DAE / IIA - 2017

MATH - 233 APPLIED MATHEMATICS - II

PAPER 'A' PART - A (OBJECTIVE)

Time : 30 Minutes   Marks : 15

Q.1: *Encircle the correct answer.*

1. ~~$\tan^{-1}(\tan\theta) = ?$~~

[a] $\frac{1}{1+\theta^2}$ [b] $\frac{1}{1+x^2}$

[c] θ [d] $\sec^2\theta$

2. $\frac{d}{dx}(\ell n \sin x) = ?$

[a] $\cot x$ [b] $\frac{1}{\sin x}\ell n \sin x$

[c] $\ell n \cos x$ [d] $\tan x$

3. If $\frac{dy}{dx}$ does not change sign before and after a point where it vanished then that is point of:

[a] Maxima   [b] Minima

[c] Inflection   [d] None of these

4. The sum of all variable divided by their number is called;

[a] Median [b] Arithmetic Mean

[c] Mode [d] Geometric Mean

5. ~~$\frac{d}{dx}(e^{\sin x}) = ?$~~

[a] $e^{\cos x}$ [b] $\cos x e^{\sin x}$

[c] $\sin x e^{\sin x - 1}$ [d] $\sin x e^{\sin x}$

6. For a decreasing function $\frac{dy}{dx}$ is:

[a] +ve   [b] −ve

[c] zero   [d] None of these

7. A function is minimum at a point if its 2^{nd} derivative is:

[a] +ve   [b] −ve

[c] zero   [d] None of these

8. ~~$\lim_{x\to\frac{\pi}{2}} \frac{1}{\cos\theta} = ?$~~

[a] 0 [b] ∞ [c] 1 [d] $2/\pi$

9. $\frac{d}{dx}(2\cos 3x) = ?$

[a] $6\sin 3x$ [b] $-6\sin 3x$

[c] $-6\cos 3x$ [d] $6\cos 3x$

10. The result obtained from an experiment or a trial is called:

[a] Sample space [b] An event

[c] Out come   [d] Population

11. A function which is given in terms of the independent variable is called:

[a] Even   [b] Explicit

[c] Periodic   [d] Implicit

12. If $y = \frac{x+1}{x}$, then $\frac{dy}{dx} = ?$

[a] $-\frac{1}{x^2}$ [b] $\frac{x+1}{x^2}$ [c] $\frac{2}{x^2}$ [d] $\frac{x^2-1}{x^2}$

13. Given $f(x) = \frac{1}{x} - 1$ then $f(2) = ?$

[a] 1   [b] 2   [c] $-\frac{1}{2}$ [d] 3

14. Subset of population is called:

[a] Raw data   [b] Secondary data

[c] Population   [d] Sample

15. mx^{m-1} is the differential w.r.t. x of:

[a] $m(m-1)x^{m-2}$ [b] $(m-1)x^{m-2}$

[c] x^m [d] mx^m

Answer Key

1	c	2	a	3	c	4	b	5	b
6	b	7	a	8	b	9	b	10	c
11	b	12	a	13	c	14	d	15	a

****

DAE / IIA - 2017

MATH - 233 APPLIED MATHEMATICS - II

PAPER 'B' PART - B (SUBJECTIVE)

Time : 2 : 30Hrs Marks : 60

Section - I

Q.1. Write short answers to any Eighteen (18) questions.

