DAE / IIA - 2016

MATH - 233 APPLIED MATHEMATICS - II

PAPER 'A' PART - A (OBJECTIVE)

Time : 30 Minutes Marks : 15

Q.1 : *Encircle the correct answer.*

1. A function $f(x) = x^2 + 2x + 3$ is:

[a] Odd [b] Even
[c] Implicit [d] Explicit

2. $\lim_{x \to 2}(x - 1) = ?$

[a] 1 [b] 2
[c] 3 [d] 4

3. $\lim_{\theta \to \frac{\pi}{2}} \frac{\sin\theta}{\theta} = ?$

[a] 1 [b] $\frac{\pi}{2}$ [c] $\frac{2}{\pi}$ [d] $\frac{1}{2}$

4. $\frac{d}{dx}(ax + b)^2 = ?$

[a] $2(ax + b)$ [b] $2a(ax + b)$
[c] $\frac{(ax + b)^3}{3}$ [d] $2(ax + b)b$

5. If $y = u^2$ and $u = x$ then $\frac{dy}{dx} = ?$

[a] $2x$ [b] u^2
[c] x [d] $2x^2$

6. $\frac{d}{dx}(\sin x^3) = ?$

[a] $\cos x^3$ [b] $-\cos x^3$
[c] $3x \cos x^3$ [d] $3x^2 \cos x^3$

7. $\frac{d}{dx}(\tan^{-1} x^2) = ?$

[a] $\frac{1}{1 + x^2}$ [b] $\frac{1}{1 - x^2}$
[c] $\frac{2x}{1 + x^4}$ [d] $\frac{2x}{1 - x^4}$

8. $\frac{d}{dx}(e^{3x}) = ?$

[a] e^{3x-1} [b] e^{x-1} [c] $3e^{3x}$ [d] $3xe^{3x}$

9. $\frac{d}{dx}[\ell n(x^2 + 1)] = ?$

[a] $\frac{x}{x^2 + 1}$ [b] $\frac{2x}{x^2 + 1}$
[c] $\ell n(2x + 1)$ [d] $2x\,\ell n(2x + 1)$

10. If $\frac{dy}{dx}$ changes sign from $+ve$ to $-ve$ then it is a point of:

[a] Maxima [b] Minima
[c] Inflection [d] None of these

11. A function is maximum at a point if its 2^{nd} derivative is:

[a] $+ve$ [b] $-ve$
[c] zero [d] None of these

12. Second derivative of 'x^2' is:

[a] 2 [b] $2x$ [c] zero [d] $2x^2$

13. A set of data is called:

[a] Continuous data
[b] Discontinuous
[c] Population [d] Sample

14. When n is odd the $\left(\frac{n+1}{2}\right)^{th}$ value is called:

[a] Mean [b] Mode
[c] Median [d] Harmonic Mean

15. A single performance of an experiment is called:

[a] An event [b] Trail
[c] Out come [d] Sample space

Answer Key

1	d	2	a	3	c	4	b	5	a
6	d	7	c	8	c	9	b	10	a
11	b	12	a	13	c	14	c	15	b

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DAE / IIA - 2016

MATH - 233 APPLIED MATHEMATICS - II

PAPER 'B' PART - B (SUBJECTIVE)

Time : 2 : 30 Hrs Marks : 60

Section - I

Q.1. Write short answers to any Eighteen (18) questions.

1. Find the value of

$$f\left(\frac{1}{x}\right), \text{ if } f(x)=\frac{1}{x^2+4}$$

Sol. $f(x)=\dfrac{1}{x^2+4}$

Replace x by $1/x$, we have :

$$f\left(\frac{1}{x}\right)=\frac{1}{\left(\frac{1}{x}\right)^2+4}$$

$$=\frac{1}{\frac{1}{x^2}+4}=\frac{1}{\frac{1+4x^2}{x^2}}=\boxed{\frac{x^2}{1+4x^2}}$$

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2. Show that the function $f(x)=x^4-7x^2+7$ is an even function of x.

