

DAE / IIA - 2016

MATH-212 APPLIED MATHEMATICS -II

PART - A (OBJECTIVE)

Time : 30 Minutes Marks : 20

Q.1: Encircle the correct answer.

1. Given $f(x) = \frac{1}{x} - 1$ then $f(2) = ?$

- [a] 1 [b] 2
[c] $-\frac{1}{2}$ [d] 3

2. $\lim_{\theta \rightarrow \frac{\pi}{2}} \frac{\sin \theta}{\theta} =$

- [a] 1 [b] $\frac{\pi}{2}$
[c] $\frac{2}{\pi}$ [d] $\frac{1}{2}$

3. $\frac{d}{dx}(2x+3)^4 =$

- [a] $8(2x+3)^3$ [b] $4(2x+3)^3$
[c] $(2x+3)^3$ [d] $4(2x+3)^2$

4. Second derivative of x^2 is;

- [a] 2 [b] $2x$
[c] zero [d] $2x^2$

5. $\frac{d}{dx}(\sin x^3) = ?$

- [a] $\cos x^3$ [b] $-\cos x^3$
[c] $3x \cos x^3$ [d] $3x^2 \cos x^3$

6. $\frac{d}{dx}(\sin^{-1} x) = ?$

- [a] $\frac{1}{\sqrt{x^2-1}}$ [b] $\frac{-1}{\sqrt{1-x^2}}$
[c] $\frac{\sin^{-1} x}{\sqrt{1-x^2}}$ [d] $\frac{1}{\sqrt{1-x^2}}$

7. $\frac{d}{dx}(a^x) = ?$

- [a] $a^x \ell \ln a$ [b] xa^{x-1}
[c] a^{x-1} [d] a^x

8. $\frac{d}{dx}(x^a) = ?$

- [a] ax^{a-1} [b] x^{a-1}
[c] ax^a [d] x^a

9. For an increasing function $\frac{dy}{dx}$ is:

- [a] +ve [b] -ve
[c] zero [d] None of these

10. $\int (x^3) dx =$

- [a] $\frac{x^4}{4}$ [b] $\frac{x^4}{3}$
[c] $3x^2$ [d] $4x^4$

11. $\int \left(\frac{\cos x}{\sin x} \right) dx =$

- [a] $\ell \ln \cos x$ [b] $\ell \ln \sin x$
[c] $\ell \ln \cot x$ [d] $\frac{\cos^2 x}{2}$

12. $\int (e^{2x}) dx = ?$

- [a] $\frac{e^{2x}}{2}$ [b] $\frac{e^{x^2}}{2}$
[c] $2e^{2x}$ [d] $\frac{e^{2x+1}}{2}$

13. $\int \left(\frac{1}{\sqrt{1-x^2}} \right) dx = ?$

- [a] $\sin^{-1} x$ [b] $\cos^{-1} x$
[c] $\sec^{-1} x$ [d] $\tan^{-1} x$

14. $\int_0^1 (1) dx =$

- [a] -1 [b] 0
[c] 1 [d] 2

15. Distance between (4, 3) and (7, 5) is:

- [a] 25 [b] $\sqrt{13}$
 [c] 5 [d] 7

16. Slope of the line through (x_1, y_1) and (x_2, y_2) :

- [a] $\frac{x_1 + x_2}{y_1 + y_2}$ [b] $\frac{y_2 + y_1}{x_2 + y_1}$
 [c] $\frac{y_2 - y_1}{x_2 - x_1}$ [d] $\frac{y_2 - y_1}{x_2 + x_1}$

17. Slope of y-axis is:

- [a] 0 [b] ∞
 [c] 1 [d] -1

18. $y = mx + c$ is the:

- [a] Slope intercept form
 [b] Intercepts form
 [c] Point - Slope form
 [d] Two - Points form

19. Center of the circle

$x^2 + y^2 - 2x - 4y = 8$ is:

- [a] (1, 2) [b] (2, 4)
 [c] (-1, -2) [d] (-2, -4)

20. Radius of the circle

$(x - 1)^2 + (y - 2)^2 = 16$ is:

- [a] 2 [b] 1
 [c] 4 [d] 16

Answer Key

1	c	2	c	3	a	4	a	5	d
6	d	7	a	8	a	9	a	10	a
11	b	12	a	13	a	14	c	15	b
16	c	17	b	18	a	19	a	20	c

DAE / IIA - 2016

MATH - 212 APPLIED MATHEMATICS - II

PART - B (SUBJECTIVE)

Time : 2 : 30 Hrs

Marks : 60

Section - I

Q.1 : Write short answers to any Twenty Five (25) of the following questions. 25 × 2 = 50

1. if $f(x) = \frac{x^2 - 3}{x + 4}$, find $f(-3)$

Sol. $f(x) = \frac{x^2 - 3}{x + 4}$

Put $x = -3$, we have :

$$f(-3) = \frac{(-3)^2 - 3}{-3 + 4} = \frac{9 - 3}{1} = \boxed{6}$$

2. Is the function even, odd or neither? $f(x) = x\sqrt{x^2 - 1}$

Sol. As, $f(x) = x\sqrt{x^2 - 1}$

Replace x by $-x$, we have :

$$f(-x) = -x\sqrt{(-x)^2 - 1}$$

$$f(-x) = -x\sqrt{x^2 - 1}$$

$$f(-x) = -f(x)$$

Hence $f(x)$ is an odd function.

3. Evaluate $\lim_{x \rightarrow -2} \frac{x^2}{x + 1}$

Sol. $\lim_{x \rightarrow -2} \frac{x^2}{x + 1} = \frac{(-2)^2}{-2 + 1} = \frac{4}{-1} = \boxed{-4}$

4. Evaluate $\lim_{\theta \rightarrow 0} \frac{\sin 7\theta}{\theta}$

Sol. $\lim_{\theta \rightarrow 0} \frac{\sin 7\theta}{\theta}$ $\left(\frac{0}{0}\right)$ form

$$= \lim_{\theta \rightarrow 0} 7 \times \frac{\sin 7\theta}{7\theta}$$

$$= 7(1) = \boxed{7}$$

5. Differentiate w.r.t. 'x'

$$-5 + 3x - \frac{3}{2}x^2 - 7x^3$$

Sol. $\frac{d}{dx} \left(-5 + 3x - \frac{3}{2}x^2 - 7x^3 \right)$
 $= 0 + 3(1) - \frac{3}{2}(2x) - 7(3x^2)$
 $= \boxed{3 - 3x - 21x^2}$

6. Differentiate $(x^2 + 3x + 9)^{3/2}$

w.r.t. 'x'.

Sol. $\frac{d}{dx} \left((x^2 + 3x + 9)^{3/2} \right)$
 $= \frac{3}{2}(x^2 + 3x + 9)^{3/2-1} \left(\frac{d}{dx}(x^2 + 3x + 9) \right)$
 $= \frac{3}{2}(x^2 + 3x + 9)^{1/2} (2x + 3(1) + 0)$
 $= \boxed{\frac{3}{2}(x^2 + 3x + 9)^{1/2} (2x + 3)}$

7. Find the derivative $\sin x^n$ w.r.t. 'x'.

Sol. $\frac{d}{dx}(\sin x^n) = \cos x^n \frac{d}{dx}(x^n)$
 $= \cos x^n \cdot nx^{n-1} \left(\frac{d}{dx}(x) \right)$
 $= n \cos x^n \cdot x^{n-1} \cdot (1) = \boxed{nx^{n-1} \cos x^n}$

8. Find $\frac{dy}{dx}$ if $xy + y^2 = 2$

Sol. $xy + y^2 = 2$

Differentiate both sides w.r.t. 'x':

$$\frac{d}{dx}(xy + y^2) = \frac{d}{dx}(2)$$

$$\left(\frac{d}{dx}(x) \right) \cdot y + x \left(\frac{d}{dx}(y) \right) + 2y \frac{d}{dx}(y) = 0$$

$$1 \cdot y + x \frac{dy}{dx} + 2y \frac{dy}{dx} = 0$$

$$\frac{dy}{dx}(x + 2y) = -y \Rightarrow \boxed{\frac{dy}{dx} = -\frac{y}{x + 2y}}$$