1. If $f(x) = \log x$, prove that:

$f(x^a) = af(x)$

Sol. L.H.S. $= f(x^a)$

$= \log x^a = a\log x$

$= af(x) = \text{R.H.S.}$ **Proved.**

---

2. If $f(x) = \dfrac{1}{1-x}$, then find $f[f(5)]$

Sol. As, $f(x) = \dfrac{1}{1-x} \rightarrow (i)$

Put x = 5 in eq.(i)

$$f(5) = \frac{1}{1-5} = \frac{1}{-4} = -\frac{1}{4}$$

Put x = f(5) in eq.(i)

$$f[f(5)] = \frac{1}{1-\left(-\frac{1}{4}\right)}$$

$$= \frac{1}{\frac{4+1}{4}} = \frac{1}{\frac{5}{4}} = \boxed{\frac{4}{5}}$$

---

3. Find: $\lim\limits_{x\to 0} \dfrac{\sqrt{1+x}-1}{x}$

Sol. $\lim\limits_{x\to 0} \dfrac{\sqrt{1+x}-1}{x}$ $\left(\dfrac{0}{0}\right)$ form

$$= \lim_{x\to 0} \frac{\sqrt{1+x}-1}{x} \times \frac{\sqrt{1+x}+1}{\sqrt{1+x}+1}$$

$$= \lim_{x\to 0} \frac{\left(\sqrt{1+x}\right)^2 - (1)^2}{x\left(\sqrt{1+x}+1\right)}$$

$$= \lim_{x\to 0} \frac{\cancel{1}+x-\cancel{1}}{x\left(\sqrt{1+x}+1\right)}$$

$$= \lim_{x\to 0} \frac{x}{x\left(\sqrt{1+x}+1\right)}$$

$$= \lim_{x\to 0} \frac{1}{\sqrt{1+x}+1} = \frac{1}{\sqrt{1+0}+1}$$

$$= \frac{1}{\sqrt{1}+1} = \frac{1}{1+1} = \boxed{\frac{1}{2}}$$

---

4. ~~**Evaluate:** $\lim\limits_{x\to 0} \dfrac{\sin x^\circ}{x}$~~

Sol. $\lim\limits_{x\to 0} \dfrac{\sin x^\circ}{x}$

$$= \lim_{x\to 0} \frac{\sin \frac{\pi x}{180}}{x}$$

$$= \lim_{x\to 0} \frac{\sin \frac{\pi x}{180}}{\frac{\pi x}{180}} \times \frac{\pi}{180}$$

$$= \frac{\pi}{180} \lim_{x\to 0} \frac{\sin \frac{\pi x}{180}}{\frac{\pi x}{180}}$$

$$= \frac{\pi}{180}(1) = \boxed{\frac{\pi}{180}}$$

---

5. ~~**Differentiate $x^{2/3}$ by ab-initio method.**~~

Sol. Let, $y = x^{2/3} \rightarrow (i)$

Step-I:

then $y + \delta y = (x+\delta x)^{2/3} \rightarrow (ii)$

Step-II:

Subtracting eq. (i) from eq. (ii), we have:

$$y+\delta y-y=(x+\delta x)^{2/3}-x^{2/3}$$

$$\delta y=x^{2/3}\left(1+\frac{\delta x}{x}\right)^{2/3}-x^{2/3}$$

$$\delta y=x^{2/3}\left[1+\left(\frac{2}{3}\right)\left(\frac{\delta x}{x}\right)+\frac{1}{2!}\left(\frac{2}{3}\right)\left(\frac{2}{3}-1\right)\left(\frac{\delta x}{x}\right)^2+\ldots\right]-x^{2/3}$$

$$\delta y=x^{2/3}+\left(\frac{2}{3}\right)x^{2/3}\frac{\delta x}{x}+\frac{1}{2}\left(\frac{2}{3}\right)\left(-\frac{1}{3}\right)x^{2/3}\frac{\delta x^2}{x^2}+\ldots-x^{2/3}$$

$$\delta y=\frac{2}{3}\frac{\delta x}{x^{1-\frac{2}{3}}}-\frac{2}{18}\frac{\delta x^2}{x^{2-\frac{2}{3}}}+\ldots$$

$$\delta y=\delta x\left(\frac{2}{3x^{1/3}}-\frac{\delta x}{9x^{4/3}}+\ldots\right)$$

