

DAE / IA - 2019

MATH- 233 APPLIED MATHEMATICS-II

PAPER 'A' PART - A (OBJECTIVE)

Time : 30 Minutes

Marks : 15

Q.1: Encircle the correct answer.

1.  $\lim_{x \rightarrow 3} \sqrt{25 - x^2}$   
 [a] 5 [b] 3 [c] 4 [d] 0
2.  $\lim_{x \rightarrow \frac{\pi}{3}} (\cos x) = ?$   
 [a]  $\sqrt{3}/2$  [b]  $1/2$  [c] 0 [d]  $1/\sqrt{2}$
3. Derivative of  $3x^2 + x^{-1/2}$  w.r.t. x is:  
 [a]  $6x + 1/2x^{-1/2}$  [b]  $6x - 1/2x^{-1/2}$   
 [c]  $6x - 1/2x^{1/2}$  [d]  $6x + 1/2x^{-1/2}$
4. Derivative  $x^2 - x^{-2}$  w.r.t. x is:  
 [a]  $2x - 2x^{-2}$  [b]  $2x + 2x^{-3}$   
 [c]  $2x - 4x^{-2}$  [d]  $2x - 2x^{-3}$
5. If  $u = t^2 - 3$  then  $\frac{du}{dt} = ?$   
 [a] 2t [b]  $2t - 3$  [c]  $t^{-2}$  [d]  $2t^{-2}$
6.  $\frac{d}{dx} (\sec 3x) = ?$   
 [a]  $3 \sec 3x \tan 3x$  [b]  $\sec 3x \tan 3x$   
 [c]  $3 \sec 3x \cot 3x$  [d]  $-3 \sec 3x \tan 3x$
7.  ~~$\frac{d}{dx} (\sin^{-1} x) = ?$~~   
 [a]  $\frac{1}{\sqrt{x^2 - 1}}$  [b]  $\frac{-1}{\sqrt{1 - x^2}}$   
 [c]  $\frac{\sin^{-1} x}{\sqrt{1 - x^2}}$  [d]  $\frac{1}{\sqrt{1 - x^2}}$
8.  ~~$\frac{d}{dx} \sin^{-1}(\sqrt{x}) = ?$~~   
 [a]  $\frac{1}{\sqrt{1 - x^2}}$  [b]  $\frac{1}{\sqrt{1 + x^2}}$   
 [c]  $\frac{1}{2\sqrt{x}}$   $\frac{1}{\sqrt{1 - x}}$  [d]  $\frac{1}{2\sqrt{x}}$   $\frac{1}{1 + x}$

9.

~~$\frac{d}{dx} (\sin^{-1} \frac{1}{x}) = ?$~~

- [a]  $\frac{1}{\sqrt{1 - x^2}}$  [b]  $\frac{-1}{x\sqrt{x^2 - 1}}$   
 [c]  $\frac{1}{x\sqrt{x^2 - 1}}$  [d]  $\frac{-1}{\sqrt{x^2 - 1}}$
10. If  $\frac{dy}{dx}$  does not change sign before and after a point where it vanished then that is point of:  
 [a] Maxima [b] Minima  
 [c] Inflection [d] None of these
  11. A function is maximum at a point if  $\frac{dy}{dx}$  change sign from:  
 [a] +ve to -ve [b] -ve to +ve  
 [c] does not change sign [d] zero
  12. A quantity whose value remains fixed is called;  
 [a] Constant [b] Variable  
 [c] Parameter [d] Function
  13. Mean, Median and Mode are the types of:  
 [a] Average [b] Function  
 [c] Variable [d] Constant
  14. The probability of occurring one when a perfect die is rolled:  
 [a]  $\frac{5}{6}$  [b]  $\frac{2}{3}$  [c]  $\frac{1}{3}$  [d]  $\frac{1}{6}$
  15. From a well shuffled pack of 52 cards, a card is drawn at random, the probability that it is king card:  
 [a]  $\frac{1}{13}$  [b]  $\frac{1}{26}$  [c]  $\frac{1}{14}$  [d]  $\frac{1}{52}$

Answer Key

1	c	2	b	3	b	4	b	5	a
6	a	7	d	8	c	9	c	10	c
11	a	12	a	13	a	14	d	15	a

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DAE / IA - 2019

MATH- 233 APPLIED MATHEMATICS - II

PAPER 'B' PART - B (SUBJECTIVE)

Time : 2 : 30 Hrs

Marks : 60

Section - I

Q.1. Write short answers to any Eighteen (18) questions.

