DAE / IIA - 2017

MATH - 212 APPLIED MATHEMATICS - II

PART - A (OBJECTIVE)

Time : 30 Minutes Marks : 20

Q.1 : *Encircle the correct answer.*

1. A function $f(x) = x^2 + 2x + 3$ is:

[a] Odd [b] Even
[c] Implicit [d] Explicit

2. $\lim_{x \to 0} (1 + x)^{1/x} = ?$

[a] 0 [b] 1
[c] e [d] e^2

3. $\frac{d}{dx}(ax+b)^2 = ?$

[a] $2(ax+b)$ [b] $2a(ax+b)$
[c] $\frac{(ax+b)^3}{3}$ [d] $2(ax+b)b$

4. If $y = \frac{x+1}{x}$, then $\frac{dy}{dx} = ?$

[a] $-\frac{1}{x^2}$ [b] $\frac{x+1}{x^2}$
[c] $\frac{2}{x^2}$ [d] $\frac{x^2-1}{x^2}$

5. $\frac{d}{dx}(\operatorname{cosec} 3x) =$

[a] $-\operatorname{cosec} 3x \cot 3x$
[b] $-3 \operatorname{cosec} 3x \cot 3x$
[c] $\cot 3x$ [d] $\operatorname{cosec} 3x$

6. $\frac{d}{dx}(a^x) = ?$

[a] $a^x \ln a$ [b] xa^{x-1}
[c] a^{x-1} [d] a^x

7. $\frac{d}{dx}\{\ln(x^2+1)\} = ?$

[a] $\frac{2x}{x^2+1}$ [b] $\frac{x}{x^2+1}$
[c] $\ln(2x+1)$ [d] $2x\ln(2x+1)$

8. A function is maximum at a point if its 2^{nd} derivative is:

[a] $+ve$ [b] $-ve$
[c] zero [d] None of these

9. $\int (e^{2x})\, dx = ?$

[a] $\frac{e^{2x}}{2}$ [b] $\frac{e^{x^2}}{2}$
[c] $2e^{2x}$ [d] $\frac{e^{2x+1}}{2}$

10. $\int \left(\frac{a+x}{x}\right) dx = ?$

[a] $a \ln x + x$ [b] $\frac{(ax+b)^2}{2}$
[c] $\ln x + a$ [d] $x + a$

11. $\int (ax+b)^3\, dx = ?$

[a] $3(ax+b)^2$ [b] $3a(ax+b)^2$
[c] $\frac{(ax+b)^3}{4a}$ [d] $\frac{(ax+b)^4}{4a}$

12. $\int (\operatorname{cosec} x)\, dx = ?$

[a] $\ln(\operatorname{cosec} x - \cot x)$
[b] $\ln \sec x$
[c] $\ln(\operatorname{cosec} x + \cot x)$
[d] $\cos x$

13. $\int_0^{\pi/4} (\sec^2 x)\, dx = ?$

[a] 1 [b] 2
[c] 0 [d] 3

14. $\int_1^2 (3x^2)\, dx = ?$

[a] 7 [b] 8
[c] 6 [d] 9

15. Equation of line in slope intercept form is:

[a] $\frac{x}{a}+\frac{y}{b}=1$

[b] $y=mx+c$

[c] $y-y_1=m(x-x_1)$

[d] $y+y_1=m(x+x_1)$

16. When two lines are perpendicular:

[a] $m_1=m_2$ [b] $m_1m_2=-1$

[c] $m_1=-m_2$ [d] $m_1m_2=1$

17. The slope of x-axis is:

[a] $0°$ [b] $30°$

[c] $45°$ [d] $60°$

18. Give three points are collinear if their slopes are:

[a] Equal [b] Unequal

[c] $m_1m_2=-1$ [d] $m_1m_2=1$

19. Center of the circle $(x-1)^2+(y-2)^2=16$ is:

[a] $(1,2)$ [b] $(2,1)$

[c] $(4,0)$ [d] $(-1,-2)$

20. If the radius $r^2=g^2+f^2-c$ is negative, the circle is:

[a] Real [b] Imaginary

[c] Point [d] None of these

Answer Key

1	d	2	c	3	b	4	a	5	b
6	b	7	a	8	b	9	a	10	a
11	d	12	a	13	c	14	a	15	b
16	b	17	a	18	a	19	a	20	b