Sol. $f(x)=x^4-7x^2+7$

Replace x by $-x$, we have :

$$f(-x)=(-x)^4-7(-x)^2+7$$

$$f(-x)=x^4-7x^2+7$$

$$f(-x)=f(x)$$

Hence f(x) is an **even** function. **Proved.**

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3. Evaluate: $\lim\limits_{x\to -2}\dfrac{x^2}{x+1}$

Sol. $\lim\limits_{x\to -2}\dfrac{x^2}{x+1}=\dfrac{(-2)^2}{-2+1}=\dfrac{4}{-1}=\boxed{-4}$

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4. ~~Evaluate the limit:~~ $\lim\limits_{x\to 0}\dfrac{\sin^2 x}{x^3}$

Sol. $\lim\limits_{x\to 0}\dfrac{\sin^2 x}{x^3}$ $\left(\dfrac{0}{0}\right)$ form

$$=\lim_{x\to 0}\left(\frac{\sin^2 x}{x^2}\cdot\frac{1}{x}\right)$$

$$=\left(\lim_{x\to 0}\frac{\sin x}{x}\right)^2\cdot\lim_{x\to 0}\frac{1}{x}$$

$$=(1)^2\times\frac{1}{0}=\boxed{\infty}$$

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5. Find $\dfrac{dy}{dx}$ if $y=x^3+x^2+2x+3$

Sol. Differentiate both sides w.r.t. 'x' :

$$\frac{d}{dx}(y)=\frac{d}{dx}\left(x^3+x^2+2x+3\right)$$

$$\frac{dy}{dx}=3x^2+2x+2(1)+0$$

$$\boxed{\frac{dy}{dx}=3x^2+2x+2}$$

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6. Find $\dfrac{dy}{dx}$ if $\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}=1$

Sol. $\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}=1$

Differentiate both sides w.r.t. 'x' :

$$\frac{d}{dx}\left(\frac{x^2}{a^2}+\frac{y^2}{b^2}\right)=\frac{d}{dx}(1)$$

$$\frac{2x}{a^2}+\frac{2y}{b^2}\frac{dy}{dx}=0$$

$$\frac{2y}{b^2}\frac{dy}{dx}=-\frac{2x}{a^2}$$

$$\frac{dy}{dx}=-\frac{2x}{a^2}\cdot\frac{b^2}{2y}\Rightarrow\boxed{\frac{dy}{dx}=-\frac{b^2x}{a^2y}}$$

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7. If $y=\left(3x^2+2x+9\right)^7$, find $\dfrac{dy}{dx}$

Sol. $y=\left(3x^2+2x+9\right)^7$

Differentiate both sides w.r.t. 'x' :

$$\frac{d}{dx}(y)=\frac{d}{dx}\left(3x^2+2x+9\right)^7$$

$$\frac{dy}{dx}=7\left(3x^2+2x+9\right)^6\left[\frac{d}{dx}\left(3x^2+2x+9\right)\right]$$

$$\frac{dy}{dx}=7\left(3x^2+2x+9\right)^6\left[3(2x)+2(1)+0\right]$$

$$\frac{dy}{dx}=7\left(3x^2+2x+9\right)^6(6x+2)$$

$$\boxed{\frac{dy}{dx}=7(6x+2)\left(3x^2+2x+9\right)^6}$$

8. Find the derivative of $\left(ax^2+b\right)\left(cx^2+d\right)$ w.r.t. 'x'.

Sol. $\frac{d}{dx}\left[\left(ax^2+b\right)\left(cx^2+d\right)\right]$

{using Product Rule}

$$=\left(\frac{d}{dx}\left(ax^2+b\right)\right)\left(cx^2+d\right)+\left(ax^2+b\right)\left(\frac{d}{dx}\left(cx^2+d\right)\right)$$

$$=\left(a(2x)+0\right)\left(cx^2+d\right)+\left(ax^2+b\right)\left(c(2x)+0\right)$$

