DAE / IA - 2019

MATH - 212 APPLIED MATHEMATICS - II

PART - A (OBJECTIVE)

Time: 30 Minutes Marks: 20

Q.1: *Encircle the correct answer.*

1. $\lim\limits_{\theta \to \frac{\pi}{2}} \frac{\sin\theta}{\theta} =$

[a] 1 [b] $\frac{\pi}{2}$

[c] $\frac{2}{\pi}$ [d] $\frac{1}{2}$

2. $\lim\limits_{x \to \frac{\pi}{2}} \frac{1}{\cos\theta} = ?$

[a] 0 [b] ∞

[c] 1 [d] $\frac{2}{\pi}$

3. If $y = u^2$ and $u = x$ then $\frac{dy}{dx} = ?$

[a] $2x$ [b] u^2

[c] x [d] $2x^2$

4. If $y = \frac{x+1}{x}$, then $\frac{dy}{dx} = ?$

[a] $-\frac{1}{x^2}$ [b] $\frac{x+1}{x^2}$

[c] $\frac{2}{x^2}$ [d] $\frac{x^2-1}{x^2}$

5. ~~$\frac{d}{dx}(e^{3x}) =$~~

[a] e^{3x-1} [b] e^{x-1}

[c] $3e^{3x}$ [d] $3xe^{3x}$

6. ~~$\frac{d}{dx}(x^a) = ?$~~

[a] ax^{a-1} [b] x^{n-1}

[c] ax^a [d] x^a

7. A function is maximum at a point if its 2^{nd} derivative is:

[a] $+ve$ [b] $-ve$

[c] zero [d] None of these

8. If 2^{nd} derivative is $-ve$ at a point, then function is:

[a] Maximum [b] Minimum

[c] Point of inflection

[d] None of these

9. $\int (\sec x)\, dx = ?$

[a] $\tan x$ [b] $\frac{\sec^2 x}{2}$

[c] $\ell n(\sec x + \tan x)$

[d] $\sec x \tan x$

10. $\int (ax+b)dx = ?$

[a] $\frac{(ax+b)^2}{2a}$ [b] $\frac{(ax+b)^2}{2}$

[c] $\ell n(ax+b)$ [d] $a(ax+b)$

11. $\int (x\sec^2 x)dx = ?$

[a] $x\tan x$

[b] $x\tan x + \ell n\sec x$

[c] $\tan x$

[d] $x\tan x - \ell n\sec x$

12. $\int (xe^x)dx = ?$

[a] $xe^x + e^x$ [b] $xe^x - e^x$

[c] e^x [d] $\frac{x^2}{2}e^x$

13. ~~$\int_0^1 \left(\frac{1}{x^2+1}\right)dx = ?$~~

[a] 0 [b] 1

[c] $\frac{\pi}{4}$ [d] $-\frac{\pi}{4}$

14. ~~$\int_0^1 \frac{1}{x\sqrt{x^2-1}}dx = ?$~~

[a] -1 [b] 1

[c] 0 [d] $-\frac{\pi}{2}$

15. $y - y_1 = m(x - x_1)$ is the:

[a] Slope intercept form

[b] Intercepts form

[c] Point - Slope form

[d] Two - Points form

16. When two lines are parallel:

[a] $m_1 = m_2$ [b] $m_1 m_2 = -1$

[c] $m_1 m_2 = 1$ [d] $m_1 = -m_2$

17. y – intercept of the line $3x + 4y - 12 = 0$:

[a] -4 [b] 3

[c] 4 [d] -3

18. Midpoint of $A(2, 5)$ & $B(7, -3)$:

[a] $\left(\frac{9}{2}, 1\right)$ [b] $\left(1, \frac{9}{2}\right)$

[c] $\left(1, \frac{2}{9}\right)$ [d] $\left(\frac{2}{9}, 1\right)$

19. Center of the circle $(x-1)^2 + (y-2)^2 = 16$ is:

[a] $(1, 2)$ [b] $(2, 1)$

[c] $(4, 0)$ [d] $(-1, -2)$

20. For a point circle, the radius will be:

[a] 1 [b] -1

[c] 0 [d] Infinity

Answer Key

1	c	2	b	3	a	4	a	5	c
6	a	7	b	8	a	9	c	10	a
11	d	12	b	13	a	14	d	15	c
16	a	17	b	18	a	19	a	20	c