9. Find $\frac{dy}{dx}$ $x = t + 2, y = 2t^2 + 2$

Sol. Differentiate both sides w.r.t. 't':

$$\frac{d}{dt}(x) = \frac{d}{dt}(t + 2) \quad \left| \quad \frac{d}{dt}(y) = \frac{d}{dt}(2t^2 + 2) \right.$$

$$\frac{dx}{dt} = 1 + 0$$

$$\frac{dy}{dt} = 2(2t) + 0$$

$$\frac{dt}{dx} = 1$$

$$\frac{dy}{dt} = 4t$$

using chain rule: $\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$

$$\frac{dy}{dx} = (4t)(1) \Rightarrow \boxed{\frac{dy}{dx} = 4t}$$

10. Differentiate $\sin^{-1} \sqrt{x}$ w.r.t. 'x'.

Sol. $\frac{d}{dx}(\sin^{-1} \sqrt{x})$
 $= \frac{1}{\sqrt{1 - (\sqrt{x})^2}} \frac{d}{dx}(\sqrt{x})$
 $= \frac{1}{\sqrt{1 - x}} \cdot \frac{1}{2}(x)^{1/2-1} \frac{d}{dx}(x)$
 $= \frac{1}{\sqrt{1 - x}} \cdot \frac{1}{2}(x)^{-1/2}(1)$
 $= \frac{1}{\sqrt{1 - x}} \cdot \frac{1}{2\sqrt{x}} = \boxed{\frac{1}{2\sqrt{x}\sqrt{1 - x}}}$

11. Differentiate $\frac{x}{\ln x}$ w.r.t. 'x'.

Sol. $\frac{d}{dx} \left(\frac{x}{\ln x} \right)$ {using Quotient Rule}

$$= \frac{\ln x \cdot \left(\frac{d}{dx}(x) \right) - x \left(\frac{d}{dx}(\ln x) \right)}{(\ln x)^2}$$

$$= \frac{\ln x \cdot (1) - x \left(\frac{1}{x} \right)}{(\ln x)^2} = \boxed{\frac{\ln x - 1}{(\ln x)^2}}$$

12. Find $\frac{dy}{dx}$ for $e^{\sqrt{x+1}}$

Sol. Let $y = e^{\sqrt{x+1}}$
Differentiate both sides w.r.t. 'x':

$$\frac{d}{dx}(y) = \frac{d}{dx}(e^{\sqrt{x+1}})$$

$$\frac{dy}{dx} = e^{\sqrt{x+1}} \cdot \left(\frac{d}{dx}(\sqrt{x+1}) \right)$$

$$\frac{dy}{dx} = e^{\sqrt{x+1}} \cdot \frac{1}{2}(x+1)^{-\frac{1}{2}} \left(\frac{d}{dx}(x+1) \right)$$

$$\frac{dy}{dx} = e^{\sqrt{x+1}} \cdot \frac{1}{2\sqrt{x+1}} (1+0)$$

$$\boxed{\frac{dy}{dx} = \frac{e^{\sqrt{x+1}}}{2\sqrt{x+1}}}$$

13. Differentiate $\sin x$ w.r.t. $\tan x$

Sol. Let, $y = \sin x$ & $t = \tan x$
Differentiate both equations
both sides w.r.t. 'x':

$$\frac{d}{dx}(y) = \frac{d}{dx}(\sin x) \quad \left\{ \begin{array}{l} \frac{d}{dt}(t) = \frac{d}{dx}(\tan x) \\ \frac{dt}{dx} = \sec^2 x \\ \frac{dx}{dt} = \frac{1}{\sec^2 x} \end{array} \right.$$

$$\frac{dy}{dx} = \cos x$$

By using Chain's Rule:

$$\frac{dy}{dt} = \frac{dy}{dx} \cdot \frac{dx}{dt} = (\cos x) \left(\frac{1}{\sec^2 x} \right)$$

$$\frac{dy}{dt} = (\cos x)(\cos^2 x) \Rightarrow \boxed{\frac{dy}{dt} = \cos^3 x}$$

