DAE / IA - 2019

MATH - 233 APPLIED MATHEMATICS - II

PAPER 'B' PART - A (OBJECTIVE)

Time : 30 Minutes Marks : 15

Q.1: *Encircle the correct answer.*

1. $\int (ax+b)\,dx = ?$

[a] $\frac{(ax+b)^2}{2a}$ [b] $\frac{(ax+b)^2}{2}$

[c] $\ell n(ax+b)$ [d] $a(ax+b)$

2. $\int \left(\frac{a+x}{x}\right) dx = ?$

[a] $a\,\ell nx + x$ [b] $\frac{(ax+b)^2}{2}$

[c] $\ell nx + a$ [d] $x + a$

3. $\int \left(\frac{\operatorname{cosec}^2 x}{\cot x}\right) dx = ?$

[a] $-\ell n \cot x$ [b] $\ell n \cot x$

[c] $\frac{\cot^2 x}{2}$ [d] $\ell n\left(\operatorname{cosec}^2 x\right)$

4. $\int \left(\frac{1}{\sqrt{1-x^2}}\right) dx = ?$

[a] $\sin^{-1} x$ [b] $\cos^{-1} x$

[c] $\sec^{-1} x$ [d] $\tan^{-1} x$

5. $\int \left(\frac{-1}{\sqrt{1-x^2}}\right) dx = ?$

[a] $\sin^{-1} x$ [b] $\cos^{-1} x$

[c] $\sec^{-1} x$ [d] $\sqrt{1-x^2}$

6. $\int \left(\frac{e^x}{1+e^x}\right) dx = ?$

[a] $1+e^x$ [b] $\ell n\left(1+e^x\right)$

[c] e^x [d] $\frac{\left(1+e^x\right)^2}{2}$

7. $\int_1^3 \left(e^{2x}\right) dx = ?$

[a] $e^6 - e^2$ [b] $\frac{e^{2x}}{2}$

[c] $\frac{1}{2}\left(e^6 + e^2\right)$ [d] $\frac{1}{2}\left(e^6 - e^2\right)$

8. $\int_1^2 \left(3x^2\right) dx = ?$ [a] 7 [b] 8 [c] 6 [d] 9

9. An equation involving one or more derivative of a function is called:

[a] Quadratic [b] Linear

[c] Differential [d] Cubic

10. Order of differential equation

$\left(\frac{d^3y}{dx^3}\right)^2 + \frac{dy}{dx} + y = 0$ is:

[a] 2 [b] 1 [c] 0 [d] 3

11. ~~If an odd function, then Fourier coefficient~~ 'a_n' ~~is;~~

[a] 0 [b] 1 [c] -1 [d] 2

12. ~~If an even function, the Fourier coefficient~~ 'b_n' ~~is:~~

[a] 0 [b] 1 [c] -1 [d] 2

13. $L^{-1}\left(\frac{1}{S}\right) = ?$ [a] 1 [b] 2 [c] 3 [d] 4

14. $L^{-1}\left(\frac{1}{S^2}\right) = ?$ [a] 1 [b] t [c] t^2 [d] $\frac{t}{2}$

15. $L^{-1}\left\{\frac{S}{S^2+\omega^2}\right\} = ?$

[a] $\sin \omega t$ [b] $\cos \omega t$

[c] $\sin \frac{t}{\omega}$ [d] $\cos \frac{t}{\omega}$

Answer Key

1	a	2	a	3	a	4	a	5	b
6	b	7	b	8	a	9	c	10	a
11	b	12	a	13	a	14	b	15	b

****

DAE / IA - 2019

MATH - 233 APPLIED MATHEMATICS - II

PAPER 'B' PART - B (SUBJECTIVE)

Time : 2 : 30Hrs Marks : 60

Section - I

Q.1. Write short answers to any Eighteen (18) questions.

