DAE / IA - 2018

MATH - 212 APPLIED MATHEMATICS - II

PART - A (OBJECTIVE)

Time : 30 Minutes Marks : 20

Q.1: *Encircle the correct answer.*

1. If $f(x) = 3^x - 1$, then $f(3) = ?$

[a] 27 [b] 8
[c] 26 [d] 16

2. $\lim\limits_{x \to \frac{\pi}{2}} \frac{1}{\cos\theta} = ?$

[a] 0 [b] ∞
[c] 1 [d] $\frac{2}{\pi}$

3. $\frac{d}{dx}\left(\frac{1}{x}\right) = ?$

[a] $-\frac{1}{x^2}$ [b] $\frac{1}{x^2}$
[c] $-\frac{1}{x^3}$ [d] $\frac{2}{x}$

4. If $y = u^2$ and $u = x$ then $\frac{dy}{dx} = ?$

[a] $2x$ [b] u^2
[c] x [d] $2x^2$

5. $\frac{d}{dx}\left(\tan x^2\right) = ?$

[a] $2x\sec^2 x^2$ [b] $\sec^2 x^2$
[c] $\sec x^2$ [d] $\sec^2 x$

6. $\frac{d}{dx}\left(\ln \sin x\right) = ?$

[a] $\cot x$ [b] $\frac{1}{\sin x}\ln \sin x$
[c] $\ln \cos x$ [d] $\tan x$

7. $\xcancel{\frac{d}{dx}\left(e^{\sin x}\right) = ?}$

[a] $e^{\cos x}$ [b] $\cos x e^{\sin x}$
[c] $\sin x e^{\sin x - 1}$ [d] $\sin x e^{\sin x}$

8. If 2^{nd} derivative is $-ve$ at a point, then function is:

[a] Maximum [b] Minimum
[c] Point of inflection
[d] None of these

9. $\int\left(\sqrt{x}\right)dx =$

[a] $\frac{1}{2}x^{1/2}$ [b] $\frac{2x^{1/2}}{3}$
[c] $\frac{2}{3}x^{3/2}$ [d] $\frac{1}{2x^{1/2}}$

10. $\int\left(\sec x\right) dx = ?$

[a] $\tan x$ [b] $\frac{\sec^2 x}{2}$
[c] $\ln(\sec x + \tan x)$
[d] $\sec x \tan x$

11. $\int\left(\sin^4 x \cos x\right)dx = ?$

[a] $\frac{\sin^5 x}{5}$ [b] $\frac{\sin^5 x \cos x}{5}$
[c] $\frac{\cos^2 x}{2}$ [d] $-\sin x \cos x$

12. $\xcancel{\int\left(\frac{e^x}{1+e^x}\right)dx = ?}$

[a] $1 + e^x$ [b] $\ln\left(1+e^x\right)$
[c] e^x [d] $\frac{\left(1+e^x\right)^2}{2}$

13. $\xcancel{\int_1^3\left(e^{2x}\right)dx = ?}$

[a] $e^6 - e^2$ [b] $\frac{e^{2x}}{2}$
[c] $\frac{1}{2}\left(e^6 + e^2\right)$ [d] $\frac{1}{2}\left(e^6 - e^2\right)$

14. $\cancel{\int_0^1\left(\frac{1}{x^2+1}\right)dx = ?}$

[a] 0 [b] 1

[c] $\frac{\pi}{4}$ [d] $-\frac{\pi}{4}$

15. Slope of the line $\frac{x}{a}+\frac{y}{b}=1$ is:

[a] $\frac{a}{b}$ [b] $\frac{b}{a}$

[c] $-\frac{b}{a}$ [d] $-\frac{a}{b}$

16. Point $(-4,-5)$ lies in the quadrant:

[a] 1^{st} [b] 2^{nd}

[c] 3^{rd} [d] 4^{th}

17. $y=2$ is a line parallel to:

