DAE / IIA - 2016

MATH - 233 APPLIED MATHEMATICS - II

PAPER 'B' PART - A (OBJECTIVE)

Time : 30 Minutes Marks : 15

Q.1: *Encircle the correct answer.*

1. $\int\left(x^3\right)dx = ?$

[a] $\frac{x^4}{4}$ [b] $\frac{x^4}{3}$ [c] $3x^2$ [d] $4x^4$

2. $\int(ax+b)\,dx = ?$

[a] $\frac{(ax+b)^2}{2a}$ [b] $\frac{(ax+b)^2}{2}$

[c] $\ell n(ax+b)$ [d] $a(ax+b)$

3. $\int\left(\tan x\sec^2 x\right)dx = ?$

[a] $\ell n\tan x$ [b] $\frac{\tan^2 x}{2}$

[c] $\frac{\sec^2 x}{3}$ [d] $\sec x\tan x$

4. $\xcancel{\int\left(\frac{1}{\sqrt{1-x^2}}\right)dx = ?}$

[a] $\sin^{-1}x$ [b] $\cos^{-1}x$

[c] $\sec^{-1}x$ [d] $\tan^{-1}x$

5. $\xcancel{\int\left(\frac{e^x}{1+e^x}\right)dx = ?}$

[a] $1+e^x$ [b] $\ell n\left(1+e^x\right)$

[c] e^x [d] $\frac{\left(1+e^x\right)^2}{2}$

6. $\int_0^1(1)\,dx = ?$

[a] -1 [b] 0 [c] 1 [d] 2

7. $\int_0^{\pi/2}(\cot x)\,dx = ?$

[a] -1 [b] 1 [c] 0 [d] $\pi/2$

8. Solution of differential equation $\frac{dy}{dx} = -y$ is:

[a] $y = ce^{-x}$ [b] $y = ce^{x}$

[c] $y = e^x + c$ [d] $y = e^{-x+c}$

9. Order of differential equation $\left(\frac{d^3y}{dx^3}\right)^2 + \frac{dy}{dx} + y = 0$ is:

[a] 2 [b] 1 [c] 0 [d] 3

10. If a function $f(-x) = -f(x)$ then function is:

[a] Even [b] Odd

[c] Linear [d] Constant

11. Laplace transform of the function $f(t) = e^t$ is:

[a] $\frac{1}{s-1}$ [b] $\frac{1}{s}$ [c] $s-1$ [d] s

12. ~~The inverse Laplace transform~~ $\xcancel{L^{-1}\left(\frac{1}{S}\right)}$ ~~is equal to:~~

[a] 1 [b] 2 [c] 3 [d] 4

13. The period of $\sin x$ is:

[a] π [b] 2π [c] $-\pi$ [d] -2π

14. $\xcancel{\int\left(xe^x\right)dx = ?}$

[a] $xe^x + e^x$ [b] $xe^x - e^x$

[c] e^x [d] $\frac{x^2}{2}e^x$

15. $\int\left(\frac{1}{\sqrt{x}}\right)dx = ?$

[a] $2\sqrt{x}$ [b] $-2\sqrt{x}$ [c] $\frac{1}{x}$ [d] $-\frac{1}{x}$

Answer Key

1	a	2	a	3	b	4	b	5	b
6	c	7	c	8	a	9	b	10	b
11	a	12	a	13	b	14	b	15	a

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DAE / IIA - 2016

MATH - 233 APPLIED MATHEMATICS - II

PAPER 'B' PART - B (SUBJECTIVE)

Time: 2:30 Hrs Marks: 60

Section - I

Q.1. Write short answersto any Eighteen (18) questions.

1. Evaluate $\int\left(ax^n+bx^m\right)dx$

Sol. $\int\left(ax^n+bx^m\right)dx$

$$=\boxed{\frac{ax^{n+1}}{n+1}+\frac{bx^{m+1}}{m+1}+c}$$

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2. Evaluate

$$\int\left(x^2+3x+4\right)^3(2x+3)\,dx$$

Sol. $\int\left(x^2+3x+4\right)^3(2x+3)dx$

$$=\boxed{\frac{\left(x^2+3x+4\right)^4}{4}+c}$$

---

3. Evaluate $\int 8(2x+1)^3\,dx$

Sol. $\int 8(2x+1)^3\,dx$

$$=4\int(2x+1)^3(2)\,dx$$

$$=4\frac{(2x+1)^4}{4}+c \qquad \left\{\begin{matrix}\text{using}\\ \text{Rule-I}\end{matrix}\right\}$$