1. Find $\int\left(e^{3x}+e^{5x}\right)dx$

Sol. $\int\left(e^{3x}+e^{5x}\right)dx$

$$=\boxed{\frac{e^{3x}}{3}+\frac{e^{5x}}{5}+c}$$ {Using formula # 05 from page # 282}

2. Evaluate $\int\left(3x^2+2x+1\right)dx$

Sol. $\int\left(3x^2+2x+1\right)dx$

$\int\left(3x^2+2x+1\right)dx$

$$=3\frac{x^3}{3}+2\frac{x^2}{2}+x+c$$

$$=\boxed{x^3+x^2+x+c}$$

3. Evaluate $\int\left(\frac{1+x}{x}\right)dx$

Sol. $\int\left(\frac{1+x}{x}\right)dx$

$$=\int\left(\frac{1}{x}+\frac{x}{x}\right)dx$$

$$=\int\left(\frac{1}{x}+1\right)dx=\boxed{\ell n\,x+x+c}$$

4. Find $\int\left(e^{x}+e^{2x}+e^{3x}\right)dx$

Sol. $=\int e^{x}dx+\int e^{2x}dx+\int e^{3x}dx$

$$=\boxed{e^{x}+\frac{e^{2x}}{2}+\frac{e^{3x}}{3}+c}$$

5. Find $\int\sqrt{x}\left(x^5+\frac{1}{x}\right)dx$

Sol. $\int\sqrt{x}\left(x^5+\frac{1}{x}\right)dx$

$$=\int\left(x^{5+\frac{1}{2}}+x^{\frac{1}{2}-1}\right)dx$$

$$=\int\left[x^{11/2}+x^{-1/2}\right]dx$$

$$=\frac{x^{13/2}}{13/2}+\frac{x^{1/2}}{1/2}+c$$

$$=\boxed{\frac{2}{13}x^{13/2}+2x^{1/2}+c}$$

6. Evaluate $\int\left(\sqrt{x}+\frac{1}{\sqrt{x}}\right)^2dx$

Sol. $\int\left(\sqrt{x}+\frac{1}{\sqrt{x}}\right)^2dx$

$$=\int\left[\left(\sqrt{x}\right)^2+2\left(\sqrt{x}\right)\left(\frac{1}{\sqrt{x}}\right)+\left(\frac{1}{\sqrt{x}}\right)^2\right]dx$$

$$=\int\left[x+2+\frac{1}{x}\right]dx$$

$$=\int\left(x+2+x^{-1}\right)dx=\boxed{\frac{x^2}{2}+2x+\ell nx+c}$$

7. Evaluate $\int\frac{1}{2}\left(e^{\frac{1}{2}x}-e^{-\frac{1}{2}x}\right)dx$

Sol. $\int\frac{1}{2}\left(e^{\frac{1}{2}x}-e^{-\frac{1}{2}x}\right)dx$

$$=\frac{1}{2}\left(\frac{e^{\frac{1}{2}x}}{\frac{1}{2}}-\frac{e^{-\frac{1}{2}x}}{-\frac{1}{2}}\right)+c$$

$$=\boxed{e^{\frac{1}{2}x}+e^{-\frac{1}{2}x}+c}$$

23. Find $\int \frac{1}{25+x^2}\,dx$

Sol. $\int \frac{1}{25+x^2}\,dx$

$= \int \frac{1}{(5)^2+(x)^2}\,dx$

$= \boxed{\frac{1}{5}\tan^{-1}\left(\frac{x}{5}\right)+c}$ {Using formula # 17 from page # 282}