****

DAE / IA - 2019

MATH - 212 APPLIED MATHEMATICS - II

PART - B (SUBJECTIVE)

Time : 2 : 30 Hrs Marks : 60

Section - I

Q.1 : Write short answers to any Twenty Five (25) of the follwing questions. **25 × 2 = 50**

1. Find $\lim_{n\to\infty}\left(1+\frac{1}{n+1}\right)^n$

Sol. $\lim_{n\to\infty}\left(1+\frac{1}{n+1}\right)^n \rightarrow (i)$

$$\boxed{\text{As } \lim_{n\to\infty}\frac{1}{n+1} = 0 = \lim_{n\to\infty}\frac{1}{n}}$$

So, by replacing $\frac{1}{n+1}$ with $\frac{1}{n}$ in eq.(i), we have

$$= \lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n = \boxed{e}$$

2. ~~**Evaluate:** $\lim_{x\to 0}\frac{\sin x^\circ}{x}$~~

Sol. $\lim_{x\to 0}\frac{\sin x^\circ}{x}$

$$= \lim_{x\to 0}\frac{\sin\frac{\pi x}{180}}{x}$$

$$= \lim_{x\to 0}\frac{\sin\frac{\pi x}{180}}{\frac{\pi x}{180}} \times \frac{\pi}{180}$$

$$= \frac{\pi}{180}\lim_{x\to 0}\frac{\sin\frac{\pi x}{180}}{\frac{\pi x}{180}}$$

$$= \frac{\pi}{180}(1) = \boxed{\frac{\pi}{180}}$$

3. Find $\lim\limits_{x\to 0}\left(1+\dfrac{x}{3}\right)^{1/x}$.

Sol. $\lim\limits_{x\to 0}\left(1+\dfrac{x}{3}\right)^{1/x}$

$$=\lim_{x\to 0}\left(1+\frac{x}{3}\right)^{\frac{3}{x}\times\frac{1}{3}}$$

$$=\left[\lim_{x\to 0}\left(1+\frac{x}{3}\right)^{3/x}\right]^{1/3}=\boxed{e^{1/3}}$$

4. Evaluate: $\lim\limits_{x\to 0}\dfrac{\tan x}{x}$

Sol. $\lim\limits_{x\to 0}\dfrac{\tan x}{x}\ \left(\dfrac{0}{0}\right)$ form

$$=\lim_{x\to 0}\frac{\sin x}{x.\cos x}\quad\left\{\because \tan x=\frac{\sin x}{\cos x}\right\}$$

$$=\lim_{x\to 0}\frac{\sin x}{x}.\lim_{x\to 0}\frac{1}{\cos x}$$

$$=(1).\frac{1}{\cos 0}=\frac{1}{1}=\boxed{1}$$

5. Find $\dfrac{dy}{dx}$ if $x^{2/3}+y^{2/3}=a^{2/3}$

Sol. Differentiate both sides w.r.t. 'x' :

$$\frac{d}{dx}\left(x^{2/3}+y^{2/3}\right)=\frac{d}{dx}\left(a^{2/3}\right)$$

$$\frac{2}{3}x^{-1/3}+\frac{2}{3}y^{-1/3}\frac{dy}{dx}=0$$

$$\frac{2}{3}y^{-1/3}\frac{dy}{dx}=-\frac{2}{3}x^{-1/3}$$

$$\frac{dy}{dx}=\left(-\frac{2}{3}x^{-1/3}\right).\left(\frac{3}{2y^{-1/3}}\right)$$

$$\frac{dy}{dx}=-\frac{x^{-1/3}}{y^{-1/3}}\quad\Rightarrow\quad\boxed{\frac{dy}{dx}=-\frac{y^{1/3}}{x^{1/3}}}$$