[a] $x-\text{axis}$ [b] $y-\text{axis}$

[c] $y=x$ [d] $x=3$

18. Midpoint of $A(2,5)$ & $B(7,-3)$:

[a] $\left(\frac{9}{2},1\right)$ [b] $\left(1,\frac{9}{2}\right)$

[c] $\left(1,\frac{2}{9}\right)$ [d] $\left(\frac{2}{9},1\right)$

19. Radius of the circle $(x-1)^2+(y-2)^2=16$ is:

[a] 2 [b] 1

[c] 4 [d] 16

20. For a point circle, the radius will be:

[a] 1 [b] -1

[c] 0 [d] Infinity

Answer Key

1	b	2	b	3	a	4	a	5	c
6	c	7	b	8	a	9	c	10	c
11	a	12	b	13	d	14	c	15	c
16	c	17	a	18	a	19	c	20	d

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DAE / IA - 2018

MATH - 212 APPLIED MATHEMATICS - II

PART - B (SUBJECTIVE)

Time : 2:30 Hrs Marks : 60

Section - I

Q.1 : Write short answers to any Twenty Five (25) of the follwing questions. $\boxed{25\times 2=50}$

1. **If $f(x)=3x^2-7x+4$, then find $f\left(\frac{1}{x}\right)$.**

Sol. As, $f(x)=3x^2-7x+4$

Replace 'x' by '$1/x$', we have :

$$f\left(\frac{1}{x}\right)=3\left(\frac{1}{x}\right)^2-7\left(\frac{1}{x}\right)+4$$

$$=\frac{3}{x^2}-\frac{7}{x}+4=\boxed{\frac{3-7x+4x^2}{x^2}}$$

2. **Find $\lim\limits_{x\to 3}\frac{x-3}{x^2-9}$**

Sol. $\lim\limits_{x\to 3}\frac{x-3}{x^2-9}$ $\left(\frac{0}{0}\right)$ Form

$$=\lim_{x\to 3}\frac{x-3}{x^2-(3)^2}=\lim_{x\to 3}\frac{\cancel{(x-3)}}{\cancel{(x-3)}(x+3)}$$

$$=\lim_{x\to 3}\frac{1}{x+3}=\frac{1}{3+3}=\boxed{\frac{1}{6}}$$

3. **Show that the function $f(x)=x^4-7x^2+7$ is an even function of 'x'.**

Sol. $f(x)=x^4-7x^2+7$

Replace 'x' by '−x', we have :

$$f(-x)=(-x)^4-7(-x)^2+7$$

$$f(-x)=x^4-7x^2+7$$

$$f(-x)=f(x)$$

Hence $f(x)$ is an $\boxed{\text{even}}$ function.

4. ~~Find~~ $\xcancel{\lim_{x\to 0}\left(1+\frac{x}{3}\right)^{1/x}}$.

Sol. $\lim_{x\to 0}\left(1+\frac{x}{3}\right)^{1/x}=\lim_{x\to 0}\left(1+\frac{x}{3}\right)^{\frac{3}{x}\times\frac{1}{3}}$

$$=\left[\lim_{x\to 0}\left(1+\frac{x}{3}\right)^{3/x}\right]^{1/3}=\boxed{e^{1/3}}$$

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5. **If $y = u^n$ & $u = \left(3x^3 - 7x^2 + x + 1\right)$**

find $\frac{dy}{dx}$.

Sol. As, $y=u^n$

$\Rightarrow$ $y=\left(3x^3-7x^2+x+1\right)^n$

Differentiate both sides w.r.t. 'x':

$$\frac{d}{dx}(y)=\frac{d}{dx}\left(3x^3-7x^2+x+1\right)^n$$

$$\frac{dy}{dx}=n\left(3x^3-7x^2+x+1\right)^{n-1}\frac{d}{dx}\left(3x^3-7x^2+x+1\right)$$

$$\frac{dy}{dx}=n\left(3x^3-7x^2+x+1\right)^{n-1}\left(3\left(3x^2\right)-7\left(2x\right)+1+0\right)$$

$$\boxed{\frac{dy}{dx}=n\left(3x^3-7x^2+x+1\right)^{n-1}\left(9x^2-14x+1\right)}$$