1. Find the value of  $\lim_{x \rightarrow 2} \frac{x^3 - 8}{x^2 - 3x + 2}$

**Sol.**  $\lim_{x \rightarrow 2} \frac{x^3 - 8}{x^2 - 3x + 2} \left( \frac{0}{0} \right)$  form

$$= \lim_{x \rightarrow 2} \frac{(x)^3 - (2)^3}{x^2 - 2x - x + 2}$$

$$= \lim_{x \rightarrow 2} \frac{(x-2)(x^2 + 2x + 4)}{x(x-2) - 1(x-2)}$$

$$= \lim_{x \rightarrow 2} \frac{\cancel{(x-2)}(x^2 + 2x + 4)}{\cancel{(x-2)}(x-1)}$$

$$= \lim_{x \rightarrow 2} \frac{(x^2 + 2x + 4)}{(x-1)}$$

$$= \frac{(2)^2 + 2(2) + 4}{2-1}$$

$$= \frac{4+4+4}{1} = \boxed{12}$$

2. Find the value of  $\lim_{x \rightarrow 1} \frac{x^2 + x - 2}{x^2 - x}$

**Sol.**  $\lim_{x \rightarrow 1} \frac{x^2 + x - 2}{x^2 - x} \left( \frac{0}{0} \right)$  form

$$= \lim_{x \rightarrow 1} \frac{x^2 + 2x - x - 2}{x(x-1)}$$

$$= \lim_{x \rightarrow 1} \frac{x(x+2) - 1(x+2)}{x(x-1)}$$

$$= \lim_{x \rightarrow 1} \frac{(x+2)(x-1)}{x(x-1)}$$

$$= \lim_{x \rightarrow 1} \frac{x+2}{x} = \frac{1+2}{1} = \frac{3}{1} = \boxed{3}$$

3. Find:  $\lim_{x \rightarrow 0} \frac{\sqrt{1+x} - 1}{x}$

**Sol.**  $\lim_{x \rightarrow 0} \frac{\sqrt{1+x} - 1}{x} \left( \frac{0}{0} \right)$  form

$$= \lim_{x \rightarrow 0} \frac{\sqrt{1+x} - 1}{x} \times \frac{\sqrt{1+x} + 1}{\sqrt{1+x} + 1}$$