6. If $ax^2+by^2+2hxy=0$, find $\dfrac{dy}{dx}$

Sol. As, $ax^2+by^2+2hxy=0$

Differentiate both sides w.r.t. 'x' :

$$\frac{d}{dx}\left(ax^2+by^2+2hxy\right)=\frac{d}{dx}(0)$$

$$a(2x)+b(2y)\frac{dy}{dx}+2h\left[\left(\frac{d}{dx}(x)\right)y+x\left(\frac{d}{dx}(y)\right)\right]=0$$

$$2ax+2by\frac{dy}{dx}+2h\left(1.y+x\frac{dy}{dx}\right)=0$$

$$2ax+2by\frac{dy}{dx}+2hy+2hx\frac{dy}{dx}=0$$

$$2by\frac{dy}{dx}+2hx\frac{dy}{dx}=-2ax-2hy$$

$$2\frac{dy}{dx}(by+2h)=-2(ax+hy)$$

$$\frac{dy}{dx}=\frac{-2(ax+hy)}{2(by+2h)}$$

$$\boxed{\frac{dy}{dx}=-\frac{(ax+hy)}{(by+hx)}}$$

7. Differentiate $\dfrac{x^3}{1+x^3}$ w.r.t. x^3

Sol. Let $y=\dfrac{x^3}{1+x^3}$ and $t=x^3$

Differentiate both sides w.r.t. 'x' :

$$\frac{d}{dx}(y)=\frac{d}{dx}\left(\frac{x^3}{1+x^3}\right)\{\text{using Quotient Rule}\}$$

$$\frac{dy}{dx}=\frac{(1+x^3)\left(\frac{d}{dx}(x^3)\right)-x^3\left(\frac{d}{dx}(1+x^3)\right)}{(1+x^3)^2}$$

$$\frac{dy}{dx}=\frac{(1+x^3)(3x^2)-x^3(0+3x^2)}{(1+x^3)^2}$$

$$\frac{dy}{dx}=\frac{3x^2+3x^5-3x^5}{(1+x^3)^2}=\frac{3x^2}{(1+x^3)^2}$$

$$\frac{d}{dx}(t)=\frac{d}{dx}(x^3)$$

$$\frac{dt}{dx}=3x^2\Rightarrow\frac{dx}{dt}=\frac{1}{3x^2}$$

using chain rule : $\dfrac{dy}{dt}=\dfrac{dy}{dx}\times\dfrac{dx}{dt}$

$$\frac{dy}{dt}=\frac{3x^2}{(1+x^3)^2}.\frac{1}{3x^2}=\boxed{\frac{1}{(1+x^3)^2}}$$

8. ~~**Find $\frac{dy}{dx}$ if $x = u + \frac{1}{u}$, $y = u - \frac{1}{u}$**~~

Sol. As, $x = u + \frac{1}{u}$ & $y = u - \frac{1}{u}$

Differentiate both sides w.r.t. 'u':

$$\frac{d}{du}(x) = \frac{d}{du}\left(u + \frac{1}{u}\right)$$
$$\frac{dx}{du} = \frac{d}{du}\left(u + u^{-1}\right)$$
$$\frac{dx}{du} = 1 + (-1)u^{-2}$$
$$\frac{dx}{du} = 1 - \frac{1}{u^2} = \frac{u^2 - 1}{u^2}$$
$$\frac{du}{dx} = \frac{u^2}{u^2 - 1}$$

$$\frac{d}{du}(y) = \frac{d}{du}\left(u - \frac{1}{u}\right)$$
$$\frac{dy}{du} = \frac{d}{du}\left(u - u^{-1}\right)$$
$$\frac{dy}{du} = 1 - (-1)u^{-2}$$
$$\frac{dy}{du} = 1 + \frac{1}{u^2}$$
$$\frac{dy}{du} = \frac{u^2 + 1}{u^2}$$

using chain rule :

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx} = \frac{u^2 + 1}{u^2} \times \frac{u^2}{u^2 - 1} = \boxed{\frac{u^2 + 1}{u^2 - 1}}$$