---

9. Find $\int\left(x^2\cos x^3\right)dx$

Sol. $\int\left(x^2\cos x^3\right)dx$

$= \frac{1}{3}\int\left(\cos x^3 3x^2\right)dx$

Put $x^3 = t$

$\frac{d}{dx}\left(x^3\right) = \frac{d}{dx}(t)$

$3x^2 = \frac{dt}{dx}$

$3x^2\,dx = dt$

$= \frac{1}{3}\int(\cos t)\,dt$

$= \frac{1}{3}\sin t + c$

$= \boxed{\frac{1}{3}\sin x^3 + c}$

---

10. Find $\int\left(\frac{1}{x(\ell n\, x)}\right)dx$

Sol. $\int\left(\frac{1}{x(\ell n\, x)}\right)dx$

Put $\ell nx = t$

$\frac{1}{x}dx = dt$

$= \int\frac{1}{t}\,dt$

$= \ell n\,|t| + c$

$= \boxed{\ell n\,|\ell nx| + c}$

---

11. Find $\int \frac{\sin^{-1}x}{\sqrt{1-x^2}}\,dx$

Sol. $\int \frac{\sin^{-1}x}{\sqrt{1-x^2}}\,dx$

$= \int \sin^{-1}x\left(\frac{1}{\sqrt{1-x^2}}\right)dx$

$= \boxed{\frac{\left(\sin^{-1}x\right)^2}{2}+c}$ {using Rule-I}

---

12. Find $\int\left(\cos^3 x\right)dx$

Sol. $\int\left(\cos^3 x\right)dx$

$= \int\left(\cos^2 x.\cos x\right)dx$

$= \int\left(1-\sin^2 x\right)\cos x\,dx$

$= \int \cos x\,dx - \int \sin^2 x.\cos x\,dx$

$= \boxed{\sin x - \frac{\sin^3 x}{3} + c}$

---

13. Find $\int\left(\cos^4 x.\sin x\right)dx$

Sol. $\int\left(\cos^4 x.\sin x\right)dx$

$= -\int \cos^4 x(-\sin x)\,dx$

$= \boxed{-\frac{\cos^5 x}{5}+c}$ {using Rule-I}

---

14. Find value of $\int_0^{\pi/4}\left(\cos^2 x\right)dx$

Sol. $\int_0^{\pi/4}\left(\cos^2 x\right)dx$

$= \int_0^{\pi/4}\left(\frac{1+\cos 2x}{2}\right)dx \quad \because \left\{\cos^2 x = \frac{1+\cos 2x}{2}\right\}$

$= \frac{1}{2}\left[x + \frac{\sin 2x}{2}\right]_0^{\pi/4}$

$$=\frac{1}{2}\left[\left\{\frac{\pi}{4}+\frac{\sin 2\left(\pi/4\right)}{2}\right\}-\left\{0+\frac{\sin 2(0)}{2}\right\}\right]$$

$$=\frac{1}{2}\left[\left\{\frac{\pi}{4}+\frac{\sin 2(45^\circ)}{2}\right\}-\left\{\frac{\sin 2(0^\circ)}{2}\right\}\right]$$

$$=\frac{1}{2}\left[\frac{\pi}{4}+\frac{\sin 90^\circ}{2}-\frac{\sin 0^\circ}{2}\right]$$

$$=\frac{1}{2}\left[\frac{\pi}{4}+\frac{1}{2}-\frac{0}{2}\right]\left\{\begin{array}{c}\text{using calculator}\\ \sin(90^\circ)=1\ \&\ \sin(0^\circ)=0\end{array}\right\}$$

$$=\frac{1}{2}\left(\frac{\pi}{4}+\frac{1}{2}\right)=\frac{1}{2}\left(\frac{\pi+2}{4}\right)=\boxed{\frac{1}{8}(\pi+2)}$$

15. Find value of $\int_0^{\pi/2}\left(\tan^2 x\right)dx$

Sol. $\int_0^{\pi/2}\left(\tan^2 x\right)dx$

$$=\int_0^{\pi/2}\left(\sec^2 x-1\right)dx \because \left\{\begin{array}{l}\tan^2 x\\ =\sec^2 x-1\end{array}\right\}$$

$$=\left[\tan x-x\right]_0^{\pi/2}$$

$$=\left[\tan\left(\frac{\pi}{2}\right)-\frac{\pi}{2}\right]-\left[\tan 0-0\right]$$