$$=\boxed{(2x+1)^4+c}$$

---

4. Evaluate $\int\left(\cos^2 x\right)dx$

Sol. $\int\left(\cos^2 x\right)dx$

$$=\int\frac{1+\cos 2x}{2}dx$$

$$=\frac{1}{2}\int(1+\cos 2x)\,dx$$

$$=\boxed{\frac{1}{2}\left[x+\frac{\sin 2x}{2}\right]+c}$$

---

5. Evaluate $\int\left(\frac{\sin 2x}{\sin x}\right)dx$

Sol. $\int\left(\frac{\sin 2x}{\sin x}\right)dx$

$$=\int\left(\frac{2\sin x\cos x}{\sin x}\right)dx \because \left\{\begin{matrix}\sin 2x\\ =2\sin x\cos x\end{matrix}\right\}$$

$$=2\int(\cos x)dx=\boxed{2\sin x+c}$$

---

6. Evaluate $\int\left(\tan x-\sec^2 x\right)dx$

Sol. $\int\left(\tan x-\sec^2 x\right)dx$

$$=\boxed{\ell n\sec x-\tan x+c}$$

---

7. Find $\int\frac{(\ell nx)^3}{x}\,dx$

Sol. $\int\frac{(\ell nx)^3}{x}\,dx$

$$=\int(\ell nx)^3.\left(\frac{1}{x}\right)dx=\boxed{\frac{(\ell nx)^4}{4}+c}$$

---

8. Evaluate $\int\frac{1}{2}\left(e^{\frac{1}{2}x}-e^{-\frac{1}{2}x}\right)dx$

Sol. $\int\frac{1}{2}\left(e^{\frac{1}{2}x}-e^{-\frac{1}{2}x}\right)dx$

$$=\frac{1}{2}\left(\frac{e^{\frac{1}{2}x}}{1/2}-\frac{e^{-\frac{1}{2}x}}{-1/2}\right)+c=\boxed{e^{\frac{1}{2}x}+e^{-\frac{1}{2}x}+c}$$

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9. Evaluate $\int\frac{dx}{\sqrt{25-16x^2}}$

Sol. $\int\frac{dx}{\sqrt{25-16x^2}}$

$$=\int\frac{dx}{\sqrt{16\left(\frac{25}{16}-x^2\right)}}=\int\frac{dx}{4\sqrt{\left(\frac{5}{4}\right)^2-x^2}}$$

$$=\frac{1}{4}\int\frac{\frac{5}{4}\cos\theta\,d\theta}{\sqrt{\left(\frac{5}{4}\right)^2-\left(\frac{5}{4}\sin\theta\right)^2}}$$

$$=\frac{1}{4}\times\frac{5}{4}\int\frac{\cos\theta\,d\theta}{\sqrt{\left(\frac{5}{4}\right)^2\left(1-\sin^2\theta\right)}}$$

$$=\frac{5}{16}\int\frac{\cos\theta\,d\theta}{\sqrt{\left(\frac{5}{4}\right)^2\cos^2\theta}}$$

$$=\frac{5}{16}\int\left(\frac{\cos\theta}{\frac{5}{4}\cos\theta}\right)d\theta$$

Put $x=\frac{5}{4}\sin\theta,$

$$\frac{d}{dx}(x)=\frac{d}{dx}\left(\frac{5}{4}\sin\theta\right)$$

$$1=\left(\frac{5}{4}\cos\theta\right)\frac{d\theta}{dx}$$

$$dx=\left(\frac{5}{4}\cos\theta\right)d\theta$$

As, $\frac{5}{4}\sin\theta=x$

$$\Rightarrow\quad \sin\theta=\frac{4x}{5}$$

$$\Rightarrow\quad \theta=\sin^{-1}\left(\frac{4x}{5}\right)$$

$$=\frac{1}{4}\int 1d\theta=\frac{1}{4}\theta+c=\boxed{\frac{1}{4}\sin^{-1}\left(\frac{4x}{5}\right)+c}$$