$$= \lim_{x \rightarrow 0} \frac{(\sqrt{1+x})^2 - (1)^2}{x(\sqrt{1+x} + 1)}$$

$$= \lim_{x \rightarrow 0} \frac{1+x-1}{x(\sqrt{1+x} + 1)}$$

$$= \lim_{x \rightarrow 0} \frac{x}{x(\sqrt{1+x} + 1)}$$

$$= \lim_{x \rightarrow 0} \frac{1}{\sqrt{1+x} + 1} = \frac{1}{\sqrt{1+0} + 1}$$

$$= \frac{1}{\sqrt{1} + 1} = \frac{1}{1+1} = \boxed{\frac{1}{2}}$$

4. Find the value of  $\lim_{x \rightarrow 0} \frac{\sqrt{4+x} - 2}{x}$

**Sol.**  $\lim_{x \rightarrow 0} \frac{\sqrt{4+x} - 2}{x} \left( \frac{0}{0} \right)$  form

$$= \lim_{x \rightarrow 0} \frac{(\sqrt{4+x})^2 - (2)^2}{x(\sqrt{4+x} + 2)}$$

$$= \lim_{x \rightarrow 0} \frac{4+x-4}{x(\sqrt{4+x} + 2)}$$

$$= \lim_{x \rightarrow 0} \frac{x}{x(\sqrt{4+x} + 2)}$$

$$= \lim_{x \rightarrow 0} \frac{1}{\sqrt{4+x} + 2} = \frac{1}{\sqrt{4+0} + 2}$$

$$= \frac{1}{\sqrt{4} + 2} = \frac{1}{2+2} = \boxed{\frac{1}{4}}$$

~~5. Find  $\frac{dy}{dx}$  if  $x = t+2$ ,  $y = 2t^2 + 2$~~

**Sol.** Differentiate both sides w.r.t. 't':

$$\frac{d}{dt}(x) = \frac{d}{dt}(t+2) \quad \left| \quad \frac{d}{dt}(y) = \frac{d}{dt}(2t^2+2) \right.$$

$$\frac{dx}{dt} = 1+0=1 \quad \left| \quad \frac{dy}{dt} = 2(2t)+0=4t \right.$$

using chain rule:  $\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$

$$\frac{dy}{dx} = (4t)(1) \Rightarrow \boxed{\frac{dy}{dx} = 4t}$$

**6. Find  $\frac{dy}{dx}$  if  $x = \theta^2 - \theta - 1, y = 2\theta^2 + \theta + 1$**

**Sol.** Differentiate both sides w.r.t. 'θ':

$$\frac{d}{d\theta}(x) = \frac{d}{d\theta}(\theta^2 - \theta - 1) \quad \left| \quad \frac{d}{d\theta}(y) = \frac{d}{d\theta}(2\theta^2 + \theta + 1) \right.$$

$$\frac{dx}{d\theta} = 2\theta - 1 - 0 \quad \left| \quad \frac{dy}{d\theta} = 2(2\theta) + 1 + 0 \right.$$

$$\frac{dx}{d\theta} = \frac{1}{2\theta - 1} \quad \left| \quad \frac{dy}{d\theta} = 4\theta + 1 \right.$$

using chain rule:  $\frac{dy}{dx} = \frac{dy}{d\theta} \times \frac{d\theta}{dx}$

$$\frac{dy}{dx} = (4\theta + 1) \left( \frac{1}{2\theta - 1} \right) = \boxed{\frac{4\theta + 1}{2\theta - 1}}$$

**7. Find  $\frac{dy}{dx}$  if  $x = u + \frac{1}{u}, y = u - \frac{1}{u}$**

**Sol.** As,  $x = u + \frac{1}{u}$  &  $y = u - \frac{1}{u}$

Differentiate both sides w.r.t. 'u':

$$\frac{d}{du}(x) = \frac{d}{du}\left(u + \frac{1}{u}\right) \quad \left| \quad \frac{d}{du}(y) = \frac{d}{du}\left(u - \frac{1}{u}\right) \right.$$

$$\frac{dx}{du} = \frac{d}{du}(u + u^{-1}) \quad \left| \quad \frac{dy}{du} = \frac{d}{du}(u - u^{-1}) \right.$$

$$\frac{dx}{du} = 1 + (-1)u^{-2} \quad \left| \quad \frac{dy}{du} = 1 - (-1)u^{-2} \right.$$

$$\frac{dx}{du} = 1 - \frac{1}{u^2} = \frac{u^2 - 1}{u^2} \quad \left| \quad \frac{dy}{du} = 1 + \frac{1}{u^2} \right.$$

$$\frac{du}{dx} = \frac{u^2}{u^2 - 1} \quad \left| \quad \frac{dy}{du} = \frac{u^2 + 1}{u^2} \right.$$

using chain rule:

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx} = \frac{u^2 + 1}{u^2} \times \frac{u^2}{u^2 - 1} = \boxed{\frac{u^2 + 1}{u^2 - 1}}$$