Step-III: Dividing both sides by 'δx':

$$\frac{\delta y}{\delta x}=\frac{2}{3x^{1/3}}-\frac{\delta x}{9x^{4/3}}+\ldots$$

Step-IV:

Taking limit $\delta x\to 0$ on both sides:

$$\lim_{\delta x\to 0}\frac{\delta y}{\delta x}=\lim_{\delta x\to 0}\left(\frac{2}{3x^{1/3}}-\frac{\delta x}{9x^{4/3}}+\ldots\right)$$

$$\frac{dy}{dx}=\frac{2}{3x^{1/3}}-\frac{0}{9x^{4/3}}+\ldots$$

$$\frac{dy}{dx}=\frac{2}{3x^{1/3}}-0+\ldots=\boxed{\frac{2}{3x^{1/3}}}$$

6. If $y=\sqrt{\dfrac{a+x}{a-x}}$, find $\dfrac{dy}{dx}$

Sol. $$\frac{d}{dx}(y)=\frac{d}{dx}\left(\sqrt{\frac{a+x}{a-x}}\right)$$

$$\frac{dy}{dx}=\frac{1}{2}\left(\frac{a+x}{a-x}\right)^{-1/2}\left(\frac{d}{dx}\left(\frac{a+x}{a-x}\right)\right)$$

{using Quotient Rule}

$$\frac{dy}{dx}=\frac{1}{2}\left(\frac{a-x}{a+x}\right)^{1/2}\left(\frac{(a-x)\left(\frac{d}{dx}(a+x)\right)-(a+x)\left(\frac{d}{dx}(a-x)\right)}{(a-x)^2}\right)$$

$$\frac{dy}{dx}=\frac{(a-x)(0+1)-(a+x)(0-1)}{2\sqrt{a+x}\,(a-x)^{2-1/2}}$$

$$\frac{dy}{dx}=\frac{a-x+a+x}{2\sqrt{a+x}\,(a-x)^{3/2}}$$

$$\frac{dy}{dx}=\frac{2a}{2\sqrt{a+x}\sqrt{a-x}\,(a-x)}$$

$$\frac{dy}{dx}=\boxed{\frac{a}{(a-x)\sqrt{a^2-x^2}}}$$

7. Find $\dfrac{dy}{dx}$, If $\dfrac{1}{\sqrt{x}}+\dfrac{1}{\sqrt{y}}=\dfrac{1}{\sqrt{a}}$

Sol. Differentiate both sides w.r.t. 'x':

$$\frac{d}{dx}\left(\frac{1}{\sqrt{x}}+\frac{1}{\sqrt{y}}\right)=\frac{d}{dx}\left(\frac{1}{\sqrt{a}}\right)$$

$$\frac{d}{dx}\left(x^{-1/2}+y^{-1/2}\right)=0$$

$$\frac{-1}{2}x^{-3/2}+\left(-\frac{1}{2}\right)y^{-3/2}\frac{dy}{dx}=0$$

$$-\frac{1}{2}y^{-3/2}\frac{dy}{dx}=\frac{1}{2}x^{-3/2}$$

$$\frac{dy}{dx}=\left(\frac{x^{-3/2}}{2}\right)\left(-\frac{2}{y^{-3/2}}\right)$$

$$\frac{dy}{dx}=-\frac{y^{3/2}}{x^{3/2}}\Rightarrow\boxed{\frac{dy}{dx}=-\left(\frac{y}{x}\right)^{3/2}}$$

$$\frac{dy}{dx}=\frac{x^{-1/2}}{y^{-3/2}}=\left(\frac{x}{y}\right)^{-3/2}=-\left(\frac{y}{x}\right)^{-3/2}$$