9. **Find the differential co-efficient of $e^{\tan^{-1} x}$.**

Sol. $$\frac{d}{dx}\left(e^{\tan^{-1} x}\right)$$
$$= e^{\tan^{-1} x} . \frac{d}{dx}\left(\tan^{-1} x\right)$$
$$= e^{\tan^{-1} x} . \frac{1}{1 + x^2} = \boxed{\frac{e^{\tan^{-1} x}}{1 + x^2}}$$

10. ~~**Find the derivative of $e^{-2\log x}$ w.r.t. 'x'.**~~

Sol. $$\frac{d}{dx}\left(e^{-2\log x}\right)$$
$$= \frac{d}{dx}\left(e^{\log x^{-2}}\right)$$
$$= \frac{d}{dx}\left(x^{-2}\right) = -2x^{-3} = \boxed{\frac{-2}{x^3}}$$

11. ~~**Find $\frac{d}{dx}\left(e^{2x} \cos 2x\right)$**~~

Sol. $$\frac{d}{dx}\left(e^{2x} \cos 2x\right)$$
$$= \left(\frac{d}{dx}\left(e^{2x}\right)\right)\cos 2x + e^{2x}\left(\frac{d}{dx}(\cos 2x)\right)$$
$$= e^{2x}\left(\frac{d}{dx}(2x)\right)\cos 2x + e^{2x}(-\sin 2x)\left(\frac{d}{dx}(2x)\right)$$
$$= e^{2x}(2)\cos 2x - e^{2x}\sin 2x(2)$$
$$= \boxed{2e^{2x}(\cos 2x - \sin 2x)}$$

12. ~~**Find the value of $\frac{d}{dx}\left(x^x\right)$**~~

Sol. Let $y = x^x$

Taking 'ℓn' on both sides:

$$\ell n(y) = \ell n\left(x^x\right)$$
$$\ell n(y) = x(\ell n x)\left\{\begin{matrix}\text{using logrithm law} \\ \ell n(m^n) = n\,\ell n(m)\end{matrix}\right\}$$
$$\frac{d}{dx}(\ell n y) = \frac{d}{dx}\left(x(\ell n x)\right)\left\{\begin{matrix}\text{using by} \\ \text{Product Rule}\end{matrix}\right\}$$
$$\frac{1}{y}\frac{dy}{dx} = \left(\frac{d}{dx}(x)\right)\ell n x + x\left(\frac{d}{dx}(\ell n x)\right)$$
$$\frac{dy}{dx} = y\left[(1)\ell n x + x\left(\frac{1}{x}\right)\right]$$
$$\frac{dy}{dx} = x^x[\ell n x + 1] \Rightarrow \frac{dy}{dx} = \boxed{x^x[1 + \ell n x]}$$

13. **Find the acceleration of the moving particle given according to the law $V^2 = 4S - 10$, where S and V have their usual meaning.**

Sol. $V^2 = 4S - 10$

Differentiate both sides w.r.t. 't':

$$\frac{d}{dt}\left(V^2\right) = \frac{d}{dt}(4S - 10)$$
$$2V\frac{dV}{dt} = 4\frac{dS}{dt} - 0$$
$$2V(a) = 4V \because \left\{\begin{matrix} v = \frac{ds}{dt} \\ \& \ a = \frac{dv}{dt}\end{matrix}\right\}$$
$$a = \frac{4V}{2V} = \boxed{2}$$

14. **If $s=\log(t)$, , find the velocity and acceleration at $t=3\sec$.**

Sol. $s=\log(t)$

Differentiate both sides w.r.t. 't':

$$\frac{d}{dt}(s)=\frac{d}{dt}(\log t)$$

$$v=\frac{1}{t}\rightarrow(i)$$

Diff. again both sides w.r.t. 't':

$$\frac{d}{dt}(v)=dt\left(\frac{1}{t}\right)$$

$$a=-\frac{1}{t^2}\rightarrow(ii)$$

Put $t=3$ in eq.(i) & eq.(ii),

we get:

$$\boxed{v\Big|_{t=3}=\frac{1}{3}\ m/s}\ \&$$

$$a\Big|_{t=3}=-\frac{1}{(3)^2}=\boxed{\frac{-1}{9}m/\sec^2}$$

15. **The distance x meters moved by a particle in t seconds is given by $x=t^3+3t^2+4$. Find the velocity and accelerating after 3 seconds.**