**8. Differentiate  $\frac{x^3}{1+x^3}$  w.r.t.  $x^3$**

**Sol.** Let  $y = \frac{x^3}{1+x^3}$  and  $t = x^3$

Differentiate both sides w.r.t. 'x':

$$\frac{d}{dx}(y) = \frac{d}{dx}\left(\frac{x^3}{1+x^3}\right) \quad \left\{ \text{using Quotient Rule} \right\}$$

$$\frac{dy}{dx} = \frac{(1+x^3)\left(\frac{d}{dx}(x^3)\right) - x^3\left(\frac{d}{dx}(1+x^3)\right)}{(1+x^3)^2}$$

$$\frac{dy}{dx} = \frac{(1+x^3)(3x^2) - x^3(0+3x^2)}{(1+x^3)^2}$$

$$\frac{dy}{dx} = \frac{3x^2 + 3x^5 - 3x^5}{(1+x^3)^2} = \frac{3x^2}{(1+x^3)^2}$$

$$\frac{d}{dx}(t) = \frac{d}{dx}(x^3)$$

$$\frac{dt}{dx} = 3x^2 \Rightarrow \frac{dx}{dt} = \frac{1}{3x^2}$$

using chain rule:

$$\frac{dy}{dt} = \frac{dy}{dx} \times \frac{dx}{dt} = \frac{3x^2}{(1+x^3)^2} \times \frac{1}{3x^2} = \boxed{\frac{1}{(1+x^3)^2}}$$

**9. Differentiate  $\frac{x^2}{1+x^2}$  w.r.t.  $x^2$ .**

**Sol.** Let,  $y = \frac{x^2}{1+x^2}$  and  $t = x^2$

Differentiate both sides w.r.t. 'x':

$$\frac{d}{dx}(y) = \frac{d}{dx}\left(\frac{x^2}{1+x^2}\right) \quad \left\{ \text{using Quotient Rule} \right\}$$

$$\frac{dy}{dx} = \frac{(1+x^2)\frac{d}{dx}(x^2) - x^2\frac{d}{dx}(1+x^2)}{(1+x^2)^2}$$

$$\frac{dy}{dx} = \frac{(1+x^2)(2x) - x^2(0+2x)}{(1+x^2)^2}$$

$$\frac{dy}{dx} = \frac{2x + 2x^3 - 2x^3}{(1+x^2)^2} \Rightarrow \frac{dy}{dx} = \frac{2x}{(1+x^2)^2}$$

$$\frac{d}{dx}(t) = \frac{d}{dx}(x^2)$$

$$\frac{dt}{dx} = 2x \Rightarrow \frac{dx}{dt} = \frac{1}{2x}$$

using chain rule:  $\frac{dy}{dt} = \frac{dy}{dx} \times \frac{dx}{dt}$

$$\frac{dy}{dt} = \frac{2x}{(1+x^2)^2} \times \frac{1}{2x} = \frac{1}{(1+x^2)^2}$$

**10.** Show that if  $x = a\theta^2$ ,  $y = 2a\theta$

then  ~~$\frac{dy}{dx} = 2a = 0$~~

**Sol.** As,  $x = a\theta^2$  &  $y = 2a\theta$

Differentiate both sides w.r.t. 'θ':

$$\frac{d}{d\theta}(x) = \frac{d}{d\theta}(a\theta^2) \quad \left| \quad \frac{d}{d\theta}(y) = \frac{d}{d\theta}(2a\theta) \right.$$

$$\frac{dx}{d\theta} = a(2\theta) = 2a\theta \quad \left| \quad \frac{dy}{d\theta} = 2a(1) \right.$$

$$\frac{d\theta}{dx} = \frac{1}{2a\theta} \quad \left| \quad \frac{dy}{d\theta} = 2a \right.$$

using chain rule:

$$\frac{dy}{dx} = \frac{dy}{d\theta} \times \frac{d\theta}{dx} = (2a) \left( \frac{1}{2a\theta} \right) = \frac{1}{\theta}$$