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6. **Find $\frac{dy}{dx}$ if $x^{2/3}+y^{2/3}=a^{2/3}$**

Sol. Differentiate both sides w.r.t. 'x':

$$\frac{d}{dx}\left(x^{2/3}+y^{2/3}\right)=\frac{d}{dx}\left(a^{2/3}\right)$$

$$\frac{2}{3}x^{-1/3}+\frac{2}{3}y^{-1/3}\frac{dy}{dx}=0$$

$$\frac{2}{3}y^{-1/3}\frac{dy}{dx}=-\frac{2}{3}x^{-1/3}$$

$$\frac{dy}{dx}=\left(-\frac{2}{3}x^{-1/3}\right)\cdot\left(\frac{3}{2y^{-1/3}}\right)$$

$$\frac{dy}{dx}=-\frac{x^{-1/3}}{y^{-1/3}}\quad\Rightarrow\quad\boxed{\frac{dy}{dx}=-\frac{y^{1/3}}{x^{1/3}}}$$

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7. **Find: $\frac{d}{dx}\left(\frac{1}{(ax+b)^m}\right)$**

Sol. $\frac{d}{dx}\left(\frac{1}{(ax+b)^m}\right)$

$$=\frac{d}{dx}(ax+b)^{-m}$$

$$=-m\,(ax+b)^{-m-1}\ \frac{d}{dx}(ax+b)$$

$$=-m\frac{1}{(ax+b)^{m+1}}\cdot\left(a(1)+0\right)$$

$$=\boxed{-\frac{am}{(ax+b)^{m+1}}}$$

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8. **If $y=1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+\frac{x^4}{4!}+\ldots,$**

then show that $\frac{dy}{dx}=y$

Sol. $y=1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+\frac{x^4}{4!}+\ldots$

Differentiate both sides w.r.t. 'x':

$$\frac{d}{dx}(y)=\frac{d}{dx}\left(1+x+\frac{x^2}{2}+\frac{x^3}{6}+\frac{x^4}{24}+\ldots\right)$$

$$\frac{dy}{dx}=0+1+\frac{2x}{2}+\frac{3x^2}{6}+\frac{4x^3}{24}+\ldots$$

$$\frac{dy}{dx}=1+x+\frac{x^2}{2}+\frac{x^3}{6}+\ldots$$

$$\frac{dy}{dx}=1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+\ldots$$

$\boxed{\frac{dy}{dx}=y}$ **Proved.**

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9. **Find the derivative of $x\cot x$ w.r.t. 'x'.**

Sol. $\frac{d}{dx}(x\cot x)\ \left\{\begin{smallmatrix}\text{using}\\ \text{Product Rule}\end{smallmatrix}\right\}$

$$=\left(\frac{d}{dx}(x)\right)\cot x+x\left(\frac{d}{dx}(\cot x)\right)$$

$$= 1.\cot x + x\left(-\text{cosec}^2 x\right)$$

$$= \boxed{\cot x - x\cos ec^2 x}$$

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10. ~~Find the derivative of~~ $\xcancel{\sin^{-1}\left(\frac{x}{a}\right)}$

Sol. $\frac{d}{dx}\left(\sin^{-1}\left(\frac{x}{a}\right)\right)$

$$= \frac{1}{\sqrt{1-\left(\frac{x}{a}\right)^2}} \cdot \frac{d}{dx}\left(\frac{x}{a}\right)$$

$$= \frac{1}{\sqrt{1-\frac{x^2}{a^2}}} \times \left(\frac{1}{a}\right) = \frac{1}{\sqrt{\frac{a^2-x^2}{a^2}}} \cdot \frac{1}{a}$$

$$= \frac{1}{\frac{\sqrt{a^2-x^2}}{a}} \cdot \frac{1}{a} = \boxed{\frac{1}{\sqrt{a^2-x^2}}}$$

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11. $\xcancel{\text{Find } \frac{dy}{dx} \text{ if } x = a\sec\theta,\ y = b\tan\theta}$