8. Find $\dfrac{dy}{dx}$, If $ax^2+by^2+2hxy=0$

Sol. Differentiate both sides w.r.t. 'x':

$$\frac{d}{dx}\left(ax^2+by^2+2hxy\right)=\frac{d}{dx}(0)$$

$$a(2x)+b\left(2y\frac{d}{dx}(y)\right)+2h\left(\frac{d}{dx}(x)y+x\frac{d}{dx}(y)\right)=0$$

$$2ax+2by\frac{dy}{dx}+2h\left(1.y+x\frac{dy}{dx}\right)=0$$

$$2ax+2by\frac{dy}{dx}+2hy+2hx\frac{dy}{dx}=0$$

$$2by\frac{dy}{dx}+2hx\frac{dy}{dx}=-2ax-2hy$$

$$2\frac{dy}{dx}(by+hx)=-2(ax+hy)$$

$$\frac{dy}{dx}=\frac{-2(ax+hy)}{2(by+hx)}\Rightarrow\boxed{\frac{dy}{dx}=-\frac{ax+hy}{by+hx}}$$

9. ~~Differentiate $\frac{x^3}{1+x^3}$ w.r.t. x^3~~

Sol. Let $y=\frac{x^3}{1+x^3}$ and $t=x^3$

Differentiate both sides w.r.t. 'x':

$$\frac{d}{dx}(y)=\frac{d}{dx}\left(\frac{x^3}{1+x^3}\right)\{\text{using Quotient Rule}\}$$

$$\frac{dy}{dx}=\frac{(1+x^3)\left(\frac{d}{dx}(x^3)\right)-x^3\left(\frac{d}{dx}(1+x^3)\right)}{(1+x^3)^2}$$

$$\frac{dy}{dx}=\frac{(1+x^3)(3x^2)-x^3(0+3x^2)}{(1+x^3)^2}$$

$$\frac{dy}{dx}=\frac{3x^2+3x^5-3x^5}{(1+x^3)^2}=\frac{3x^2}{(1+x^3)^2}$$

$$\frac{d}{dx}(t)=\frac{d}{dx}(x^3)$$

$$\frac{dt}{dx}=3x^2\Rightarrow\frac{dx}{dt}=\frac{1}{3x^2}$$

using chain rule: $\frac{dy}{dt}=\frac{dy}{dx}\times\frac{dx}{dt}$

$$\frac{dy}{dt}=\frac{3x^2}{(1+x^3)^2}\cdot\frac{1}{3x^2}=\boxed{\frac{1}{(1+x^3)^2}}$$

10. Differentiate $\sqrt{\sin\sqrt{x}}$ w.r.t. 'x'.

Sol. $\frac{d}{dx}\left(\sqrt{\sin\sqrt{x}}\right)$

$$=\frac{1}{2}\left(\sin\sqrt{x}\right)^{\frac{1}{2}-1}\frac{d}{dx}\left(\sin\sqrt{x}\right)$$

$$=\frac{1}{2}\left(\sin\sqrt{x}\right)^{-\frac{1}{2}}\left(\cos\sqrt{x}\right)\frac{d}{dx}\left(\sqrt{x}\right)$$

$$=\frac{1}{2\sqrt{\sin\sqrt{x}}}\left(\cos\sqrt{x}\right)\left(\frac{1}{2}x^{\frac{1}{2}-1}\right)$$

$$=\frac{\cos\sqrt{x}}{2\sqrt{\sin\sqrt{x}}}\left(\frac{1}{2}x^{-\frac{1}{2}}\right)$$

$$=\frac{\cos\sqrt{x}}{2\sqrt{\sin\sqrt{x}}}\left(\frac{1}{2\sqrt{x}}\right)=\boxed{\frac{\cos\sqrt{x}}{4\sqrt{x}\sqrt{\sin\sqrt{x}}}}$$