---

10. **Evaluate $\int\left(x\sec^2x\right)dx$**

Sol. $\int\left(x\sec^2x\right)dx$

Integrating by parts:

taking $u=x$ & $v=\sec^2x$

$$=x\int\sec^2xdx-\int\left[\frac{d}{dx}(x)\int\sec^2xdx\right]dx$$

$$=x\tan x-\int(1\cdot\tan x)dx$$

$$=x\tan x-\int(\tan x)dx$$

$$=\boxed{x\tan x-\ell n\sec x+c}$$

---

11. **Evaluate $\int(x\ell nx)\,dx$**

Sol. $\int(x\ell nx)\,dx=\int(\ell nx.x)dx$

Integrating by parts:

taking $u=\ell nx$ & $v=x$

$$=\ell nx\int x\,dx-\int\left\{\frac{d}{dx}(\ell nx)\int x\,dx\right\}dx$$

$$=\ell nx.\frac{x^2}{2}-\int\frac{1}{x}.\frac{x^2}{2}dx$$

$$=\frac{x^2}{2}\ell nx-\frac{1}{2}\int x\,dx$$

$$=\frac{x^2}{2}\ell nx-\frac{1}{2}.\frac{x^2}{2}+c$$

$$=\boxed{\frac{x^2}{2}\ell nx-\frac{1}{4}x^2+c}$$

---

12. **Evaluate $\int_1^3\frac{1}{x+1}dx$**

Sol. $\int_1^3\frac{1}{x+1}dx$

$$=\left[\ell n(x+1)\right]_1^3\quad\left\{\begin{matrix}\textbf{using}\\\textbf{Rule-II}\end{matrix}\right\}$$

$$=\ell n(3+1)-\ell n(1+1)$$

$$=\ell n(4)-\ell n(2)$$

$$=\ell n\left(\frac{4}{2}\right)=\boxed{\ell n2}$$

---

13. **Evaluate $\int_0^{\pi/6}\left(\sec^2x\right)dx$**

Sol. $\int_0^{\pi/6}\left(\sec^2x\right)dx$

$$=\left[\tan x\right]_0^{\pi/6}=\tan\left(\frac{\pi}{6}\right)-\tan(0)$$

$$=\tan(30^\circ)-\tan(0^\circ)$$

$$=\frac{1}{\sqrt{3}}-0=\boxed{\frac{1}{\sqrt{3}}}$$

---

14. **Find the area bounded by the curve $y=x^3+3x^2$ the x-axis, and the lines $x=0$ and $x=2$.**

Sol. Area $=\int_a^b y\,dx$

$$A=\int_0^2\left(x^3+3x^2\right)dx$$

$$A=\left[\frac{x^4}{4}+3\frac{x^3}{3}\right]_0^2$$

$$A = \left[\frac{x^4}{4} + x^3\right]_0^2$$

$$A = \left(\frac{(2)^4}{4} + (2)^3\right) - \left(\frac{(0)^4}{4} + (0)^3\right)$$

$$A = \left(\frac{16}{4} + 8\right) - (0 + 0)$$

$$A = 4 + 8 = \boxed{12 \text{ sq. unit}}$$

---

15. **Define differential equation.**

Sol. An equation involving derivatives or differentials is called a differential equation.

Example: $\frac{dy}{dx} + 2x = 0$

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16. **Solve the differential equation**

$$\mathbf{dy = sec^2 x\ dx}$$

Sol. $dy = \sec^2 x \ dx$

Integrating both sides, we have:

$$\int(1)dy = \int\left(\sec^2 x\right)dx$$

$$\boxed{y = \tan x + c}$$

---

17. **Write down the order and degree of differential equation**

$$\mathbf{\frac{dy}{dx} + 2y = 0}$$

Sol. The highest derivative is one, therefore order = 1 The highest power of highest derivative is one, therefore degree = 1

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18. **What are Fourier coefficients.**

Sol. Constants $a_{0,}$ a_n and b_n present in the Fourier series are called Fourier coefficients.

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19. **Define Even Function.**

Sol. A function $f(x)$ is said to be an **Even function** of 'x'.

$$\text{If for}: x = -x \ \Rightarrow f(-x) = f(x)$$

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20. **Find Laplace transform of a constant 'K'.**

Sol. $L\{K\} = K\,L\{1\} = K\left(\frac{1}{s}\right) = \boxed{\frac{K}{s}}$

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21. **Write Laplace transform of e^{at}.**

Sol. $L\left\{e^{at}\right\} = \boxed{\frac{1}{s-a}}$

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22. ~~**What is inverse Laplace transform of**~~ $\cancel{\frac{2}{s^3}}$ **?**

Sol. $L^{-1}\left\{\frac{2}{s^3}\right\} = L^{-1}\left\{\frac{2!}{s^{2+1}}\right\} = \boxed{t^2}$

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23. **Find** $\int\left(\frac{\cot x}{\ell n \sin x}\right)dx$

