \int (1-\cos^2 x)\sin x \, dx$

$= \int (\sin x - \cos^2 x \sin x)dx$

$= \int [\sin x + \cos^2 x(-\sin x)]dx$

$= \boxed{-\cos x + \frac{\cos^3 x}{3} + c}$

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10. Integrate $\int (\sin^{-1} x) dx$

Sol. $\int (\sin^{-1} x)dx = \int \sin^{-1} x.(1)dx$

Integrating by parts:

taking $u = \sin^{-1}x$ & $v = 1$

$= \sin^{-1} x \int 1dx - \int \left\{\frac{d}{dx}(\sin^{-1} x)\int 1dx\right\}dx$

$= \sin^{-1} x.(x) - \int \frac{1}{\sqrt{1-x^2}}(x)dx$

$= x\sin^{-1} x - \int \frac{x}{\sqrt{1-x^2}}dx$

$= x\sin^{-1} x - \frac{1}{-2}\int (1-x^2)^{-1/2}(-2x)dx$

$= x\sin^{-1} x + \frac{1}{2}\frac{(1-x^2)^{1/2}}{1/2} + c$

$= \boxed{x\sin^{-1} x + \sqrt{1-x^2} + c}$

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11. Find $6\int x^2 e^{x^3} dx$

Sol. $6\int x^2 e^{x^3} dx$

$= 6\int e^{x^3}(x^2)dx$

$= 6\int e^t \frac{dt}{3}$

$= \frac{6}{3}\int e^t dt$

$= 2e^t + c$

$= \boxed{2e^{x^3} + c}$

Put $x^3 = t$

$\frac{d}{dx}(x^3) = \frac{d}{dx}(t)$

$3x^2 = \frac{dt}{dx}$

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12. Integrate $\int \frac{\cos(\ell nx)}{x} dx$

Sol. $\int \frac{\cos(\ell nx)}{x} dx$

$= \int \cos(\ell nx).\left(\frac{1}{x}\right) dx$

$$\text{Put } \ell nx = t \Rightarrow \frac{d}{dx}(\ell nx) = \frac{d}{dx}(t)$$
$$\frac{1}{x} = \frac{dt}{dx} \Rightarrow \left(\frac{1}{x}\right)dx = dt$$

$$= \int \cos t\, dt = \sin t + c = \boxed{\sin(\ell nx) + c}$$

13. Find $\int \frac{1}{25 + x^2}\, dx$

Sol. $\int \frac{1}{25 + x^2}\, dx$

$$= \int \frac{1}{(5)^2 + (x)^2}\, dx$$

$$= \boxed{\frac{1}{5}\tan^{-1}\left(\frac{x}{5}\right) + c} \quad \left\{\begin{array}{l}\text{Using formula \# 17} \\ \text{from page \# 282}\end{array}\right\}$$

14. Calculate the Integral

$$\int_0^{\pi/4} \left(\sin^2 x\right) dx$$

Sol. $\int_0^{\pi/4} \left(\sin^2 x\right) dx$

$$= \int_0^{\pi/4} \left(\frac{1 - \cos 2x}{2}\right) dx \because \left\{\sin^2 x = \frac{1 - \cos 2x}{2}\right\}$$

$$= \frac{1}{2}\int_0^{\pi/4} (1 - \cos 2x)\, dx$$

$$= \frac{1}{2}\left[x - \frac{\sin 2x}{2}\right]_0^{\pi/4} \quad \left\{\begin{array}{l}\text{Using formula \# 01 \& 08} \\ \text{from page \# 282}\end{array}\right\}$$

$$= \frac{1}{2}\left[\left(\frac{\pi}{4} - \frac{\sin 2\left(\pi/4\right)}{2}\right) - \left(0 - \frac{\sin 2(0)}{2}\right)\right]$$

$$= \frac{1}{2}\left[\frac{\pi}{4} - \frac{\sin 2(45°)}{2} + \frac{\sin 2(0°)}{2}\right]$$

$$= \frac{1}{2}\left[\frac{\pi}{4} - \frac{\sin(90°)}{2} + \frac{\sin(0°)}{2}\right]$$

$$= \frac{1}{2}\left[\frac{\pi}{4} - \frac{1}{2} + \frac{0}{2}\right] = \frac{1}{2}\left(\frac{\pi - 2}{4}\right) = \boxed{\frac{\pi - 2}{8}}$$