14. Find the derivative of $x \cot x$ w.r.t. 'x'.

Sol. $\frac{d}{dx}(x \cot x) \left\{ \begin{array}{l} \text{using} \\ \text{Product Rule} \end{array} \right\}$
 $= \left(\frac{d}{dx}(x) \right) \cot x + x \left(\frac{d}{dx}(\cot x) \right)$
 $= 1 \cdot \cot x + x(-\operatorname{cosec}^2 x)$
 $= \boxed{\cot x - x \operatorname{cosec}^2 x}$

15. Differentiate $\frac{x}{x^2+1}$ w.r.t. 'x'

Sol. Differentiate w.r.t. 'x':

$$\frac{d}{dx} \left(\frac{x}{x^2+1} \right) \left\{ \text{using Quotient Rule} \right\}$$

$$= \frac{(x^2+1) \left(\frac{d}{dx}(x) \right) - x \left(\frac{d}{dx}(x^2+1) \right)}{(x^2+1)^2}$$

$$= \frac{(x^2+1)(1) - x(2x+0)}{(x^2+1)^2}$$

$$= \frac{x^2+1-2x^2}{(x^2+1)^2} = \boxed{\frac{1-x^2}{(x^2+1)^2}}$$

16. Find the critical values (turning points) for 'x' of the function $5x^2 - 4x + 9$

Sol. Let $y = 5x^2 - 4x + 9$
Differentiate both sides w.r.t. 'x':

$$\frac{d}{dx}(y) = \frac{d}{dx}(5x^2 - 4x + 9)$$

$$\frac{dy}{dx} = 5(2x) - 4(1) + 0$$

$$\frac{dy}{dx} = 10x - 4$$

For critical values, put $\frac{dy}{dx} = 0$

$$10x - 4 = 0$$

$$10x = 4$$

$$x = \frac{4}{10} \Rightarrow \boxed{x = \frac{2}{5}}$$

17. Evaluate $\int (3x^2 + 2x + 1) dx$

Sol. $\int (3x^2 + 2x + 1) dx$
 $= 3 \frac{x^3}{3} + 2 \frac{x^2}{2} + x + c$
 $= \boxed{x^3 + x^2 + x + c}$

18. Evaluate $\int (e^x + e^{-x})^2 dx$

Sol.
$$\int (e^x + e^{-x})^2 dx$$

$$= \int \left[(e^x)^2 + (e^{-x})^2 + 2(e^x)(e^{-x}) \right] dx$$

$$= \int [e^{2x} + e^{-2x} + 2] dx$$

$$= \frac{e^{2x}}{2} + \frac{e^{-2x}}{-2} + 2x + c$$

$$= \boxed{\frac{1}{2}e^{2x} - \frac{1}{2}e^{-2x} + 2x + c}$$

19. Evaluate $\int (\sin x - \cos x)^2 dx$

Sol.
$$\int (\sin x - \cos x)^2 dx$$

$$= \int (\sin^2 x + \cos^2 x - 2 \sin x \cos x) dx$$

$$= \int (1 - \sin 2x) dx \because \begin{cases} \sin^2 x + \cos^2 x = 1 \\ \sin 2x = 2 \sin x \cos x \end{cases}$$

$$= x - \left(\frac{-\cos 2x}{2} \right) + c$$

$$= \boxed{x + \frac{1}{2} \cos 2x + c}$$

20. Find $\int (2x+9)^{-5/2} dx$

Sol.
$$\int (2x+9)^{-5/2} dx$$

$$= \frac{1}{2} \int (2x+9)^{-5/2} (2) dx$$

$$= \frac{1}{2} \left[\frac{(2x+9)^{-3/2}}{-3/2} \right] + c \quad \left\{ \begin{array}{l} \text{using} \\ \text{Rule-1} \end{array} \right\}$$

$$= \boxed{-\frac{1}{3} (2x+9)^{-3/2} + c}$$

21. Evaluate $\int (\cos^4 x \sin x) dx$

Sol.
$$\int (\cos^4 x \sin x) dx$$

$$= -\int \cos^4 x (-\sin x) dx$$

$$= -\frac{\cos^5 x}{5} + c = \boxed{-\frac{1}{5} \cos^5 x + c}$$

22. Evaluate $\int \left(\frac{x}{x^2+1} \right) dx$

Sol.
$$\int \left(\frac{x}{x^2+1} \right) dx$$

$$= \frac{1}{2} \int \left(\frac{2x}{x^2+1} \right) dx = \boxed{\frac{1}{2} \ln(x^2+1) + c}$$