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DAE / IIA - 2017

MATH - 212 APPLIED MATHEMATICS - II

PART - B (SUBJECTIVE)

Time :2:30Hrs Marks:60

Section - I

Q.1: Write short answers to any Twenty Five (25) of the follwing questions. **25 × 2 = 50**

1. If $f(x)=\frac{1}{1-x}$, then find $f[f(5)]$

Sol. As, $f(x)=\frac{1}{1-x} \rightarrow (i)$

Put $x = 5$ in eq.(i)

$$f(5)=\frac{1}{1-5}=\frac{1}{-4}=-\frac{1}{4}$$

Put $x = f(5)$ in eq.(i)

$$f[f(5)]=\frac{1}{1-\left(-\frac{1}{4}\right)}$$

$$=\frac{1}{\frac{4+1}{4}}=\frac{1}{\frac{5}{4}}=\boxed{\frac{4}{5}}$$

2. Find: $\lim_{x\to 0}\frac{\sqrt{1+x}-1}{x}$

Sol. $\lim_{x\to 0}\frac{\sqrt{1+x}-1}{x}$ $\left(\frac{0}{0}\right)$ form

$$=\lim_{x\to 0}\frac{\sqrt{1+x}-1}{x}\times\frac{\sqrt{1+x}+1}{\sqrt{1+x}+1}$$

$$=\lim_{x\to 0}\frac{\left(\sqrt{1+x}\right)^2-(1)^2}{x\left(\sqrt{1+x}+1\right)}$$

$$=\lim_{x\to 0}\frac{\cancel{1}+x-\cancel{1}}{x\left(\sqrt{1+x}+1\right)}$$

$$=\lim_{x\to 0}\frac{x}{x\left(\sqrt{1+x}+1\right)}$$

$$=\lim_{x\to 0}\frac{1}{\sqrt{1+x}+1}=\frac{1}{\sqrt{1+0}+1}$$

$$=\frac{1}{\sqrt{1}+1}=\frac{1}{1+1}=\boxed{\frac{1}{2}}$$

3. ~~Evaluate: $\lim\limits_{x\to 0}\frac{\tan x}{x}$~~

Sol. $\lim\limits_{x\to 0}\frac{\tan x}{x}\ \left(\frac{0}{0}\right)$ form

$=\lim\limits_{x\to 0}\frac{\sin x}{x.\cos x}\quad\left\{\because \tan x=\frac{\sin x}{\cos x}\right\}$

$=\lim\limits_{x\to 0}\frac{\sin x}{x}.\lim\limits_{x\to 0}\frac{1}{\cos x}$

$=(1).\frac{1}{\cos 0}=\frac{1}{1}=\boxed{1}$

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4. **Show that the function $f(x)=2x^3-9x$ is even function of x.**

Sol. $f(x)=2x^3-9x$

Replace 'x' by '−x', we have:

$f(-x)=2(-x)^3-9(-x)$

$f(-x)=-2x^3+9x$

$f(-x)=-(2x^3-9x)$

$f(-x)=-f(x)$

Hence $f(x)$ is an $\boxed{\text{Odd}}$ function.

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5. **Find $\frac{dy}{dx}$ if $\sqrt{x}+\sqrt{y}=5$**

Sol. $\sqrt{x}+\sqrt{y}=5$

Differentiate both sides w.r.t. 'x':

$\frac{d}{dx}\left(\sqrt{x}+\sqrt{y}\right)=\frac{d}{dx}(5)$

$\frac{1}{2}x^{-1/2}+\frac{1}{2}y^{-1/2}\frac{d}{dx}(y)=0$

$\frac{1}{2\sqrt{x}}+\frac{1}{2\sqrt{y}}\frac{dy}{dx}=0$

$\frac{1}{2\sqrt{y}}\frac{dy}{dx}=-\frac{1}{2\sqrt{x}}$

$\frac{dy}{dx}=-\left(\frac{1}{2\sqrt{x}}\right)\left(2\sqrt{y}\right)$

$\frac{dy}{dx}=-\frac{\sqrt{y}}{\sqrt{x}}\ \Rightarrow\ \boxed{\frac{dy}{dx}=-\sqrt{\frac{y}{x}}}$