$$\text{L.H.S.} = y \frac{dy}{dx} - 2a$$

$$= 2a\theta \left( \frac{1}{\theta} \right) - 2a$$

$$= 2a - 2a = 0 = \text{R.H.S.} \quad \text{Proved.}$$

**11.** Differentiate  $\cos^2(ax+b)$  w.r.t. 'x'.

**Sol.**  $\frac{d}{dx}(\cos^2(ax+b))$

$$= 2\cos(ax+b) \left( \frac{d}{dx} \cos(ax+b) \right)$$

$$= 2\cos(ax+b) \cdot (-\sin(ax+b)) \left( \frac{d}{dx}(ax+b) \right)$$

$$= -2\sin(ax+b)\cos(ax+b) \cdot (a(1)+0)$$

$$= \boxed{-a \sin 2(ax+b)}$$

**12.** Differentiate  $\operatorname{cosec}^2 3x$  w.r.t. 'x'.

**Sol.**  $\frac{d}{dx}(\operatorname{cosec}^2 3x)$

$$= 2 \operatorname{cosec} 3x \left( \frac{d}{dx}(\operatorname{cosec} 3x) \right)$$

$$= 2 \operatorname{cosec} 3x (-\operatorname{cosec} 3x \cot 3x) \frac{d}{dx}(3x)$$

$$= -2 \operatorname{cosec}^2 3x \cot 3x (3(1))$$

$$= \boxed{-6 \operatorname{cosec}^2 3x \cot 3x}$$

**13.** Differentiate  $\sec\sqrt{a+bx}$  w.r.t. 'x'.

**Sol.**  $\frac{d}{dx}(\sec\sqrt{a+bx})$

$$= \sec\sqrt{a+bx} \tan\sqrt{a+bx} \frac{d}{dx}(\sqrt{a+bx})$$

$$= \sec\sqrt{a+bx} \tan\sqrt{a+bx} \times \frac{1}{2}(a+bx)^{-1/2} \frac{d}{dx}(a+bx)$$

$$= \sec\sqrt{a+bx} \tan\sqrt{a+bx} \times \frac{1}{2\sqrt{a+bx}}(0+b(1))$$

$$= \boxed{\frac{b \sec\sqrt{a+bx} \tan\sqrt{a+bx}}{2\sqrt{a+bx}}}$$

**14.** Differentiate  $\sin(\tan x)$  w.r.t. 'x'.

**Sol.**  $\frac{d}{dx}[\sin(\tan x)]$

$$= \cos(\tan x) \cdot \left( \frac{d}{dx}(\tan x) \right)$$

$$= \boxed{\cos(\tan x) \sec^2 x}$$

**15.** Differentiate  $\cot^3(3x+1)$

**Sol.**  $\frac{d}{dx}(\cot^3(3x+1))$   
 $= 3 \cot^2(3x+1) \left( \frac{d}{dx}(\cot(3x+1)) \right)$   
 $= 3 \cot^2(3x+1) \cdot (-\operatorname{cosec}^2(3x+1)) \left( \frac{d}{dx}(3x+1) \right)$   
 $= -3 \cot^2(3x+1) \operatorname{cosec}^2(3x+1) (3(1)+0)$   
 $= \boxed{-9 \cot^2(3x+1) \operatorname{cosec}^2(3x+1)}$

**16. Differentiate  $\sin[\sin(\cos x)]$  w.r.t. 'x'.**

**Sol.**  $\frac{d}{dx}[\sin[\sin(\cos x)]]$   
 $= \cos[\sin(\cos x)] \frac{d}{dx}[\sin(\cos x)]$   
 $= \cos[\sin(\cos x)] \cdot [\cos(\cos x)] \cdot \frac{d}{dx}(\cos x)$   
 $= \cos[\sin(\cos x)] [\cos(\cos x)] \cdot (-\sin x)$   
 $= \boxed{-\cos[\sin(\cos x)] [\cos(\cos x)] [\sin x]}$

**17. Find the derivative of  $x^2 \tan x$ .**

**Sol.**  $\frac{d}{dx}(x^2 \tan x)$  {using Product Rule}  
 $= \left( \frac{d}{dx}(x^2) \right) \tan x + x^2 \left( \frac{d}{dx}(\tan x) \right)$   
 $= \boxed{2x \tan x + x^2 \sec^2 x}$