$$=\tan(90^\circ)-\frac{\pi}{2}-\tan(0^\circ)$$

$$=\infty-\frac{\pi}{2}-0=\boxed{\infty}$$

16. Find value of

$$\int_0^{\pi/2}\left(\sin^2 x\cos x\right)dx.$$

Sol. $\int_0^{\pi/2}\left(\sin^2 x\cos x\right)dx$

$$=\left[\frac{\sin^3 x}{3}\right]_0^{\pi/2}\left\{\begin{array}{c}\textbf{using}\\ \textbf{Rule-I}\end{array}\right\}$$

$$=\frac{1}{3}\left[\sin^3\left(\frac{\pi}{2}\right)-\sin^3(0)\right]$$

$$=\frac{1}{3}\left[\sin^3(90^\circ)-\sin^3(0^\circ)\right]$$

$$=\frac{1}{3}\left[(1)^3-(0)^3\right]$$

$$=\frac{1}{3}(1-0)$$

$$=\boxed{\frac{1}{3}}$$

17. Find value of $\int_0^{\pi}\left(x\cos x\right)dx$

Sol. $\int_0^{\pi}\left(x\cos x\right)dx$

Integrating by parts:

taking $u=x$ & $v=\cos x$

$$=x\int_0^{\pi}(\cos x)\,dx-\int_0^{\pi}\left(\frac{d}{dx}(x)\int(\cos x)\,dx\right)dx$$

$$=\left[x\sin x\right]_0^{\pi}-\int_0^{\pi}1.\sin x\,dx$$

$$=\left(\pi\sin\pi-0\sin 0\right)-\left[-\cos x\right]_0^{\pi}$$

$$=(0-0)+\left[\cos x\right]_0^{\pi}$$

$$=\cos\pi-\cos 0=-1-1=\boxed{-2}$$

18. Solve the differential equation

$$x^2\frac{dy}{dx}=\cos^2 y$$

Sol. $x^2\frac{dy}{dx}=\cos^2 y$

$$\frac{1}{\cos^2 y}dy=\frac{1}{x^2}dx$$

Integrating both sides, we have:

$$\int\sec^2 y\,dy=\int x^{-2}dx$$

$$\tan y=\frac{x^{-1}}{-1}+c$$

$$\tan y=-x^{-1}+c$$

$$\boxed{\tan y+x^{-1}=c}$$

19. **Find the solution of**

$$\frac{dy}{dx} = -\sin x + 3x^2$$

Sol.
$$\frac{dy}{dx} = -\sin x + 3x^2$$

$$dy = \left(-\sin x + 3x^2\right)dx$$

Integrating both sides, we have :

$$\int 1\,dy = \int\left(-\sin x + 3x^2\right)dx$$

$$y = -(-\cos x) + 3\left(\frac{x^3}{3}\right) + c$$

$$\boxed{y = \cos x + x^3 + c}$$

---

20. **Find the solution of**

$$\frac{dy}{dx} = 1 + x + y + xy$$

Sol.
$$\frac{dy}{dx} = 1 + x + y + xy$$

$$\frac{dy}{dx} = (1+x) + y(1+x)$$

$$\frac{dy}{dx} = (1+x)(1+y)$$

$$\frac{1}{(1+y)}dy = (1+x)dx$$

Integrating both sides, we have :

$$\int\left(\frac{1}{1+y}\right)dy = \int(1+x)dx$$

$$\boxed{\ell n(1+y) = x + \frac{x^2}{2} + c}$$

---

21. **Find the value of**

$$\cos^2 x\frac{dy}{dx} + \cos^2 y = 0$$

Sol.
$$\cos^2 x\frac{dy}{dx} + \cos^2 y = 0$$

$$\cos^2 x\frac{dy}{dx} = -\cos^2 y$$

$$\frac{1}{\cos^2 y}dy = -\frac{1}{\cos^2 x}dx$$

Integrating both sides, we have :

$$\int\frac{1}{\cos^2 y}dy = -\int\frac{1}{\cos^2 x}dx$$

$$\int \sec^2 y\,dy = -\int \sec^2 x\,dx$$

$$\tan y = -\tan x + c$$

$$\boxed{\tan y + \tan x = c}$$

---

22. **Find the value of** $\frac{dy}{dx} = \sqrt{\frac{1-y^2}{1-x^2}}$

Sol.
$$\frac{dy}{dx} = \sqrt{\frac{1-y^2}{1-x^2}}$$

$$\frac{1}{\sqrt{1-y^2}}dy = \frac{1}{\sqrt{1-x^2}}dx$$

Integrating both sides, we have :