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6. **If $y=\left(3x^2+2x+9\right)^7$, find $\frac{dy}{dx}$**

Sol. $y=\left(3x^2+2x+9\right)^7$

Differentiate both sides w.r.t. 'x':

$\frac{d}{dx}(y)=\frac{d}{dx}\left(3x^2+2x+9\right)^7$

$\frac{dy}{dx}=7\left(3x^2+2x+9\right)^6\left[\frac{d}{dx}\left(3x^2+2x+9\right)\right]$

$\frac{dy}{dx}=7\left(3x^2+2x+9\right)^6\left[3(2x)+2(1)+0\right]$

$\frac{dy}{dx}=7\left(3x^2+2x+9\right)^6(6x+2)$

$\boxed{\frac{dy}{dx}=7(6x+2)\left(3x^2+2x+9\right)^6}$

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7. ~~**Differentiate $\frac{x^2}{1+x^2}$ w.r.t. x^2.**~~

Sol. Let, $y=\frac{x^2}{1+x^2}$ and $t=x^2$

Differentiate both sides w.r.t. 'x':

$\frac{d}{dx}(y)=\frac{d}{dx}\left(\frac{x^2}{1+x^2}\right)$ {using Quotient Rule}

$\frac{dy}{dx}=\frac{\left(1+x^2\right)\frac{d}{dx}\left(x^2\right)-x^2\frac{d}{dx}\left(1+x^2\right)}{\left(1+x^2\right)^2}$

$\frac{dy}{dx}=\frac{\left(1+x^2\right)(2x)-x^2(0+2x)}{\left(1+x^2\right)^2}$

$\frac{dy}{dx}=\frac{2x+2x^3-2x^3}{\left(1+x^2\right)^2}\Rightarrow\frac{dy}{dx}=\frac{2x}{\left(1+x^2\right)^2}$

$\frac{d}{dx}(t)=\frac{d}{dx}\left(x^2\right)$

$\frac{dt}{dx}=2x\ \Rightarrow\ \frac{dx}{dt}=\frac{1}{2x}$

using chain rule: $\frac{dy}{dt}=\frac{dy}{dx}\times\frac{dx}{dt}$

$\frac{dy}{dt}=\frac{2x}{\left(1+x^2\right)^2}\times\frac{1}{2x}=\boxed{\frac{1}{\left(1+x^2\right)^2}}$

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8. If $y=\sqrt{1+x^2}$, show that $y\dfrac{dy}{dx}=x$

Sol. $y=\sqrt{1+x^2}$

Differentiate both sides w.r.t. 'x' :

$$\frac{d}{dx}(y)=\frac{d}{dx}\left(\sqrt{1+x^2}\right)$$

$$\frac{dy}{dx}=\frac{1}{2}\left(1+x^2\right)^{-1/2}\left(\frac{d}{dx}\left(1+x^2\right)\right)$$

$$\frac{dy}{dx}=\frac{1}{2\sqrt{1+x^2}}(0+2x)$$

$$\frac{dy}{dx}=\frac{x}{\sqrt{1+x^2}}$$

$$\text{L.H.S.}=y\frac{dy}{dx}$$

$$=\sqrt{1+x^2}\left(\frac{x}{\sqrt{1+x^2}}\right)$$

$=x=\text{R.H.S.}$ **Proved.**

9. Find the derivative of $\cos(\cot x)$

Sol. $\dfrac{d}{dx}\left(\cos\left(\cot x\right)\right)$

$$=-\sin(\cot x)\frac{d}{dx}(\cot x)$$

$$=-\sin(\cot x)\left(-\operatorname{cosec}^2 x\right)$$

$$=\boxed{\sin(\cot x)\left(\operatorname{cosec}^2 x\right)}$$

10. ~~Find $\dfrac{d}{dx}\left(e^{2x}\cos 2x\right)$~~

Sol. $\dfrac{d}{dx}\left(e^{2x}\cos 2x\right)$

$$=\left(\frac{d}{dx}\left(e^{2x}\right)\right)\cos 2x+e^{2x}\left(\frac{d}{dx}(\cos 2x)\right)$$