15. Evaluate $\int_0^3 \sqrt[3]{(3x - 1)^2}\, dx$

Sol. $\int_0^3 \sqrt[3]{(3x - 1)^2}\, dx$

$$= \int_0^3 (3x - 1)^{2/3}\, dx$$

$$= \frac{1}{3}\int_0^3 (3x - 1)^{2/3}(3)\, dx$$

$$= \frac{1}{3}\left[\frac{(3x - 1)^{5/3}}{5/3}\right]_0^3 \quad \left\{\begin{array}{l}\textbf{using} \\ \textbf{Rule-I}\end{array}\right\}$$

$$= \frac{1}{3} \times \frac{3}{5}\left[(3x - 1)^{5/3}\right]_0^3$$

$$= \frac{1}{5}\left[(3(3) - 1)^{5/3} - (3(0) - 1)^{5/3}\right]$$

$$= \frac{1}{5}\left[(8)^{5/3} - (-1)^{5/3}\right]$$

$$= \frac{1}{5}\left[32 - (-1)\right] = \boxed{\frac{33}{5}}$$

16. Evaluate $\int_0^{\pi/2} \frac{\cos x}{3 + 4\sin x}\, dx$

Sol. $\int_0^{\pi/2} \frac{\cos x}{3 + 4\sin x}\, dx$

$$= \frac{1}{4}\int_0^{\pi/2} \frac{4\cos x}{3 + 4\sin x}\, dx$$

$$= \frac{1}{4}\left[\ell n(3 + 4\sin x)\right]_0^{\pi/2} \quad \left\{\begin{array}{l}\textbf{using} \\ \textbf{Rule-II}\end{array}\right\}$$

$$= \frac{1}{4}\left[\ell n\left(3 + 4\sin\left(\pi/2\right)\right) - \ell n(3 + 4\sin(0))\right]$$

$$= \frac{1}{4}\left[\ell n(3 + 4(1)) - \ell n(3 + 4(0))\right]$$

$$= \frac{1}{4}\left[\ell n(7) - \ell n(3)\right]$$

$$= \boxed{\frac{1}{4}\ell n\left(\frac{7}{3}\right)} \quad \left\{\begin{array}{l}\text{By using logrithm law} \\ \ell n\left(\frac{m}{n}\right) = \ell n(m) - \ell n(n)\end{array}\right\}$$

17. ~~Evaluate $\int_0^1 \left(x e^x\right) dx$~~

Sol. $\int_0^1 \left(x\ e^x\right)dx$

Integrating by parts :

taking $u = x$ & $v = e^x$

$$= x\int_0^1 e^x dx - \int_0^1\left(\frac{d}{dx}(x)\int e^x dx\right)dx$$

$$= \left[x\ e^x\right]_0^1 - \int_0^1\left(1.\ e^x\right)dx$$

$$= \left[1e^1 - 0e^0\right] - \left[e^x\right]_0^1$$

$$= \left[e - 0\right] - \left[e^1 - e^0\right]$$

$$= \cancel{e} - \cancel{e} + 1 = \boxed{1}$$

18. Find the general solution

$$\mathbf{x^2\frac{dy}{dx} = \frac{1}{y^2 + \sqrt{y}}}$$

Sol. $x^2\frac{dy}{dx} = \frac{1}{y^2 + \sqrt{y}}$

$$\left(y^2 + \sqrt{y}\right)dy = \frac{1}{x^2}dx$$

Integrating both sides, we have :

$$\int\left(y^2 + \sqrt{y}\right)dy = \int\left(\frac{1}{x^2}\right)dx$$

$$\frac{y^3}{3} + \frac{y^{3/2}}{3/2} = \frac{x^{-1}}{-1} + c$$

$$\boxed{\frac{1}{3}y^3 + \frac{2}{3}y^{3/2} = -\frac{1}{x} + c}$$

19. Find the general solution

$$\mathbf{\left(e^x + e^{-x}\right)\frac{dy}{dx} = \left(e^x - e^{-x}\right)}$$

Sol. $\left(e^x + e^{-x}\right)\frac{dy}{dx} = \left(e^x - e^{-x}\right)$

$$dy = \left(\frac{e^x - e^{-x}}{e^x + e^{-x}}\right)dx$$

Integrating both sides, we have :