$$=2ax\left(cx^2+d\right)+\left(ax^2+b\right)2cx$$

$$=4acx^3+2adx+2bcx=\boxed{2x\left(2acx^2+ad+bc\right)}$$

9. Differentiate $\frac{x^2}{1+x^2}$ w.r.t. 'x'.

Sol. $\frac{d}{dx}\left(\frac{x^2}{1+x^2}\right)\left\{\text{using Quotient Rule}\right\}$

$$=\frac{\left(1+x^2\right)\left(\frac{d}{dx}\left(x^2\right)\right)-x^2\left(\frac{d}{dx}\left(1+x^2\right)\right)}{\left(1+x^2\right)^2}$$

$$=\frac{\left(1+x^2\right)(2x)-x^2(0+2x)}{\left(1+x^2\right)^2}$$

$$=\frac{2x+2x^3-2x^3}{\left(1+x^2\right)^2}=\boxed{\frac{2x}{\left(1+x^2\right)^2}}$$

10. Find the derivative $\sin x^n$ w.r.t. 'x'.

Sol. $\frac{d}{dx}\left(\sin x^n\right)=\cos x^n\frac{d}{dx}\left(x^n\right)$

$$=\cos x^n.nx^{n-1}\left(\frac{d}{dx}(x)\right)$$

$$=n\cos x^n.x^{n-1}.(1)=\boxed{nx^{n-1}\cos x^n}$$

11. Differentiate $\cot^3(3x+1)$

Sol. $\frac{d}{dx}\left(\cot^3(3x+1)\right)$

$$=3\cot^2(3x+1)\left(\frac{d}{dx}\left(\cot(3x+1)\right)\right)$$

$$=3\cot^2(3x+1).\left(-\operatorname{cosec}^2(3x+1)\right)\left(\frac{d}{dx}(3x+1)\right)$$

$$=-3\cot^2(3x+1)\operatorname{cosec}^2(3x+1)(3(1)+0)$$

$$=\boxed{-9\cot^2(3x+1)\operatorname{cosec}^2(3x+1)}$$

12. Differentiate $\frac{\sin x}{1-\cos x}$ w.r.t. 'x'.

Sol. $\frac{d}{dx}\left(\frac{\sin x}{1-\cos x}\right)\left\{\text{using Quotient Rule}\right\}$

$$=\frac{(1-\cos x)\left(\frac{d}{dx}(\sin x)\right)-\sin x\left(\frac{d}{dx}(1-\cos x)\right)}{(1-\cos x)^2}$$

$$=\frac{(1-\cos x)\cos x-\sin x(0-(-\sin x))}{(1-\cos x)^2}$$

$$=\frac{\cos x-\cos^2 x-\sin^2 x}{(1-\cos x)^2}$$

$$=\frac{\cos x-\left(\cos^2 x+\sin^2 x\right)}{(1-\cos x)^2}$$

$$=\frac{\cos x-1}{(1-\cos x)^2}\quad\because\left\{\cos^2 x+\sin^2 x=1\right\}$$

$$=\frac{-(1-\cos x)}{(1-\cos x)^2}=\boxed{-\frac{1}{1-\cos x}}$$

13. Find the derivative of $x^2 \tan x$.

Sol. $\frac{d}{dx}(x^2 \tan x)$ {using Product Rule}

$$= \left(\frac{d}{dx}(x^2)\right)\tan x + x^2\left(\frac{d}{dx}(\tan x)\right)$$

$$= \boxed{2x\tan x + x^2 \sec^2 x}$$

14. ~~Differentiate~~ ~~$\tan^{-1}\left(\frac{1}{x^2}\right)$~~

Sol. $\frac{d}{dx}\left(\tan^{-1}\frac{1}{x^2}\right)$

$$= \frac{1}{1+\left(\frac{1}{x^2}\right)^2} \cdot \frac{d}{dx}\left(\frac{1}{x^2}\right)$$