$$=e^{2x}\left(\frac{d}{dx}(2x)\right)\cos 2x+e^{2x}(-\sin 2x)\left(\frac{d}{dx}(2x)\right)$$

$$=e^{2x}(2)\cos 2x-e^{2x}\sin 2x(2)$$

$$=\boxed{2e^{2x}(\cos 2x-\sin 2x)}$$

11. Find the derivative of $\dfrac{\tan x}{x^2}$

Sol. $\dfrac{d}{dx}\left(\dfrac{\tan x}{x^2}\right)$ {using Quotient Rule}

$$=\frac{x^2\left(\frac{d}{dx}(\tan x)\right)-\tan x\left(\frac{d}{dx}(x^2)\right)}{\left(x^2\right)^2}$$

$$=\frac{x^2\sec^2 x-\tan x(2x)}{x^4}$$

$$=\frac{x\left[x\sec^2 x-2\tan x\right]}{x^4}=\boxed{\frac{x\sec^2 x-2\tan x}{x^3}}$$

12. Differentiate $\tan^{-1}\sqrt{x}$ w.r.t. 'x'.

Sol. $\dfrac{d}{dx}\left(\tan^{-1}\sqrt{x}\right)$

$$=\frac{1}{1+(\sqrt{x})^2}\cdot\frac{d}{dx}\left(\sqrt{x}\right)$$

$$=\frac{1}{1+x}\cdot\frac{1}{2}\left(x^{-1/2}\right)=\boxed{\frac{1}{2\sqrt{x}(1+x)}}$$

13. If $y=x^4-3x^2+4x-1$, find $\dfrac{d^2y}{dx^2}$

Sol. $y=x^4-3x^2+4x-1$

Differentiate both sides w.r.t. 'x' :

$$\frac{d}{dx}(y)=\frac{d}{dx}\left(x^4-3x^2+4x-1\right)$$

$$\frac{dy}{dx}=4x^3-3(2x)+4(1)-0$$

$$\frac{dy}{dx}=4x^3-6x+4$$

Differentiate both sides w.r.t. 'x' :

$$\frac{d}{dx}\left(\frac{dy}{dx}\right)=\frac{d}{dx}\left(4x^3-6x+4\right)$$

$$\frac{d^2y}{dx^2}=4\left(3x^2\right)-6(1)+0$$

$$\boxed{\frac{d^2y}{dx^2}=12x^2-6}$$

14. Find the derivative of $\sin^2 x \cos^3 x$ w.r.t. 'x'.

Sol. $\frac{d}{dx}\left(\sin^m x.\ \sin mx\right)$

$=\left(\frac{d}{dx}\left(\sin^m x\right)\right)\sin mx+\sin^m x\left(\frac{d}{dx}\left(\sin mx\right)\right)$

$=m\sin^{m-1}x\left(\frac{d}{dx}\left(\sin x\right)\right)\sin mx+\sin^m x.\cos mx\left(\frac{d}{dx}\left(mx\right)\right)$

$=m\sin^{m-1}x.\cos x.\ \sin mx+\sin^m x.\cos mx\left(m\right)$

$=m\sin^{m-1}x\cos x\ \sin mx+m\sin^m x\cos mx$

$=m\sin^{m-1}x\left[\cos x\sin mx+\sin x\cos mx\right]$

$=m\sin^{m-1}x\left[\sin mx.\cos x+\cos mx\ \sin x\right]$

$=m\sin^{m-1}x\ \sin\left(mx+x\right)$ $\left\{\begin{array}{l}\because\ \sin(\alpha+\beta)\\ =\sin\alpha\cos\beta+\cos\alpha\sin\beta\end{array}\right\}$

$=\boxed{m\sin^{m-1}x\ \sin\left(1+m\right)x}$

15. If $s=\sin 2t$, find the velocity at $t=\frac{\pi}{6}$.

Sol. $s=\sin 2t$

Differentiate both sides w.r.t. 't':