$$\int 1dy = \int\left(\frac{e^x - e^{-x}}{e^x + e^{-x}}\right)dx$$

$$\boxed{y = \ell n\left(e^x + e^{-x}\right) + c}$$

20. Solve the differential equation

$$\mathbf{x^2\frac{dy}{dx} = \cos^2 y}$$

Sol. $x^2\frac{dy}{dx} = \cos^2 y$

$$\frac{1}{\cos^2 y}dy = \frac{1}{x^2}dx$$

Integrating both sides, we have :

$$\int \sec^2 y dy = \int x^{-2}dx$$

$$\tan y = \frac{x^{-1}}{-1} + c$$

$$\tan y = -x^{-1} + c$$

$$\boxed{\tan y + x^{-1} = c}$$

21. Find the particular solution of the equation $\mathbf{\frac{dy}{dx} = 2xy}$, given that y = 1 when x = 0

Sol. $\frac{dy}{dx} = 2xy$

$$\frac{1}{y}dy = 2xdx$$

Integrating both sides, we have :

$$\int\frac{1}{y}dy = \int 2xdx$$

$$\ell ny = 2\frac{x^2}{2} + c$$

$$\ell ny = x^2 + c \rightarrow (i)$$

when $y = 1$ & $x = 0$

$$\ell n(1) = (0)^2 + c$$

$$0 = 0 + c \Rightarrow \boxed{c = 0}$$

So, eq.(i) becomes:

$\ell ny = x^2 + 0$

$\ell ny = x^2 \Rightarrow \boxed{y = e^{x^2}}$

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22. **Define differential equation and give example.**

Sol. An equation involving derivatives or differentials is called a differential equation.

Example: $\dfrac{dy}{dx} + 2x = 0$

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23. **What is Fourier Series?**

Sol. The infinite sum

$\dfrac{a_0}{2} + \sum_{n=1}^{\infty}(a_n \cos nx + b_n \sin nx)$ is called Fourier series.

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24. **Find Laplace transform of a constant 'k'.**

Sol. $L\{K\} = K\,L\{1\} = K\left(\dfrac{1}{s}\right) = \boxed{\dfrac{K}{s}}$

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25. ~~**Find the inverse Laplace transformation of**~~ ~~$\dfrac{5}{s-3}$~~.

Sol. $L^{-1}\left(\dfrac{5}{s-3}\right)$

$= 5L^{-1}\left\{\dfrac{1}{s-3}\right\} = \boxed{5e^{3t}}$

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26. **Write Laplace transform of e^{at}.**

Sol. $L\{e^{at}\} = \boxed{\dfrac{1}{s-a}}$

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27. **Define Laplace Transformation.**

Sol. Te Laplace transformation of a function f(t), is defined as:

$L\{f(t)\} = \int_0^{\infty} e^{-st} f(t)\,dt$, where $t > 0$

Section - II

Note: Attemp any three (3) questions $3 \times 8 = 24$

Q.2.[a] Evaluate $\int\left(\dfrac{x^4}{x+1}\right)dx$

Sol. See Q.12 of Ex# 7.1 (Page # 286)

[b] Evaluate $\int(\tan^4 x)\,dx$

Sol. See Q.1(ix) of Ex# 7.3 (Page # 299)

Q.3.[a] ~~**Integrate by substitution method**~~

~~$\int \dfrac{dx}{\sqrt{25-16x^2}}$~~

Sol. See Q.1(i) of Ex# 8.2 (Page # 327)

[b] ~~**Evaluate** $\int e^{3x}\sin 2x\,dx$~~.

Sol. See Q.5(iv) of Ex# 8.3 (Page # 356)

Q.4.[a] Solve the $\int_0^{16}\dfrac{\sqrt{x}}{1+\sqrt{x}}\,dx$

Sol. See Q.1(xi) of Ex# 9.1 (Page # 379)

[b] ~~**Find the area of the region enclosed by parabola $y = 2 - x^2$ and line $y = -x$.**~~

Sol. See Q.8 of Ex# 9.2 (Page # 392)

Q.5.[a] Find the general solution

$$x\frac{dy}{dx} = y^2 - 3y + 2$$

Sol. See Q.3 of Ex# 10 (Page # 411)

[b] Find the particular solution satisfying the given boundary conditions $2xdx - dy = x(xdy - ydx)$ given $y = 1$ when $x = -3$

Sol. See Q.16 of Ex# 10 (Page # 419)

Q.6. If $f(t) = 2\sin wt$. Find $L\{f(t)\}$.

Sol. See Q.5 of Ex# 12 (Page # 470)

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