11. **Find $\frac{dy}{dx}$ if $y=\frac{1+\tan x}{1-\tan x}$**

Sol. Differentiate both sides w.r.t. 'x':

$$\frac{d}{dx}(y)=\frac{d}{dx}\left(\frac{1+\tan x}{1-\tan x}\right)\left\{\begin{matrix}\text{using}\\\text{Quotient Rule}\end{matrix}\right\}$$

$$\frac{dy}{dx}=\frac{(1-\tan x)\left(\frac{d}{dx}(1+\tan x)\right)-(1+\tan x)\left(\frac{d}{dx}(1-\tan x)\right)}{(1-\tan x)^2}$$

$$\frac{dy}{dx}=\frac{(1-\tan x)(0+\sec^2 x)-(1+\tan x)(0-\sec^2 x)}{(1-\tan x)^2}$$

$$\frac{dy}{dx}=\frac{(1-\tan x)\sec^2 x+(1+\tan x)\sec^2 x}{(1-\tan x)^2}$$

$$\frac{dy}{dx}=\frac{\sec^2 x-\cancel{\tan x\sec^2 x}+\sec^2 x+\cancel{\tan x\sec^2 x}}{(1-\tan x)^2}$$

$$\frac{dy}{dx}=\frac{2\sec^2 x}{\left(1-\frac{\sin x}{\cos x}\right)^2}=\frac{2}{\cos^2 x\left(\frac{\cos x-\sin x}{\cos x}\right)^2}$$

$$\frac{dy}{dx}=\frac{2}{\cancel{\cos^2 x}\left(\frac{\cos^2 x+\sin^2 x-2\sin x\cos x}{\cancel{\cos^2 x}}\right)}$$

$$\frac{dy}{dx}=\frac{2}{1-2\sin x\cos x}=\boxed{\frac{2}{1-\sin 2x}}\left\{\because \sin 2x=2\sin x\cos x\right\}$$

12. ~~Find $\frac{dy}{dx}$ if $x = a\sec\theta,\ y = b\tan\theta$~~

Sol. $x = a\sec\theta,\ y = b\tan\theta$

Differentiate both equations both sides w.r.t. 'θ' :

$$\frac{d}{d\theta}(x) = \frac{d}{d\theta}(a\sec\theta)$$

$$\frac{dx}{d\theta} = a\sec\theta\tan\theta$$

$$\frac{d\theta}{dx} = \frac{1}{a\sec\theta\tan\theta}$$

$$\frac{d}{d\theta}(y) = \frac{d}{d\theta}(b\tan\theta)$$

$$\frac{dy}{d\theta} = b\sec^2\theta$$

By using Chain's Rule: $\frac{dy}{dx} = \frac{dy}{d\theta} \times \frac{d\theta}{dx}$

$$\frac{dy}{dx} = b\sec^2\theta\left(\frac{1}{a\sec\theta\tan\theta}\right) = \frac{b}{a}\frac{\sec\theta}{\tan\theta}$$

$$\frac{dy}{dx} = \frac{b}{a}\cot\theta.\sec\theta$$

$$\frac{dy}{dx} = \frac{b}{a}\frac{\cos\theta}{\sin\theta} \times \frac{1}{\cos\theta} = \boxed{\frac{b}{a}\operatorname{cosec}\theta}$$

13. **Find the derivative of $\frac{\tan x}{x^2}$**

Sol. $\frac{d}{dx}\left(\frac{\tan x}{x^2}\right)$ {using Quotient Rule}

$$= \frac{x^2\left(\frac{d}{dx}(\tan x)\right) - \tan x\left(\frac{d}{dx}(x^2)\right)}{(x^2)^2}$$

$$= \frac{x^2\sec^2 x - \tan x(2x)}{x^4}$$

$$= \frac{x\left[x\sec^2 x - 2\tan x\right]}{x^4}$$

$$= \boxed{\frac{x\sec^2 x - 2\tan x}{x^3}}$$

14. ~~Find the value of~~

~~$\frac{d}{dx}\left(\cos^{-1}\left(1 - 2x^2\right)\right)$~~

Sol. $\frac{d}{dx}\left(\cos^{-1}\left(1 - 2x^2\right)\right)$

$$= \frac{-1}{\sqrt{1 - (1 - 2x^2)^2}} \cdot \left(\frac{d}{dx}(1 - 2x^2)\right)$$