$v=\frac{ds}{dt}=\frac{d}{dt}\left(\sin 2t\right)$

$v=\cos 2t\left(\frac{d}{dt}\left(2t\right)\right)$

$v=\cos 2t\left(2\left(1\right)\right)$

$v=2\cos 2t$

At $t=\frac{\pi}{6}$

$v\Big|_{t=\frac{\pi}{6}}=2\cos 2\left(\pi/6\right)$

$v\Big|_{t=\frac{\pi}{6}}=2\left(\frac{1}{2}\right)=\boxed{1}$

16. Find the turning points of the function $x^3-3x^2-24x+10$

Sol. Let $y=x^3-3x^2-24x+10$

Differentiate both sides w.r.t. 'x':

$\frac{d}{dx}\left(y\right)=\frac{d}{dx}\left(x^3-3x^2-24x+10\right)$

$\frac{dy}{dx}=3x^2-3\left(2x\right)-24\left(1\right)+0$

$\frac{dy}{dx}=3x^2-6x-24$

For critical values, put $\frac{dy}{dx}=0$

$3x^2-6x-24=0$

Dividing each term on '3', we get:

$x^2-2x-8=0$

$x^2-4x+2x-8=0$

$x\left(x-4\right)+2\left(x-4\right)=0$

$\left(x-4\right)\left(x+2\right)=0$

Either | **OR**

$x-4=0$ | $x+2=0$

$\boxed{x=4}$ | $\boxed{x=-2}$

17. Find $\int\left(x+\frac{1}{x}\right)^2 dx$

Sol. $\int\left(x+\frac{1}{x}\right)^2 dx$

$=\int\left(x^2+\frac{1}{x^2}+2\right)dx$

$=\int\left(x^2+x^{-2}+2\right)dx$

$=\frac{x^3}{3}+\frac{x^{-1}}{-1}+2x+c$

$=\boxed{\frac{x^3}{3}-\frac{1}{x}+2x+c}$

18. Find $\int\left(\frac{x^2}{4+x^2}\right)dx$

Sol. $\int\left(\frac{x^2}{4+x^2}\right)dx$ {Improper Fraction}

$$= \int \left(1 - \frac{4}{4+x^2}\right) dx$$

$$= \int \left(1 - \frac{4}{(2)^2 + (x)^2}\right) dx$$

$$\begin{array}{r|l} x^2+4 & x^2 \quad (1) \\ & \underline{\pm x^2 \pm 4} \\ & -4 \end{array}$$

$$= x - 4.\frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right) + c$$

$$= \boxed{x - 2\tan^{-1}\left(\frac{x}{2}\right) + c}$$

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19. ~~**Find** $\int \left(e^{3x} + e^{5x}\right) dx$~~

Sol. $\int \left(e^{3x} + e^{5x}\right) dx$

$$= \boxed{\frac{e^{3x}}{3} + \frac{e^{5x}}{5} + c} \quad \left\{\begin{array}{l}\text{Using formula \# 05} \\ \text{from page \# 282}\end{array}\right\}$$

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20. **Evaluate** $\int \left(\tan^4 x + \tan^2 x\right) dx$

Sol. $\int \left(\tan^4 x + \tan^2 x\right) dx$

$$= \int \tan^2 x \left(\tan^2 x + 1\right) dx$$

$$= \int \tan^2 x.\ \sec^2 x\ dx \quad \because \left\{\begin{array}{l}1+\tan^2 x \\ =\sec^2 x\end{array}\right\}$$

$$= \frac{\tan^3 x}{3} + c = \boxed{\frac{1}{3}\tan^3 x + c}$$

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21. **Find** $\int x^4 \ \sec^2\left(x^5\right) dx$

Sol. $\int x^4 \ \sec^2\left(x^5\right) dx$

$$= \int \sec^2\left(x^5\right) x^4 dx$$

Put $x^5 = t$

$$\frac{d}{dx}\left(x^5\right) = \frac{d}{dx}(t)$$

$$5x^4 = \frac{dt}{dx}$$

$$x^4 dx = \frac{dt}{5}$$

$$= \int \left(\sec^2 t\right) \frac{dt}{5}$$

$$= \frac{1}{5}\int \left(\sec^2 t\right) dt$$

$$= \frac{1}{5}\tan t + c$$

$$= \boxed{\frac{1}{5}\tan x^5 + c}$$

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22. **Integrate** $\int \frac{\cos(\ell n x)}{x} dx$