23. Evaluate $\int \left(\frac{1}{\sqrt{x}} \sin \sqrt{x} \right) dx$

Sol.
$$\int \left(\frac{1}{\sqrt{x}} \sin \sqrt{x} \right) dx$$

Put $\sqrt{x} = t \Rightarrow \frac{d}{dx}(\sqrt{x}) = \frac{d}{dx}(t)$

$$\frac{1}{2} x^{-1/2} = \frac{dt}{dx}$$

$$\frac{1}{2\sqrt{x}} = \frac{dt}{dx} \Rightarrow \frac{1}{\sqrt{x}} dx = 2dt$$

$$= \int \sin \sqrt{x} \cdot \left(\frac{1}{\sqrt{x}} \right) dx$$

$$= \int (\sin t) (2dt)$$

$$= 2 \int (\sin t) dt$$

$$= 2(-\cos t) + c = \boxed{-2 \cos \sqrt{x} + c}$$

24. Evaluate $\int (\ell n x) dx$

Sol.
$$\int (\ell n x) dx$$

$$= \int (\ell n x \cdot 1) dx$$

Integrating by parts:
taking $u = \ell n x$ & $v = 1$

$$= \ell n x \int (1) dx - \int \left[\frac{d}{dx}(\ell n x) \int (1) dx \right] dx$$

$$= \ell n x (x) - \int \frac{1}{x} (x) dx$$

$$= x \ell n x - \int (1) dx$$

$$= x \ell n x - (x) + c = \boxed{x(\ell n x - 1) + c}$$

25. Evaluate $\int_1^3 (x^2) dx$

Sol.
$$\int_1^3 (x^2) dx$$

$$= \left[\frac{x^3}{3} \right]_1^3$$

$$= \frac{1}{3} [x^3]_1^3$$

$$= \frac{1}{3} [(3)^3 - (1)^3]$$

$$= \frac{1}{3} [27 - 1] = \boxed{\frac{26}{3}}$$

26. Evaluate $\int_0^{\pi/6} (2 \sin 2x) dx$

Sol.
$$\int_0^{\pi/6} (2 \sin 2x) dx$$

$$= 2 \left[-\frac{\cos 2x}{2} \right]_0^{\pi/6} = -[\cos 2x]_0^{\pi/6}$$

$$= -\left(\cos 2\left(\frac{\pi}{6}\right) - \cos 2(0) \right)$$

$$= -(\cos 2(30^\circ) - \cos 2(0^\circ))$$

$$= -(\cos 60^\circ - \cos 0^\circ)$$

$$= -\left(\frac{1}{2} - 1\right) \left\{ \begin{array}{l} \text{using calculator} \\ \cos 60^\circ = \frac{1}{2} \ \& \ \cos 0^\circ = 1 \end{array} \right\}$$

$$= -\left(\frac{1-2}{2}\right) = -\left(-\frac{1}{2}\right) = \boxed{\frac{1}{2}}$$

27. Evaluate $\int \left(\frac{1+x}{x}\right) dx$

Sol.
$$\int \left(\frac{1+x}{x}\right) dx$$

$$= \int \left(\frac{1}{x} + \frac{x}{x}\right) dx$$

$$= \int \left(\frac{1}{x} + 1\right) dx$$

$$= \boxed{\ell n x + x + c}$$

28. Find the distance between the points $(-3, -2)$ and $(-1, 5)$.

Sol. Distance between $(-3, -2)$ & $(-1, 5)$.

$$= \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

$$= \sqrt{(-3 - (-1))^2 + (-2 - 5)^2}$$

$$= \sqrt{(-2)^2 + (-7)^2}$$

$$= \sqrt{4 + 49} = \sqrt{53} = \boxed{7.42}$$

29. Find the mid-point of the following points A(0, -1) & B(-1, 2).

Sol. Mid - point = $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

$$= \left(\frac{0 + (-1)}{2}, \frac{-1 + 2}{2}\right)$$

$$= \left(\frac{-1}{2}, \frac{1}{2}\right)$$

30. Find the slope of a line which is perpendicular to the line joining $P_1(2, 4)$, $P_2(-2, 1)$.

Sol. Slope of line joining given point :

$$= m_1 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1 - 4}{-2 - 2} = \frac{-3}{-4} = \frac{3}{4}$$