Sol. $\int\left(\frac{\cot x}{\ell n \sin x}\right)dx$

$$= \int\left(\frac{1}{\ell n x \sin x}.\cot x\right)dx$$

$$= \int\left(\frac{1}{t}\right)dt$$

$$= \ell n(t) + c$$

Put $\ell n \sin x = t$

$$\frac{d}{dx}(\ell n \sin x) = \frac{d}{dx}(t)$$

$$\frac{1}{\sin x}\frac{d}{dx}(\sin x) = \frac{dt}{dx}$$

$$\frac{1}{\sin x}\cos x = \frac{dt}{dx}$$

$$\cot x \ dx = dt$$

$$= \boxed{\ell n(\ell n \sin x) + c}$$

---

24. **Find** $\int\left(\frac{x^2}{4+x^2}\right)dx$

Sol. $\int\left(\frac{x^2}{4+x^2}\right)dx$

$$= \int\left(1 - \frac{4}{4+x^2}\right)dx$$

$$=\int(1)\,dx-4\int\left(\frac{1}{(2)^2+(x)^2}\right)dx$$

$$=x-4.\frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right)+c$$

$$=\boxed{x-2\tan^{-1}\left(\frac{x}{2}\right)+c}$$

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25. Integrate $\int\frac{\cos^{-1}x}{\sqrt{1-x^2}}\,dx$

Sol. $\int\frac{\cos^{-1}x}{\sqrt{1-x^2}}\,dx$

$$=\int\cos^{-1}x.\frac{1}{\sqrt{1-x^2}}\,dx$$

$$=\int t(-dt)$$

$$=-\int t\,dt$$

$$=-\frac{t^2}{2}+c$$

Put $\cos^{-1}x=t$

$$\frac{d}{dx}\left(\cos^{-1}x\right)=\frac{d}{dx}(t)$$

$$\frac{-1}{\sqrt{1-x^2}}=\frac{dt}{dx}$$

$$\frac{1}{\sqrt{1-x^2}}dx=-dt$$

$$=\boxed{-\frac{1}{2}\left(\cos^{-1}x\right)^2+c}$$

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26. Integrate $\int\left(x.e^{x^2}\right)dx$

Sol. $\int x.e^{x^2}dx$

$$=\int\left(e^{x^2}\right)x\,dx$$

$$=\int\left(e^t\right)\frac{dt}{2}$$

$$=\frac{1}{2}e^t+c$$

$$=\boxed{\frac{1}{2}e^{x^2}+c}$$

Put $x^2=t$

$$\frac{d}{dx}\left(x^2\right)=\frac{d}{dx}(t)$$

$$2x=\frac{dt}{dx}$$

$$(x)dx=\frac{dt}{2}$$

27. Show that $\int_1^3\left(x^2\right)dx$

Sol. $\int_1^3\left(x^2\right)dx=\left[\frac{x^3}{3}\right]_1^3=\frac{1}{3}\left[x^3\right]_1^3$

$$=\frac{1}{3}\left[(3)^3-(1)^3\right]=\frac{1}{3}[27-1]=\boxed{\frac{26}{3}}$$

Section - II

Note: Attemp any three (3) questions $3\times 8=24$

Q.2.[a] Evaluate $\int\left(\frac{x^3-8}{x+2}\right)dx$

Sol. See Q.13 of Ex # 7.1 (Page # 287)

[b] Evaluate $\int\frac{dx}{1-\cos x}$

Sol. See Q.6 of Ex # 7.2 (Page # 292)

Q.3.[a] ~~Evaluate~~ $\cancel{\int\frac{dx}{\left(a^2-x^2\right)^{3/2}}}$

Sol. See Q.1(x) of Ex # 8.2 (Page # 334)

[b] Evaluate $\int\left(e^x\sin x\right)dx$

Sol. See Q.5(i) of Ex # 8.3 (Page # 353)

Q.4.[a] Evaluate $\int_{-2}^{0}\left(x\sqrt{2x^2+1}\right)dx$

Sol. See example # 10 of Chapter 09.

[b] ~~Compute the area bounded by the curve~~ $y=\sqrt{x}$ ~~and~~ $y=x^2$.

Sol. See Q.6 of Ex # 9.2 (Page # 390)

Q.5. Find the general solution of $dx+xydy=y^2dx+ydy$

Sol. See Q.8 of Ex # 10 (Page # 415)

Q.6. Find the Laplace transform of the function $\cos\omega t$.

Sol. See proof of Formula 06 of Chapter 12.

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