Sol. $\int \frac{\cos(\ell n x)}{x} dx$

$$= \int \cos(\ell n x).\left(\frac{1}{x}\right) dx$$

Put $\ell n x = t$

$$\frac{d}{dx}(\ell n x) = \frac{d}{dx}(t)$$

$$\frac{1}{x} = \frac{dt}{dx}$$

$$\left(\frac{1}{x}\right) dx = dt$$

$$= \int \cos t\, dt$$

$$= \sin t + c = \boxed{\sin(\ell n x) + c}$$

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23. **Evaluate** $\int \left(x\, e^x\right) dx$

Sol. $\int \left(x\, e^x\right) dx$

Integrating by parts:

taking $u = x$ & $v = e^x$

$$= x\int e^x dx - \int \left(\frac{d}{dx}(x) \int e^x dx\right) dx$$

$$= x\ e^x - \int 1.e^x\ dx$$

$$= x\, e^x - \int e^x\ dx$$

$$= x\ e^x - e^x + c = \boxed{e^x(x-1) + c}$$

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24. **Integrate** $\int \frac{\cos^{-1} x}{\sqrt{1-x^2}} dx$

Sol. $\int \frac{\cos^{-1} x}{\sqrt{1-x^2}} dx$

$$= \int \cos^{-1} x.\frac{1}{\sqrt{1-x^2}} dx$$

$$= \int t(-dt)$$
$$= -\int t\, dt$$
$$= -\frac{t^2}{2} + c$$

Put $\cos^{-1} x = t$

$\frac{d}{dx}\left(\cos^{-1} x\right) = \frac{d}{dx}(t)$

$\frac{-1}{\sqrt{1-x^2}} = \frac{dt}{dx}$

$\frac{1}{\sqrt{1-x^2}} dx = -dt$

$$= \boxed{-\frac{1}{2}\left(\cos^{-1} x\right)^2 + c}$$

25. Evaluate $\int_{\pi/6}^{\pi/3} (\sin 2x)\,dx$

Sol. $\int_{\pi/6}^{\pi/3} (\sin 2x)\,dx$

$$= \left[\frac{-\cos 2x}{2}\right]_{\pi/6}^{\pi/3} = -\frac{1}{2}\left[\cos 2x\right]_{\pi/6}^{\pi/3}$$
$$= -\frac{1}{2}\left[\cos 2\left(\frac{\pi}{3}\right) - \cos 2\left(\frac{\pi}{6}\right)\right]$$
$$= -\frac{1}{2}\left[\cos 120^\circ - \cos 60^\circ\right]$$
$$= -\frac{1}{2}\left[-\frac{1}{2} - \frac{1}{2}\right] = -\frac{1}{2}[-1] = \boxed{1/2}$$

26. Evaluate $\int_{-\pi/2}^{\pi/2} (\cos x)\,dx$

Sol. $\int_{-\pi/2}^{\pi/2} (\cos x)\,dx$

$$= [\sin x]_{-\pi/2}^{\pi/2}$$
$$= \sin\left(\frac{\pi}{2}\right) - \sin\left(-\frac{\pi}{2}\right)$$
$$= \sin(90^\circ) - \sin(-90^\circ) \quad \left\{\begin{array}{l}\frac{\pi}{2}\times\frac{180}{\pi} = 90^\circ \\ -\frac{\pi}{2}\times\frac{180}{\pi} = -90^\circ\end{array}\right.$$
$$= 1 - (-1) \quad \left\{\begin{array}{c}\text{using calculator} \\ \sin(90^\circ) = 1 \ \&\ \sin(-90^\circ) = -1\end{array}\right.$$
$$= 1 + 1 = \boxed{2}$$

27. Calculate the integral $\int_1^3 \left(x - \frac{1}{x}\right) dx$

Sol. $\int_1^3 \left(x - \frac{1}{x}\right) dx$

$$= \left[\frac{x^2}{2} - \ell n x\right]_1^3$$
$$= \left[\frac{(3)^2}{2} - \ell n(3)\right] - \left[\frac{(1)^2}{2} - \ell n(1)\right]$$
$$= \frac{9}{2} - \ell n(3) - \frac{1}{2} + 0$$
$$= \frac{9}{2} - \frac{1}{2} - \ell n(3)$$
$$= \frac{9-1}{2} - \ell n(3)$$
$$= \frac{8}{2} - \ell n(3) = \boxed{4 - \ell n(3)}$$