Slope of require line = $m_2 = ?$

As, both lines are perpendicular,

So, $m_1 m_2 = -1 \Rightarrow \left(\frac{3}{4}\right) m_2 = -1$

$$\Rightarrow m_2 = -1 \times \frac{4}{3} \Rightarrow \boxed{m_2 = -\frac{4}{3}}$$

31. Find an equation on the line with the following intercepts:

$a = 2$ & $b = -5$.

Sol. Equation of line in intercept form :

$$\frac{x}{a} + \frac{y}{b} = 1 \Rightarrow \frac{x}{2} + \frac{y}{-5} = 1$$

$$\frac{5x - 2y}{10} = 1 \Rightarrow 5x - 2y = 10$$

$$\Rightarrow \boxed{5x - 2y - 10 = 0}$$

- 32.** Show that the two lines passing through the given points are perpendicular $(8, 0)$, $(6, 6)$ and $(-3, 3)$, $(6, 6)$

Sol. Let

$$\ell_1 : (8, 0) \text{ \& } (6, 6) \quad | \quad \ell_2 : (-3, 3) \text{ \& } (6, 6)$$

$$\begin{array}{l} \text{Slope of } \ell_1 \\ = m_1 = \frac{y_2 - y_1}{x_2 - x_1} \end{array} \quad \left| \quad \begin{array}{l} \text{Slope of } \ell_2 \\ = m_2 = \frac{y_2 - y_1}{x_2 - x_1} \end{array} \right.$$

$$m_1 = \frac{6 - 0}{6 - 8} \quad \left| \quad m_2 = \frac{6 - 3}{6 - (-3)} \right.$$

$$m_1 = \frac{6}{-2} = -3 \quad \left| \quad m_2 = \frac{3}{9} = \frac{1}{3} \right.$$

$$\text{As, } m_1 m_2 = (-3) \left(\frac{1}{3} \right) = -1$$

Hence both lines ℓ_1 & ℓ_2 are

perpendicular. **Proved.**

- 33.** Reduce the given equation to slope intercept form $6x - 5y = 15$.

Sol. $6x - 5y = 15 \rightarrow (i)$

Slope - intercept form :

$$6x - 5y = 15$$

$$-5y = -6x + 15$$

$$\frac{-5y}{-5} = \frac{-6x}{-5} + \frac{15}{-5}$$

$$\boxed{y = \frac{6}{5}x - 3}$$

- 34.** Show that the points $(1, 2)$, $(7, 6)$ and $(4, 4)$ are collinear.

Sol.
$$\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = \begin{vmatrix} 1 & 2 & 1 \\ 7 & 6 & 1 \\ 4 & 4 & 1 \end{vmatrix}$$

$$= 1 \begin{vmatrix} 6 & 1 \\ 4 & 1 \end{vmatrix} - 2 \begin{vmatrix} 7 & 1 \\ 4 & 1 \end{vmatrix} + 1 \begin{vmatrix} 7 & 6 \\ 4 & 4 \end{vmatrix}$$

$$= 1(6 - 4) - 2(7 - 4) + 1(28 - 24)$$

$$= 1(2) - 2(3) + 1(4) = 2 - 6 + 4 = 0$$

Hence given points are collinear. **Proved.**

- 35.** Find the equation of circle with center $(-2, 3)$ and radius 6.