Sol. $x=t^3+3t^2+4$

Differentiate both sides w.r.t. 't':

$$\frac{d}{dt}(x)=\frac{d}{dt}(t^3+3t^2+4)$$

$$\frac{dx}{dt}=3t^2+3(2t)+0$$

$$v=3t^2+6t\rightarrow(i)$$

Diff. again both sides w.r.t. 't':

$$\frac{d}{dt}(v)=\frac{d}{dt}(3t^2+6t)$$

$$\frac{dv}{dt}=3(2t)+6(1)$$

$$a=6t+6\rightarrow(ii)$$

Put $t=3$ in eq.(i) & eq.(ii),

we get:

$$v\Big|_{t=3}=3(3)^2+6(3)$$

$$v\Big|_{t=3}=27+18=\boxed{45\ m/\sec}$$

$$a\Big|_{t=3}=6(3)+6$$

$$a\Big|_{t=3}=18+6=\boxed{24\,m/\sec^2}$$

16. **If $s=\sin 2t$, find the velocity at $t=\frac{\pi}{6}$.**

Sol. $s=\sin 2t$

Differentiate both sides w.r.t. 't':

$$v=\frac{ds}{dt}=\frac{d}{dt}(\sin 2t)$$

$$v=\cos 2t\left(\frac{d}{dt}(2t)\right)$$

$$v=\cos 2t(2(1))$$

$$v=2\cos 2t$$

At $t=\frac{\pi}{6}$

$$v\Big|_{t=\frac{\pi}{6}}=2\cos 2\left(\pi/6\right)$$

$$v\Big|_{t=\frac{\pi}{6}}=2\left(\frac{1}{2}\right)=\boxed{1}$$

17. **Evaluate $\int\left(\sqrt{x}+\frac{1}{\sqrt{x}}\right)^2dx$**

Sol. $\int\left(\sqrt{x}+\frac{1}{\sqrt{x}}\right)^2dx$

$$=\int\left[\left(\sqrt{x}\right)^2+2\left(\sqrt{x}\right)\left(\frac{1}{\sqrt{x}}\right)+\left(\frac{1}{\sqrt{x}}\right)^2\right]dx$$

$$=\int\left[x+2+\frac{1}{x}\right]dx=\boxed{\frac{x^2}{2}+2x+\ell nx+c}$$

18. Integrate $\int \sin^2 x dx$

Sol. $\int \sin^2 x dx$

$= \int \left(\frac{1-\cos 2x}{2}\right) dx \quad \because \left\{\sin^2 x = \frac{1-\cos 2x}{2}\right\}$

$= \frac{1}{2}\left[\int (1) dx - \int (\cos 2x) dx\right]$

$= \boxed{\frac{1}{2}\left[x - \frac{\sin 2x}{2}\right] + c}$

19. Evaluate $\int (\sin x - \cos x)^2 dx$

Sol. $\int (\sin x - \cos x)^2 dx$

$= \int (\sin^2 x + \cos^2 x - 2\sin x \cos x) dx$

$= \int (1 - \sin 2x) dx \because \left\{\begin{matrix} \sin^2 x + \cos^2 x = 1 \\ \sin 2x = 2\sin x \cos x \end{matrix}\right\}$