28. Integrate $\int \frac{\tan(\ell n x)}{x} dx$

Sol. $\int \frac{\tan(\ell n x)}{x} dx$

$$= \int \tan(\ell n x) \cdot \frac{1}{x} dx$$
$$= \int (\tan t)\, dt$$
$$= \ell n \sec t + c$$

Put $\ell n x = t$

$\frac{d}{dx}(\ell n x) = \frac{d}{dx}(t)$

$\frac{1}{x} = \frac{dt}{dx}$

$\frac{1}{x} dx = dt$

$$= \boxed{\ell n \sec(\ell n x) + c}$$

29. ~~**Find the equation of a line through the points $(-1, 2)$ and $(3, 4)$.**~~

Sol. Slope $= \frac{y_2 - y_1}{x_2 - x_1} = \frac{4-2}{3-(-1)} = \frac{2}{4} = \frac{1}{2}$

Equation of line in point - slope form:

$$y - y_1 = m(x - x_1)$$

$$y-2=\frac{1}{2}(x-(-1))$$
$$2(y-2)=1(x+1)$$
$$2y-4=x+1$$
$$2y-4-x-1=0$$
$$-x+2y-5=0 \Rightarrow \boxed{x-2y+5=0}$$

30. **Find the distance between $(-4,2)$ & $(0,5)$.**

Sol. $D=\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}$

$$D=\sqrt{(-4-0)^2+(2-5)^2}$$
$$D=\sqrt{(-4)^2+(-3)^2}$$
$$D=\sqrt{16+9}=\sqrt{25}=\boxed{5}$$

31. **Find an equation of the line with slope $-\frac{2}{3}$ and having y-intercept 3.**

Sol. $\text{Here}: m=-\frac{2}{3}$ & $c=3$

Equation of line in slope - intercept form :

$$y=mx+c$$
$$y=\frac{-2}{3}x+3$$

Multipling each term both sides by 3, we get :

$$3y=-2x+9$$
$$3y+2x-9=0 \Rightarrow \boxed{2x+3y-9=0}$$

32. **Show that the points (1, 9), (-2, 3) and (-5, -3) are collinear.**

Sol. $\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}=\begin{vmatrix} 1 & 9 & 1 \\ -2 & 3 & 1 \\ -5 & -3 & 1 \end{vmatrix}$

$$=1\begin{vmatrix} 3 & 1 \\ -3 & 1 \end{vmatrix}-9\begin{vmatrix} -2 & 1 \\ -5 & 1 \end{vmatrix}+1\begin{vmatrix} -2 & 3 \\ -5 & -3 \end{vmatrix}$$
$$=1(3-(-3))-9(-2-(-5))+1(6-(-15))$$
$$=1(3+3)-9(-2+5)+1(6+15)$$
$$=1(6)-9(3)+1(21)=6-27+21=0$$

Hence given points are collinear. **Proved.**

33. **Find 'k' so that the lines $x-2y+1=0$, $2x-5y+3=0$, and $5x+9y+k=0$ are concurrent.**

Sol. As the given lines are concurrent, so

$$\Rightarrow \begin{vmatrix} 1 & -2 & 1 \\ 2 & -5 & 3 \\ 5 & 9 & k \end{vmatrix}=0$$
$$\Rightarrow 1\begin{vmatrix} -5 & 3 \\ 9 & k \end{vmatrix}-(-2)\begin{vmatrix} 2 & 3 \\ 5 & k \end{vmatrix}+1\begin{vmatrix} 2 & -5 \\ 5 & 9 \end{vmatrix}=0$$
$$1(-5k-27)+2(2k-15)+1(18+25)=0$$
$$-5k-27+4k-30-43=0$$
$$-k-14=0$$
$$-k=14 \Rightarrow \boxed{k=-14}$$