$$\int\frac{1}{\sqrt{1-y^2}}dy = \int\frac{1}{\sqrt{1-x^2}}dx$$

$$\boxed{\sin^{-1} y = \sin^{-1} x + c}$$

---

23. ~~**What is the inverse transformation of**~~ $\frac{1}{s+a}$**?**

Sol.
$$L^{-1}\left\{\frac{1}{s+a}\right\}$$

$$= L^{-1}\left\{\frac{1}{s-(-a)}\right\} = \boxed{e^{-at}}$$

---

24. ~~**What is inverse Laplace transformation of the function**~~

$$\frac{4}{s^2+16}?$$

Sol.
$$L^{-1}\left\{\frac{4}{s^2+16}\right\}$$

$$= L^{-1}\left\{\frac{4}{s^2+(4)^2}\right\} = \boxed{\sin(4t)}$$

25. ~~Find~~ $L^{-1}\left\{\frac{1}{s-a}-\frac{1}{s+a}\right\}$

Sol. $L^{-1}\left\{\frac{1}{s-a}-\frac{1}{s+a}\right\}$

$=L^{-1}\left\{\frac{1}{s-a}\right\}-L^{-1}\left\{\frac{1}{s+a}\right\}$

$=\boxed{e^{at}-e^{-at}}$

26. ~~What is inverse Laplace transform of~~ $\frac{2}{s^3}$?

Sol. $L^{-1}\left\{\frac{2}{s^3}\right\}=L^{-1}\left\{\frac{2!}{s^{2+1}}\right\}=\boxed{t^2}$

27. Define Fourier Series?

Sol. A Fourier series decomposes a periodic function into sum of a set of simple oscillating functions, called sines and cosines.

Section - II

Note: Attemp any three (3) questions $3\times 8 = 24$

Q.2.[a] Evaluate: $\int\frac{dx}{1+\sin x}$

Sol. See Q.5 of Ex# 7.2 (Page # 292)

[b] Evaluate:

$$\int(\sin x+\cos x)^n\left(\cos^2 x-\sin^2 x\right)dx$$

Sol. See Q.1(xv) of Ex # 7.3 (Page # 301)

Q.3.[a] ~~Evaluate~~ $\int \ell n\left(x+\sqrt{x^2+1}\right)dx$

Sol. See Q.3(v) of Ex# 8.3 (Page # 348)

[b] ~~Evaluate~~ $\int\frac{dx}{\left(a^2-x^2\right)^{3/2}}$

Sol. See Q.1(x) of Ex # 8.2 (Page # 334)

Q.4.[a] Calculate $\int_0^{\pi/3}\frac{dx}{1-\sin x}$

Sol. See example # 07 of Chapter 09.

[b] ~~Find the area of the region enclosed by curve $y = 3 - x^2$ and the line $y = -x + 1$.~~

Sol. See Q.7 of Ex # 9.2 (Page # 391)

Q.5.[a] Find the general solution of equation:

$$dx + xydy = y^2dx + ydy$$

Sol. See Q.8 of Ex # 10 (Page # 415)

[b] Find the particular solution satisfying the given boundary conditions

$2xdx - dy = x(xdy - ydx)$ given y = 1 when x = −3

Sol. See Q.16 of Ex # 10 (Page # 419)

Q.6. Prove that:

(i) $L\left\{e^{at}\cos\omega t\right\}=\frac{s-a}{(s-a)^2+\omega^2}$

Sol. See Q.4(i) of Ex # 12 (Page # 467)

(ii) $L\left\{e^{at}\sin\omega t\right\}=\frac{\omega}{(s-a)^2+\omega^2}$

Sol. See Q.4(ii) of Ex # 12 (Page # 468)

****