$= x - \left(\frac{-\cos 2x}{2}\right) + c$

$= \boxed{x + \frac{1}{2}\cos 2x + c}$

20. Evaluate $\int \frac{\cot x}{\ell n \sin x} dx$

Sol. $\int \frac{\cot x}{\ell n \sin x} dx$

$= \int \frac{1}{\ell n x \sin x} . \cot x \, dx$

Put $\ell n \sin x = t$

$\frac{d}{dx}(\ell n \sin x) = \frac{d}{dx}(t)$

$\frac{1}{\sin x} \frac{d}{dx}(\sin x) = \frac{dt}{dx}$

$\frac{1}{\sin x} \cos x = \frac{dt}{dx}$

$\cot x \, dx = dt$

$= \int \left(\frac{1}{t}\right) dt$

$= \ell n(t) + c$

$= \boxed{\ell n(\ell n \sin x) + c}$

21. Evaluate $\int 3x\sqrt{1-2x^2} dx$

Sol. $\int 3x\sqrt{1-2x^2} dx$

$= 3\int (1-2x^2)^{1/2} (x) dx$

$= -\frac{3}{4} \int (1-2x^2)^{1/2} (-4x) dx$

$= -\frac{3}{4} \frac{(1-2x^2)^{3/2}}{3/2} + c \quad$ {using Rule-I}

$= -\frac{3}{4} . \frac{2}{3} (1-2x^2)^{3/2} + c$

$= \boxed{-\frac{1}{2} (1-2x^2)^{3/2} + c}$

22. ~~**Evaluate $\int \frac{e^{m \tan^{-1} x}}{1+x^2} dx$**~~

Sol. $\int \frac{e^{m \tan^{-1} x}}{1+x^2} dx$

$= \int e^{m \tan^{-1} x} \left(\frac{1}{1+x^2}\right) dx$

Put $\tan^{-1} x = t$

$\frac{d}{dx}(\tan^{-1} x) = \frac{d}{dx}(t)$

$\frac{1}{1+x^2} = \frac{dt}{dx}$

$\left(\frac{1}{1+x^2}\right) dx = dt$

$= \int e^{mt} dt$

$= \frac{e^{mt}}{m} + c = \boxed{\frac{1}{m} e^{m \tan^{-1} x} + c}$

23. Evaluate $\int (x \cos x) dx$

Sol. $\int (x \cos x) dx$

Integrating by parts:

taking $u = x$ & $v = \cos x$

$= x\int \cos x dx - \int \left[\frac{d}{dx}(x) \int \cos x dx\right] dx$

$= x\sin x - \int[(1)\sin x]dx$

$= x\sin x - \int \sin x\, dx$

$= x\sin x - (-\cos x) + c$

$= \boxed{x\sin x + \cos x + c}$

24. Evaluate $\int(\ell n x)dx$

Sol. $\int(\ell n x)dx$

$= \int(\ell n x \cdot 1)dx$

Integrating by parts : taking $u = \ell n x$ & $v = 1$

$= \ell n x\int(1)dx - \int\left[\frac{d}{dx}(\ell n x)\int(1)dx\right]dx$

$= \ell n x(x) - \int\frac{1}{x}.(x)dx$

$= x\ell n x - \int(1)dx$

$= x\ell n x - (x) + c = \boxed{x(\ell n x - 1) + c}$

25. Find value of $\int_0^{\pi/2}(\sin^2 x\cos x)\,dx$

Sol. $\int_0^{\pi/2}(\sin^2 x\cos x)\,dx$

$= \left[\frac{\sin^3 x}{3}\right]_0^{\pi/2}$ {using Rule-I}

$= \frac{1}{3}\left[\sin^3\left(\frac{\pi}{2}\right) - \sin^3(0)\right]$

$= \frac{1}{3}\left[\sin^3(90°) - \sin^3(0°)\right]$

$= \frac{1}{3}\left[(1)^3 - (0)^3\right]$

$= \frac{1}{3}(1-0) = \boxed{\frac{1}{3}}$

26. Find value of $\int_0^{\pi}(x\cos x)\,dx$

Sol. $\int_0^{\pi}(x\cos x)\,dx$

Integrating by parts :

taking $u = x$ & $v = \cos x$

$= x\int_0^{\pi}(\cos x)\,dx - \int_0^{\pi}\left(\frac{d}{dx}(x)\int(\cos x)dx\right)dx$