34. **What type of circle is represented by $x^2+y^2-2x+4y+8=0$**

Sol. $x^2+y^2-2x+4y+8=0$

Comparing this equation with general form of equation of circle:

$$x^2+y^2+2gx+2fy+c=0$$

$2g=-2$	$2f=4$	
$g=-\frac{2}{2}$	$f=\frac{4}{2}$	$c=8$
$g=-1$	$f=2$	

$$\text{Radius}=r=\sqrt{g^2+f^2-c}$$
$$r=\sqrt{(-1)^2+(2)^2-8}$$
$$r=\sqrt{1+4-8}=\sqrt{-3}=\boxed{\sqrt{3}i}$$

So, it is an **Imaginary circle.**

35. **Write the equation of circle with, center at (h, k) and radius 'r'.**

Sol. $\boxed{(x-h)^2+(y-k)^2=r^2}$

36. **Find center and radius of the circle**
$$x^2 + y^2 + 9x - 7y - 33 = 0$$

Sol. Comparing with general equation of circle.

$$x^2 + y^2 + 9x - 7y - 33 = 0$$

$2g = 9$, $g = \frac{9}{2}$ | $2f = -7$, $f = -\frac{7}{2}$ | $c = -33$

Center $= (-g, -f) =$

Center $= \left(-\frac{9}{2}, -\left(-\frac{7}{2}\right)\right) = \boxed{\left(-\frac{9}{2}, \frac{7}{2}\right)}$

$$\text{Radius} = r = \sqrt{g^2 + f^2 - c}$$

$$r = \sqrt{\left(\frac{9}{2}\right)^2 + \left(-\frac{7}{2}\right)^2 - (-33)}$$

$$r = \sqrt{\frac{81}{4} + \frac{49}{4} + 33}$$

$$r = \sqrt{\frac{81 + 49 + 132}{4}}$$

$$r = \sqrt{\frac{262}{4}} = \boxed{\sqrt{\frac{131}{2}}}$$

37. **Write the general form of the circle, also represent the center and radius in this form.**

Sol. $x^2 + y^2 + 2gx + 2fy + c = 0$

$\text{Center} = \boxed{(-g, -f)}$

$\& \ \text{Radius} = \boxed{\sqrt{g^2 + f^2 - c}}$

Section - II

Note: Attemp any three (3) questions $\boxed{3 \times 10 = 30}$

Q.2[a] Evaluate $\lim_{\theta \to 0} \frac{\tan\theta - \sin\theta}{\sin^3\theta}$

Sol. See Q.1(x) of Ex # 1.3 (Page # 25)

[b] **Find $\frac{dy}{dx}$ when**

$$x = \frac{a(1 - t^2)}{1 + t^2} \quad \text{and} \quad y = \frac{2bt}{1 + t^2}$$

Sol. See Q.3(iv) of Ex # 2.3 (Page # 72)

Q.3.[a] If $y = \tan\left(2\tan^{-1}\frac{x}{2}\right)$

Prove that: $\frac{dy}{dx} = 4\left(\frac{1 + y^2}{4 + x^2}\right)$.

Sol. See example # 10 of Chapter 03.

[b] **Find the maximum and minimum values of the function**

$$\frac{x^3}{3} - \frac{3x^2}{2} + 2x + 5$$

Sol. See Q.2(iv) of Ex # 4.2 (Page # 187)

Q.4.[a] Integrate $\int\left(\frac{1}{\sqrt{1 + x} - \sqrt{x}}\right)dx$

Sol. See Q.16 of Ex # 5.1 (Page # 232)

[b] Evaluate $\int (\tan x + \cot x)^2 \, dx$

Sol. See Q.10 of Ex # 5.2 (Page # 238)

Q.5.[a] Integrate $\int (x^2 \tan^{-1} x) \, dx$

Sol. See Q.2(ii) of Ex # 6.3 (Page # 285)

[b] **Show that the points A(2, 3), B(0, –1), C(-2, 1) are the vertices of an isosceles triangle.**

Sol. See Q.2[c] of Ex # 8.1 (Page # 355)

Q.6.[a] **Show that the two lines passing through the given points are perpendicular (0, -7), (8, -5) and (5, 7), (8, -5).**

Sol. See Q.1[a] of Ex # 8.3 (Page # 373)

[b] **Find the equation of the circle having (-2, 5) and (3, 4) as the end points of its diameter. Find also its center and radius.**

Sol. See Q.8[a] of Ex # 9 (Page # 447)

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