$$= \frac{-1}{\sqrt{1 - (1 - 4x^2 + 4x^4)}}.(0 - 2(2x))$$

$$= \frac{-1}{\sqrt{1 - 1 + 4x^2 - 4x^4}}(-4x)$$

$$= \frac{4x}{\sqrt{4x^2 - 4x^4}} = \frac{4x}{\sqrt{4x^2(1 - x^2)}}$$

$$= \frac{4x}{2x\sqrt{1 - x^2}} = \boxed{\frac{2}{\sqrt{1 - x^2}}}$$

15. ~~Find $\frac{d}{dx}\left(a^{x^2}\right)$~~

Sol. $\frac{d}{dx}\left(a^{x^2}\right)$

$$= a^{x^2}(\ln a)\left(\frac{d}{dx}(x^2)\right)$$

$$= a^{x^2}(\ln a)(2x) = \boxed{2x(\ln a)a^{x^2}}$$

16. ~~Find $\frac{d}{dx}\left(e^{2x}\cos 2x\right)$~~

Sol. $\frac{d}{dx}\left(e^{2x}\cos 2x\right)$

$$= \left(\frac{d}{dx}(e^{2x})\right)\cos 2x + e^{2x}\left(\frac{d}{dx}(\cos 2x)\right)$$

$$= e^{2x}\left(\frac{d}{dx}(2x)\right)\cos 2x + e^{2x}(-\sin 2x)\left(\frac{d}{dx}(2x)\right)$$

$$= e^{2x}(2)\cos 2x - e^{2x}\sin 2x(2)$$

$$= \boxed{2e^{2x}(\cos 2x - \sin 2x)}$$

17. **Differentiate $\ln\frac{x}{\sqrt{1 + x^2}}$ w.r.t. 'x'.**

Sol. $\frac{d}{dx}\left(\ell n\frac{x}{\sqrt{1+x^2}}\right)$

$=\frac{d}{dx}\left(\ell n x-\ell n\sqrt{1+x^2}\right)\left\{\begin{matrix}\text{using logrithm law}\\ \ell n\left(\frac{m}{n}\right)=\ell n(m)-\ell n(n)\end{matrix}\right\}$

$=\frac{d}{dx}\left(\ell n x-\frac{1}{2}\ell n(1+x^2)\right)$

$=\frac{1}{x}-\frac{1}{2}\cdot\frac{1}{1+x^2}\times(2x)$

$=\frac{1}{x}-\frac{x}{1+x^2}=\frac{1+x^2-x^2}{x(1+x^2)}=\boxed{\frac{1}{x(1+x^2)}}$

---

18. ~~**Find the derivative of** $x^y=y^x$~~

Sol. Taking 'ℓn' to both sides, we have:

$\ell n(x^y)=\ell n(y^x)$

$y(\ell n x)=x(\ell n y)\left\{\begin{matrix}\text{using logrithm law}\\ \ell n(m^n)=n\,\ell n(m)\end{matrix}\right\}$

Differentiate both sides w.r.t. 'x':

$\frac{d}{dx}(y(\ell n x))=\frac{d}{dx}(x(\ell n y))\left\{\begin{matrix}\text{using}\\ \text{Product Rule}\end{matrix}\right\}$

$\left(\frac{d}{dx}(y)\right)\ell n x+y\left(\frac{d}{dx}(\ell n x)\right)=\left(\frac{d}{dx}(x)\right)\ell n y+x\left(\frac{d}{dx}(\ell n y)\right)$