$= [x\sin x]_0^{\pi} - \int_0^{\pi}1.\sin x\,dx$

$= (\pi\sin\pi - 0\sin 0) - [-\cos x]_0^{\pi}$

$= (0-0) + [\cos x]_0^{\pi}$

$= \cos\pi - \cos 0 = -1 - 1 = \boxed{-2}$

27. ~~Evaluate $\int_0^1(x e^x)dx$~~

Sol. $\int_0^1(x e^x)dx$

Integrating by parts :

taking $u = x$ & $v = e^x$

$= x\int_0^1 e^x dx - \int_0^1\left(\frac{d}{dx}(x)\int e^x dx\right)dx$

$= \left[x e^x\right]_0^1 - \int_0^1(1.\ e^x)dx$

$= \left[1e^1 - 0e^0\right] - \left[e^x\right]_0^1$

$= [e - 0] - \left[e^1 - e^0\right]$

$= \not{e} - \not{e} + 1 = \boxed{1}$

28. Find the value of $\int_{-1}^{1}(3x^2 - x^3)dx$

Sol. $\int_{-1}^{1}(3x^2 - x^3)dx$

$= \left[\frac{3x^3}{3} - \frac{x^4}{4}\right]_{-1}^{1} = \left[x^3 - \frac{x^4}{4}\right]_{-1}^{1}$

$= \left[(1)^3 - \frac{(1)^4}{4}\right] - \left[(-1)^3 - \frac{(-1)^4}{4}\right]$

$= \left(1 - \frac{1}{4}\right) - \left(-1 - \frac{1}{4}\right)$

$= 1 - \frac{1}{4} + 1 + \frac{1}{4} = \boxed{2}$

29. Find the rectangular co-ordinates of the point with polar co-ordinates $(4, 30°)$.

Sol. $\text{Here}: r = 4 \;\&\; \theta = 30°$

Rectangular coordinates of points are :

$$= (r\cos\theta\,,\, r\sin\theta)$$

$$= (4\cos 30°\,,\, 4\sin 30°)$$

$$= \left(4\sqrt{\frac{3}{2}}, 4\sqrt{\frac{1}{2}}\right)$$

$$= \boxed{\left(2\sqrt{3}\,,\, 2\right)}$$

30. Show that the point $(3, \sqrt{7})$ is on a circle with center at the origin and radius 4.

Sol. $|\overline{OP}|$ = Distance between Origin $(0, 0)$ and point $(3, \sqrt{7})$.

$$|\overline{OP}| = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

$$|\overline{OP}| = \sqrt{(0-3)^2 + (0-\sqrt{7})^2}$$

$$|\overline{OP}| = \sqrt{(-3)^2 + (-\sqrt{7})^2}$$

$$|\overline{OP}| = \sqrt{9+7}$$

$$|\overline{OP}| = \sqrt{16} = \boxed{4}$$

Hence the given point $(3, \sqrt{7})$ lies on a circle. **Proved.**

31. Is the point $(0, 4)$ inside or outside the circle of radius 4 with center at $(-3, 1)$.

Sol. $|\overline{CP}|$ = Distance between Center $(-3, 1)$ and point $(0, 4)$.

$$|\overline{CP}| = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

$$|\overline{CP}| = \sqrt{(0+3)^2 + (4-1)^2}$$

$$|\overline{CP}| = \sqrt{(3)^2 + (3)^2} = \sqrt{9+9}$$

$$|\overline{CP}| = \sqrt{18} = \boxed{4.4} > 4$$

As distance between both points is greater than 4 so the point $(0, 4)$ lie **outside** the circle.