$$= \frac{1}{1+\frac{1}{x^4}} \cdot \frac{d}{dx}\left(x^{-2}\right)$$

$$= \frac{1}{\frac{x^4+1}{x^4}} \cdot \left(-2x^{-3}\right)$$

$$= \frac{x^4}{x^4+1} \cdot \frac{-2}{x^3} = \boxed{\frac{-2x}{x^4+1}}$$

15. ~~Differentiate~~ ~~$\sin^{-1} x$ w.r.t. $\cos^{-1} x$.~~

Sol. Let, $y = \sin^{-1} x$ and $t = \cos^{-1} x$

$$\frac{d}{dx}(y) = \frac{d}{dx}\left(\sin^{-1} x\right) \qquad \frac{d}{dx}(t) = \frac{d}{dx}\left(\cos^{-1} x\right)$$

$$\frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}} \qquad \frac{dt}{dx} = \frac{-1}{\sqrt{1-x^2}}$$

$$\frac{dx}{dt} = -\sqrt{1-x^2}$$

By using Chain's Rule: $\frac{dy}{dt} = \frac{dy}{dx} \times \frac{dx}{dt}$

$$\frac{dy}{dt} = \frac{1}{\sqrt{1-x^2}} \cdot \left(-\sqrt{1-x^2}\right) = \boxed{-1}$$

16. Evaluate: $\lim_{x\to 2} \frac{3x+4}{x+3}$

Sol. $\lim_{x\to 2} \frac{3x+4}{x+3} = \frac{3(2)+4}{2+3}$

$$= \frac{6+4}{5} = \frac{10}{5} = \boxed{2}$$

17. Differentiate $x\ell n\, x - x$ w.r.t. 'x'.

Sol. $\frac{d}{dx}(x\ell n\, x - x)$

$$= \left[\left(\frac{d}{dx}(x)\right)\ell n\, x + x\left(\frac{d}{dx}(\ell n\, x)\right)\right] - \frac{d}{dx}(x)$$

$$= (1)\ell n\, x + x\left(\frac{1}{x}\right) - 1$$

$$= \ell n\, x + 1 - 1$$

$$= \boxed{\ell n\, x}$$

18. ~~Find the derivative of~~ $e^{-2\log x}$ ~~w.r.t. 'x'.~~

Sol. $\frac{d}{dx}\left(e^{-2\log x}\right)$

$$= \frac{d}{dx}\left(e^{\log x^{-2}}\right)$$

$$= \frac{d}{dx}\left(x^{-2}\right)$$

$$= -2x^{-3} = \boxed{\frac{-2}{x^3}}$$

19. Define increasing function.

Sol. A function f(x) is said to be an increasing function of x, if as x increase f(x) also increase or if as x decrease f(x) also decreases.

20. Find mean of the data.

x	1	3	5	7	9
f	2	7	11	5	4

Sol.

x	f	fx
1	2	2
3	7	21
5	11	55
7	5	35
9	4	36
	$\Sigma f = 30$	$\Sigma fx = 156$

$$A.M. = \frac{\Sigma fx}{\Sigma f} = \frac{156}{30} = \boxed{5.2}$$

21. **If displacement is $s = \sin 2t$, find, its acceleration.**

Sol. $s = \sin 2t$

Differentiate both sides w.r.t. 't':

$$v = \frac{ds}{dt} = \frac{d}{dt}(\sin 2t)$$

$$v = \cos 2t\left(\frac{d}{dt}(2t)\right)$$

$$v = \cos 2t\big(2(1)\big) = 2\cos 2t$$

$$a = \frac{dv}{dt} = \frac{d}{dt}(2\cos 2t)$$

$$a = 2(-\sin 2t)\left(\frac{d}{dt}(2t)\right)$$

$$a = 2(-\sin 2t)\big(2(1)\big)$$

$$a = \boxed{-4\sin 2t}$$

22. **Define length of a class interval.**

Sol. The difference between upper and lower class boundaries is the length of class interval of that class and it is denoted by 'h'.