$\frac{dy}{dx}\ell n x+y.\frac{1}{x}=(1)\ell n y+x\frac{1}{y}\frac{dy}{dx}$

$\frac{dy}{dx}(\ell n x)-\frac{x}{y}\frac{dy}{dx}=\ell n y-\frac{y}{x}$

$\frac{dy}{dx}\left(\ell n x-\frac{x}{y}\right)=\ell n y-\frac{y}{x}$

$\frac{dy}{dx}\left[\frac{y\ell n x-x}{y}\right]=\frac{x\ell n y-y}{x}$

$\frac{dy}{dx}=\left(\frac{x\ell n y-y}{x}\right)\left(\frac{y}{y\ell n x-x}\right)$

$\frac{dy}{dx}=\boxed{\frac{y}{x}\left[\frac{x\ell n y-y}{y\ell n x-x}\right]}$

---

19. Using differential find an approximate value of $\sqrt[3]{124}$

Sol. Let $y=\sqrt[3]{x}\ \rightarrow(i)$

Where $x=125$ & $dx=\Delta x=-1$

$\frac{d}{dx}(y)=\frac{d}{dx}(\sqrt[3]{x})$

$\frac{dy}{dx}=\frac{1}{3}x^{\frac{1}{3}-1}=\frac{1}{3}x^{-2/3}=\frac{1}{3x^{2/3}}$

$dy=\frac{1}{3x^{2/3}}dx\ \rightarrow(ii)$

Put $x=125$ & $dx=\Delta x=-1$

in eq.(i) & eq.(ii)

$y=\sqrt[3]{125}=5$ and

$dy=\frac{1}{3(125)^{2/3}}(-1)=-0.0133$

$\sqrt[3]{124}=y+dy=5+(-0.0133)$

$=5-0.0133=\boxed{4.9867}$

---

20. If $y=\cos 3x+\sin 3x$, show that: $y_2+9y=0$

Sol. $y=\cos 3x+\sin 3x$

Differentiate both sides w.r.t. 'x':

$\frac{d}{dx}(y)=\frac{d}{dx}(\cos 3x+\sin 3x)$

$y_1=-\sin 3x(3)+\cos 3x(3)$

$y_1=-3\sin 3x+3\cos 3x$

Differentiate both sides w.r.t. 'x':

$\frac{d}{dx}(y_1)=\frac{d}{dx}(-3\sin 3x+3\cos 3x)$

$y_2=-3\cos 3x(3)+3(-\sin 3x)(3)$

$y_2=-9\cos 3x-9\sin 3x$

$y_2=-9(\cos 3x+\sin 3x)$

$y_2=-9y\Rightarrow y_2+9y=0$ **Proved.**

21. **If $s = \log t$, find the velocity and acceleration at $t = 3\sec$.**

Sol. $s = \log t$

Differentiate both sides w.r.t. 't':

$$\frac{d}{dt}(s) = \frac{d}{dt}(\log t)$$

$$v = \frac{1}{t} \rightarrow (i)$$

Differentiate both sides w.r.t. 't':

$$\frac{d}{dt}(v) = dt\left(\frac{1}{t}\right)$$

$$a = -\frac{1}{t^2} \rightarrow (ii)$$

Put $t = 3$ in eq.(i) & eq(ii)

$$\boxed{v = \frac{1}{3}\ m/s}$$

$$\& \quad a = -\frac{1}{(3)^2} = \boxed{\frac{-1}{9} m/sec^2}$$

22. **The arithmetic mean of 7 values is 6 find the sum of values.**

Sol. Here A.M = 6, n = 7, & $\sum x = ?$

As, A.M. = 6

$$\frac{\sum x}{n} = 6$$

$$\frac{\sum x}{7} = 6$$

$$\sum x = 6 \times 7$$

$$\boxed{\sum x = 42}$$

23. **What are mutually exclusive events?**

Sol. When there is nothing common between different events, then they are called mutually exclusive events.