Sol. Here: Center = $(h, k) = (-2, 3)$

& Radius = $r = 6$

Standard form of equation of circle :

$$(x - h)^2 + (y - k)^2 = r^2$$

Put $h = -2$, $k = 3$ & $r = 6$

$$(x + 2)^2 + (y - 3)^2 = (6)^2$$

$$(x)^2 + 2(x)(2) + (2)^2 + (y)^2 - 2(y)(3) + (3)^2 = 36$$

$$x^2 + 4x + 4 + y^2 - 6y + 9 - 36 = 0$$

$$\boxed{x^2 + y^2 + 4x - 6y - 23 = 0}$$

- 36.** Find center and radius of the circle

$$x^2 + y^2 - 6x + 6y = 0$$

Sol. $x^2 + y^2 - 6x + 6y = 0$

Comparing with general equation of circle.

$$x^2 + y^2 + 2gx + 2fy + c = 0$$

$$2g = -6$$

$$g = -\frac{6}{2} = -3 \quad \left| \quad \begin{array}{l} 2f = 6 \\ f = \frac{6}{2} = 3 \end{array} \right. \quad c = 0$$

Center = $(-g, -f)$

$$\text{Center} = -(-3), -3 = \boxed{(3, -3)}$$

$$\text{Radius} = r = \sqrt{g^2 + f^2 - c}$$

$$r = \sqrt{(-3)^2 + (3)^2 - 0} = \sqrt{9 + 9}$$

$$r = \sqrt{18} = \sqrt{9 \times 2} = \boxed{3\sqrt{2}}$$

37. What type of circle is represented by $x^2 + y^2 - 2x + 4y + 8 = 0$

Sol. $x^2 + y^2 - 2x + 4y + 8 = 0$

Comparing this equation with general form of equation of circle:

$$x^2 + y^2 + 2gx + 2fy + c = 0$$

$$2g = -2 \quad \left| \quad 2f = 4 \right.$$

$$g = -\frac{2}{2} \quad \left| \quad f = \frac{4}{2} \right. \quad c = 8$$

$$g = -1 \quad \left| \quad f = 2 \right.$$

$$\text{Radius} = r = \sqrt{g^2 + f^2 - c}$$

$$r = \sqrt{(-1)^2 + (2)^2 - 8}$$

$$r = \sqrt{1 + 4 - 8}$$

$$r = \sqrt{-3}$$

$$r = \sqrt{3i}$$

So, it is an Imaginary circle.

Section - II

Note: Attempt any three (3) questions $3 \times 10 = 30$

Q.2.[a] If $f(t) = \frac{t^4 + t^2 + 1}{t^2}$, show that

$$f\left(\frac{1}{t}\right) = f(t).$$

Sol. See Q.8 of Ex # 1.1 (Page # 6)

[b] Differentiate $\sqrt{\frac{a+x}{a-x}}$ w.r.t. 'x'.

Sol. See Q.4(v) of Ex # 2.2 (Page # 56)

Q.3.[a] Differentiate $\ell n \left(\frac{x}{\sqrt{1+x^2}} \right)$

w.r.t. 'x'.

Sol. See Q.1(iii) of Ex # 3.3 (Page # 136)

[b] Find the maximum and minimum values of the function

$$2x^3 - 3x^2 - 36x + 3$$

Sol. See Q.2(v) of Ex # 4.2 (Page # 188)

Q.4.[a] Evaluate

$$\int \left(x + \frac{1}{x} \right) \left(x^2 + \frac{1}{x^2} \right) dx$$

Sol. See Q.10 of Ex # 5.1 (Page # 230)

[b] Evaluate $\int (\tan^4 x + \tan^2 x) dx$

Sol. See Q.1(vi) of Ex # 5.3 (Page # 242)

Q.5.[a] Evaluate $\int (\ell n x)^2 dx$

Sol. See Q.3(iv) of Ex # 6.3 (Page # 291)

[b] Show that the points A(2, 2), B(6, 6) and C(11, 11) are the vertices of a right triangle.

Sol. See Q.2 [a] of Ex # 8.1 (Page # 354)

Q.6.[a] If a line ℓ_1 contains P(2, 6) and Q(0, y). Find 'y' if ℓ_1 is parallel to ℓ_2 and that the slope of $\ell_2 = \frac{3}{4}$.

Sol. See Q.2 of Ex # 8.3 (Page # 374)

[b] Find the equation of the circle concentric with the circle $x^2 + y^2 - 6x + 4y - 12 = 0$ with radius 6 units.

Sol. See Q.5 [a] of Ex # 9 (Page # 443)