23. **Find the median of 4, 3, 5, 2, 11.**

Sol. Write the value in ascending order

2, 3, 4, 5, 11

Here n = 5

$$\text{Median} = \left(\frac{n+1}{2}\right)\text{th value}$$

$$= \left(\frac{5+1}{2}\right)\text{th} = 3\text{th value}$$

$\therefore$ The median is the third value

which is $= \boxed{4}$

24. **~~Find standard deviation of the values:~~ 2, 4, 6, 8, 10.**

Sol.

x	x^2
2	4
4	16
6	36
8	64
10	100
$\sum x = 30$	$\sum x^2 = 220$

$$S.D. = \sqrt{\frac{\Sigma x^2}{n} - \left(\frac{\Sigma x}{n}\right)^2}$$

$$\sigma = \sqrt{\left(\frac{220}{5}\right) - \left(\frac{30}{5}\right)^2}$$

$$\sigma = \sqrt{44 - 36}$$

$$\sigma = \sqrt{8} = \boxed{2.83}$$

25. **If a die is rolled once, what is the probability of getting an even number?**

Sol. $S = \{1, 2, 3, 4, 5, 6\}$, $n(S) = 6$

Let A be event that even number appear.

$$A = \{2, 4, 6\}, \quad n(A) = 3$$

$$P(A) = \frac{n(A)}{n(S)} = \frac{3}{6} = \boxed{\frac{1}{2}}$$

26. **Differentiate $\sin(\ell n \tan x)$**

Sol. $$\frac{d}{dx}\big[\sin(\ell n \tan x)\big]$$

$$= \cos(\ell n \tan x)\frac{d}{dx}(\ell n \tan x)$$

$$= \cos(\ell n \tan x)\frac{1}{\tan x}\frac{d}{dx}(\tan x)$$

$$= \cos(\ell n \tan x)\frac{1}{\tan x}\sec^2 x$$

$$= \boxed{\frac{\cos(\ell n \tan x)\sec^2 x}{\tan x}}$$

27. ~~Find the value of~~

$$\frac{d}{dx}\left(\sin^{-1}x+\cos^{-1}x\right)$$

Sol. $\frac{d}{dx}\left(\sin^{-1}x+\cos^{-1}x\right)$

$$=\frac{1}{\sqrt{1-x^2}}-\frac{1}{\sqrt{1-x^2}}$$

$$=\boxed{0}$$

Section - II

Note : Attemp any three (3) questions $3\times 8 = 24$

Q.2.(a) Show that $\frac{e^x+1}{e^x-1}$ is an odd function of x.

Sol. See Q.12(i) of Ex # 1.1 (Page # 10)

(b) Evaluate $\lim_{\theta\to 0}\frac{1-\cos\theta}{\theta^2}$

Sol. See example # 11(i) of Chapter 01.

Q.3.(a) Differentiate $\sqrt{\frac{1+x}{1-x}}$ w.r.t. 'x'.

Sol. See Q.4(iv) of Ex # 2.2 (Page # 57)

(b) Differentiate $\frac{x^2}{1+x^2}$ w.r.t. x^4

Sol. See example # 16 of Chapter 02.

Q.4.(a) If $y = a\sin\theta + b\cos\theta$ show that

$$y^2+\left(\frac{dy}{d\theta}\right)^2 = a^2+b^2$$

Sol. See Q.7(i) of Ex # 3.1 (Page # 122)

(b) Find $\frac{dy}{dx}$ for the $\ell n\left(\frac{e^x+1}{e^x-1}\right)$

Sol. See Q.2(iv) of Ex # 3.3 (Page # 145)

Q.5. Discuss for relative maxima and minima of the function

$$y = x^3 - 3x^2 + 2$$

Sol. See Q.3(i) of Ex # 4.2 (Page # 198)

Q.6. Calculate mode of following data:

Weekly Wages	No. of workers
0 – 4	5
4 – 8	15
8 – 12	22
12 – 16	28
16 – 20	45
20 – 24	25
24 – 28	13
28 – 32	6

Sol. See Q.9 of Ex # 5.1 (Page # 236)

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