24. ~~**Find standard deviation of the values:**~~ **2, 4, 6, 8, 10.**

Sol.

x	x^2
2	4
4	16
6	36
8	64
10	100
$\sum x = 30$	$\sum x^2 = 220$

$$S.D. = \sqrt{\frac{\sum x^2}{n} - \left(\frac{\sum x}{n}\right)^2}$$

$$\sigma = \sqrt{\left(\frac{220}{5}\right) - \left(\frac{30}{5}\right)^2}$$

$$\sigma = \sqrt{44 - 36}$$

$$\sigma = \sqrt{8} = \boxed{2.83}$$

25. **If two coins are tossed find the probability that only one head.**

Sol. $S = \{HH,\ HT,\ TH,\ TT\}$, $n(S) = 4$

Let A be event that only one head appears. $A = \{HT,\ TH\}$

$$n(A) = 2 \therefore P(A) = \frac{n(A)}{n(S)} = \frac{2}{4} = \boxed{\frac{1}{2}}$$

26. **If two dice are rolled, what is probability of getting same number.**

Sol.

$$S = \begin{Bmatrix} (1,1),(1,2),(1,3),(1,4),(1,5),(1,6), \\ (2,1),(2,2),(2,3),(2,4),(2,5),(2,6), \\ (3,1),(3,2),(3,3),(3,4),(3,5),(3,6), \\ (4,1),(4,2),(4,3),(4,4),(4,5),(4,6), \\ (5,1),(5,2),(5,3),(5,4),(5,5),(5,6), \\ (6,1),(6,2),(6,3),(6,4),(6,5),(6,6), \end{Bmatrix}$$

$$n(S) = 36$$

Let A be event that same numbers appears. $A = \begin{Bmatrix} (1,1),(2,2),(3,3), \\ (4,4),(5,5),(6,6) \end{Bmatrix}$

$$n(A) = 6, \therefore P(A) = \frac{n(A)}{n(S)} = \frac{6}{36} = \boxed{\frac{1}{6}}$$

27. A card is drawn at random from a well shuffled pack of 52 cards. What is the probability of jack?

Sol. $n(S) = 52$

Let A be event that Jack card is drawn. $n(A) = 4$

$\therefore P(A) = \frac{n(A)}{n(S)} = \frac{4}{52} = \boxed{\frac{1}{13}}$

Section - II

Note : Attemp any three (3) questions $3 \times 8 = 24$

Q.2.(a) If $f(x) = \log\frac{1-x}{1+x}$, Prove that: $f(x) + f(y) = f\left(\frac{x+y}{1+xy}\right)$

Sol. See Q.13 of Ex # 1.1 (Page # 11)

(b) Evaluate $\lim\limits_{h \to 0} \frac{\sqrt{x+h} - \sqrt{x}}{h}$

Sol. See Q.2(iv) of Ex # 1.2 (Page # 16)

Q.3.(a) Differentiate $(ax^2 + b)(cx^2 + d)$ w.r.t. 'x'.

Sol. See Q.2(iv) of Ex # 1.2 (Page # 16)

(b) Find $\frac{dy}{dx}$ if $x = \frac{3at}{1+t^3}$, $y = \frac{3at^2}{1+t^3}$

Sol. See Q.2(v) of Ex # 2.3 (Page # 76)

Q.4.(a) Find the derivative of $(ax+b)\sqrt{1+\sin 2x}$

Sol. See Q.3(v) of Ex # 3.1 (Page # 115)

(b) Differentiate $\sec^{-1}\left(\frac{x^2+1}{x^2-1}\right)$ w.r.t. 'x'.

Sol. See Q.2(iv) of Ex # 3.2 (Page # 131)

Q.5. Prove that x^x has a minimum value at $x = \frac{1}{e}$.

Sol. See Q.7 of Ex # 4.2 (Page # 210)

Q.6.(a) Calculate mean median, mode from the following frequency table:

Height in cm	No. of Boys
59	1
58	3
57	7
56	8
55	25
54	30
53	55
52	50
51	40
50	38
49	30
48	9
47	4

Sol. See Q.10 of Ex # 5.1 (Page # 237)

****