**18. Find the turning (or critical point of the curve  $y = \sin 2x$  between 0 and  $\frac{\pi}{2}$ .**

**Sol.**  $y = \sin 2x$   
 Differentiate both sides w.r.t. 'x':  
 $\frac{d}{dx}(y) = \frac{d}{dx}(\sin 2x)$   
 $\frac{dy}{dx} = \cos 2x (2)$   
 $\frac{dy}{dx} = 2 \cos 2x$

For turning points, put  $\frac{dy}{dx} = 0$

$$2 \cos 2x = 0$$

$$\cos 2x = 0$$

$$2x = \cos^{-1}(0)$$

$$2x = \frac{\pi}{2} \Rightarrow \boxed{x = \frac{\pi}{4}}$$

**19. Find the turning points of the curve**

$$y = x^2 - 3x + 3.$$

**Sol.**  $y = x^2 - 3x + 3$

Differentiate both sides w.r.t. 'x':

$$\frac{d}{dx}(y) = \frac{d}{dx}(x^2 - 3x + 3)$$

$$\frac{dy}{dx} = 2x - 3$$

For turning points, put  $\frac{dy}{dx} = 0$

$$2x - 3 = 0$$

$$2x = 3 \Rightarrow \boxed{x = \frac{3}{2}}$$

**20. Find the turning points of the curve**

$$y = 2x^3 - 15x^2 + 36x + 10$$

**Sol.**  $y = 2x^3 - 15x^2 + 36x + 10$

Differentiate both sides w.r.t. 'x':

$$\frac{d}{dx}(y) = \frac{d}{dx}(2x^3 - 15x^2 + 36x + 10)$$

$$\frac{dy}{dx} = 2(3x^2) - 15(2x) + 36(1) + 0$$

$$\frac{dy}{dx} = 6x^2 - 30x + 36$$

For turning point, put  $\frac{dy}{dx} = 0$

$$6x^2 - 30x + 36 = 0$$

Dividing each term on '6'

$$x^2 - 5x + 6 = 0$$

$$x^2 - 3x - 2x + 6 = 0$$

$$x(x - 3) - 2(x - 3) = 0$$

$$(x - 3)(x - 2) = 0$$

Either OR

$$x - 3 = 0 \quad | \quad x - 2 = 0$$

$$\boxed{x = 3} \quad | \quad \boxed{x = 2}$$

**21. Find the extreme values of the function  $x^2 - 4x - 6$**

**Sol.** Let,  $y = x^2 - 4x - 6 \rightarrow (i)$

Differentiate both sides w.r.t. 'x':

$$\frac{d}{dx}(y) = \frac{d}{dx}(x^2 - 4x - 6)$$

$$\frac{dy}{dx} = 2x - 4(1) - 0$$

$$\frac{dy}{dx} = 2x - 4 \rightarrow (ii)$$

For critical values, put  $\frac{dy}{dx} = 0$

$$2x - 4 = 0$$

$$2x = 4$$

$$x = \frac{4}{2} \Rightarrow \boxed{x = 2}$$

Differentiate eq. (ii) both sides w.r.t. 'x':

$$\frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dx}(2x - 4)$$

$$\frac{d^2y}{dx^2} = 2(1) - 0$$

$$\frac{d^2y}{dx^2} = 2 \rightarrow (iii)$$

Put  $x = 2$  in eq. (iii) & eq. (i)

$$\frac{d^2y}{dx^2} = 2 > 0$$

$$y_{\min} = (2)^2 - 4(2) - 6$$

$$y_{\min} = 4 - 8 - 6 \Rightarrow \boxed{y_{\min} = -10}$$

**22. Calculate the median for 88.03, 94.50, 94.90, 95.05, 84.50**

**Sol.** Rearrange the data in Ascending order:  
84.50, 88.03, 94.05, 94.50, 94.90

$$\text{So, Median} = \left(\frac{n+1}{2}\right)^{\text{th}} \text{ value}$$

$$\text{Median} = \left(\frac{5+1}{2}\right)^{\text{th}} \text{ value}$$

$$\text{Median} = 3^{\text{th}} \text{ value}$$

$$\text{Hence, Median} = \boxed{94.05}$$

**23. Write the formula to find median for grouped frequency distribution.**

**Sol.** 
$$\text{Median} = \ell + \frac{h}{f} \left( \frac{n}{2} - c \right)$$

**24. Define standard deviation.**

**Sol.** Let  $X_1, X_2, X_3, \dots, X_n$  be  $n$  values of a variable  $x$ , then their standard deviation is defined as:

$$\text{S.D} = \sqrt{\frac{\sum (x - \bar{x})^2}{n}}, \quad i = 1, 2, 3, \dots, n$$

$$\text{OR S.D.} = \sqrt{\frac{\sum x^2}{n} - \left(\frac{\sum x}{n}\right)^2}$$

**25. If a dice is rolled. What is the probability that an even no divisible by 3 appears?**

**Sol.**  $S = \{1, 2, 3, 4, 5, 6\}, \quad n(S) = 6$

Let  $B$  be event that an even no. is divisible by 3 is appears.

$$B = \{6\}, \quad n(B) = 1$$

$$P(A) = \frac{n(B)}{n(S)} = \boxed{\frac{1}{6}}$$

**26. If two dice are rolled, find the probability that the sum is 7.**

**Sol.**  $S = \left\{ \begin{array}{l} (1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), \\ (2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6), \\ (3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6), \\ (4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6), \\ (5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6), \\ (6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6) \end{array} \right\}$   
 $n(S) = 36$

Let E be event that sum of 7 appears.

$$E = \left\{ \begin{array}{l} (1, 6), (2, 5), (3, 4), \\ (4, 3), (5, 2), (6, 1) \end{array} \right\}$$

$$n(E) = 6$$

$$\therefore P(E) = \frac{n(E)}{n(S)} = \frac{6}{36} = \frac{1}{6}$$

**27.** Write down the formula to find the probability of two not mutually exclusive events.

**Sol.**  $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

**Section - II**

**Note :** Attempt any three (3) questions  $3 \times 8 = 24$

**Q.2.(a)** If  $f(x) = \log\left(\frac{1-x}{1+x}\right)$ , Prove

that :  $f(x) + f(y) = f\left(\frac{x+y}{1+xy}\right)$

**Sol.** See Q.13 of Ex # 1.1 (Page # 11)

**(b)** Evaluate:  $\lim_{\theta \rightarrow 0} \frac{\tan \theta - \sin \theta}{\sin^3 \theta}$

**Sol.** See Q.1(x) of Ex # 1.3 (Page # 27)

**Q.3.(a)** If  $\frac{1-t^2}{1+t^2}, y = \frac{2t}{1+t^2}$  prove that

$$y \frac{dy}{dx} + x = 0$$

**Sol.** See Q.4(iii) of Ex # 2.3 (Page # 80)

**(b)** Find  $\frac{dy}{dx}$  if

$$x = a \left( \frac{t^2}{2} - t \right), y = b \left( \frac{t^3}{3} - \frac{t^2}{2} \right)$$

**Sol.** See Q.3(vi) of Ex # 2.3 (Page # 77)

**Q.4.(a)** Find  $\frac{dy}{dx}$  when

$$\begin{array}{l} x = a(\cos t + \sin t), \\ y = a(\sin t - t \cos t) \end{array}$$

**Sol.** See Q.8(ii) of Ex # 3.1 (Page # 124)

**(b)** Find the derivative of  $x^x + x^{\sin x}$

**Sol.** See Q.3(vii) of Ex # 3.3 (Page # 153)

**Q.5.(a)** Find the maximum and minimum (extreme) values of the function  $(x-2)^3(x-3)^2$

**Sol.** See Q.2(viii) of Ex # 4.2 (Page # 195)

**Q.6.** Calculate A.M and median from the following data.

Marks:

15,17,18,19,20,25,26,27,28,29, 30,31

Boys:

30,34,9,38,15,50,52,81,56,58,15, 62

**Sol.** See Q.3 of Ex # 5.1 (Page # 231)

**(b)** A die is thrown, find the probability that the dots on the top are prime numbers or odd numbers.

**Sol.** See Q.5 of Ex # 6.2 (Page # 268)

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