32. Find the value of 'y' so that the distance between $(1, y)$ and $(-1, 4)$ is 2.

Sol. Let $A(1, y) \;\&\; B(-1, 4)$

As, $|\overline{AB}| = 2$

$$\sqrt{(1-(-1))^2 + (y-4)^2} = 2$$

Squaring both sides, we have :

$$\left(\sqrt{(2)^2 + (y-4)^2}\right)^2 = (2)^2$$

$$4 + (y-4)^2 = 4$$

$$(y-4)^2 = 4-4$$

$$(y-4)^2 = 0$$

Taking square root on both sides :

$$\sqrt{(y-4)^2} = \sqrt{0}$$

$$y - 4 = 0 \quad \Rightarrow \quad \boxed{y = 4}$$

33. Find an equation of the line with slope $-\frac{2}{3}$ and having y-intercept 3.

Sol. $\text{Here}: m = -\frac{2}{3} \;\&\; c = 3$

Equation of line in slope - intercept form :

$$y = mx + c$$

$$y = \frac{-2}{3}x + 3$$

Multipling each term both sides by 3, we get :

$$3y = -2x + 9$$

$$3y + 2x - 9 = 0$$

$$\boxed{2x + 3y - 9 = 0}$$

34. **Define imaginary circle.**

Sol. A circle is called imaginary circle if $r > 0$.

---

35. **Define imaginary circle.**

Sol. A circle is called imaginary circle if $r = 0$.

---

36. **Define imaginary circle.**

Sol. A circle is called imaginary circle if $r < 0$.

---

37. **Find the equation of the circle which touches both the axes of 4th quadrant and has a radius of 5 units.**

Sol. As circle touches both the axes of 4^{th} – quad. & Radius $= r = 5$

So, centre $= (h, k) = (5, -5)$

Standard form of eq. of circle:

$$(x-h)^2 + (y-k)^2 = r^2$$

Put $h = 5,\ k = -5\ \&\ r = 5$

$$(x-5)^2 + (y+5)^2 = r^2$$

$$(x)^2 - 2(x)(5) + (5)^2 + (y)^2 + 2(y)(5) + (5)^2 = 25$$

$$x^2 - 10x + 25 + y^2 + 10y + 25 - 25 = 0$$

$$\boxed{x^2 + y^2 - 10x + 10y + 25 = 0}$$

Section - II

Note: Attemp any three (3) questions $\boxed{3 \times 10 = 30}$

Q.2.[a] ~~Prove that: $\lim_{\theta \to 0} \frac{\sin\theta}{\theta} = 1$, where θ is in radian.~~

Sol. See proof of theorem (Page # 20)

[b] ~~Differentiate: $\frac{x^2+a^2}{x^2-a^2}$ w.r.t. $\frac{x-a}{x+a}$~~

Sol. See Q.1(vii) of Ex # 2.4 (Page # 86)

Q.3.[a] ~~If $x = a(\cos t + \sin t)$, $y = a(\sin t - t\cos t)$, find $\frac{dy}{dx}$.~~

Sol. See Q.8(ii) of Ex # 3.1 (Page # 122)

[b] **Show that $\frac{\ell nx}{x}$ has a maximum value at $x = e$.**

Sol. See Q.15 of Ex # 5.1 (Page # 231)

Q.4.[a] **Evaluate**

$$\int \left(\frac{a\sin^3 x + b\cos^3 x}{\sin^2 x \cos^2 x} \right) dx$$

Sol. See Q.8 of Ex # 5.2 (Page # 237)

[b] ~~Evaluate $\int \frac{dx}{(a^2 - x^2)^{3/2}}$~~

Sol. See Q.1(x) of Ex # 6.2 (Page # 278)

Q.5.[a] ~~Show that area of a circle of radius 'r' is πr^2.~~

Sol. See example # 19 of Chapter 07.

[b] **Find the equation of the circle having (-2, 5) and (3, 4) as the end points of its diameter. Find also its center and radius.**

Sol. See Q.8[a] of Ex # 9 (Page # 447)

Q.6.[a] **Show that the following lines are concurrent. Also find the point of concurrency. $3x - 5y + 8 = 0$, $x + 2y - 4 = 0$ and $4x - 3y + 4 = 0$.**

Sol. See Q.3[a] of Ex # 8.5 (Page # 400)

[b] **The mid points of the sides of a triangle are at (-1, 4), (5, 2) and (2, -1). Find its vertices.**

Sol. See Q.11 of Ex # 8.2 (Page # 369)

****