Sol. $x = a\sec\theta,\ y = b\tan\theta$

Differentiate both equations both sides w.r.t. 'θ':

$$\frac{d}{d\theta}(x) = \frac{d}{d\theta}(a\sec\theta)$$

$$\frac{dx}{d\theta} = a\sec\theta\tan\theta$$

$$\frac{d\theta}{dx} = \frac{1}{a\sec\theta\tan\theta}$$

$$\frac{d}{d\theta}(y) = \frac{d}{d\theta}(b\tan\theta)$$

$$\frac{dy}{d\theta} = b\sec^2\theta$$

By using Chain's Rule : $\frac{dy}{dx} = \frac{dy}{d\theta} \times \frac{d\theta}{dx}$

$$\frac{dy}{dx} = b\sec^2\theta\left(\frac{1}{a\sec\theta\tan\theta}\right) = \frac{b}{a}\,\frac{\sec\theta}{\tan\theta}$$

$$\frac{dy}{dx} = \frac{b}{a}\cot\theta.\sec\theta$$

$$\frac{dy}{dx} = \frac{b}{a}\,\frac{\cos\theta}{\sin\theta} \times \frac{1}{\cos\theta} = \boxed{\frac{b}{a}\cos ec\theta}$$

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12. ~~Find the value of~~ $\xcancel{\frac{d}{dx}\left(x^x\right)}$

Sol. Let $y = x^x$

Taking 'ℓn' on both sides:

$$\ell n(y) = \ell n\left(x^x\right)$$

$$\ell n(y) = x(\ell n x)\left\{\begin{matrix}\text{using logrithm law} \\ \ell n\left(m^n\right) = n\ \ell n(m)\end{matrix}\right\}$$

$$\frac{d}{dx}(\ell n y) = \frac{d}{dx}\left(x(\ell n x)\right)\left\{\begin{matrix}\text{using by} \\ \text{Product Rule}\end{matrix}\right\}$$

$$\frac{1}{y}\,\frac{dy}{dx} = \left(\frac{d}{dx}(x)\right)\ell n x + x\left(\frac{d}{dx}(\ell n x)\right)$$

$$\frac{dy}{dx} = y\left[(1)\ell n x + x\left(\frac{1}{x}\right)\right]$$

$$\frac{dy}{dx} = x^x\left[\ell n x + 1\right] \Rightarrow \frac{dy}{dx} = \boxed{x^x\left[1 + \ell n x\right]}$$

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13. **If $y = \ell n x$, find y_2**

Sol. $y = \ell n x$

Differentiate both sides w.r.t. 'x':

$$\frac{d}{dx}(y) = \frac{d}{dx}(\ell n x)$$

$$y_1 = \frac{1}{x}$$

Differentiate both sides w.r.t. 'x':

$$\frac{d}{dx}(y_1) = \frac{d}{dx}\left(\frac{1}{x}\right)$$

$$y_2 = \frac{d}{dx}\left(x^{-1}\right) = -1(x)^{-2} = \boxed{\frac{-1}{x^2}}$$

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14. **Find the derivative of $x^2\sec 4x$**

Sol. $\frac{d}{dx}\left(x^2\sec 4x\right)\left\{\begin{matrix}\text{By using} \\ \text{Product Rule}\end{matrix}\right\}$

$$= \left(\frac{d}{dx}\left(x^2\right)\right)\sec 4x + x^2\left(\frac{d}{dx}(\sec 4x)\right)$$

$$= 2x\sec 4x + x^2\sec 4x\tan 4x\frac{d}{dx}(4x)$$

$$= 2x\sec 4x + x^2\sec 4x\tan 4x(4)$$

$$= 2x\sec 4x + 4x^2\sec 4x\tan 4x$$

$$= \boxed{2x\sec 4x\left(1 + 2x\tan 4x\right)}$$

15. **If displacement is $s = \sin 2t$, find, its acceleration.**

Sol. $s = \sin 2t$

Differentiate both sides w.r.t. 't':

$$v = \frac{ds}{dt} = \frac{d}{dt}(\sin 2t)$$

$$v = \cos 2t\left(\frac{d}{dt}(2t)\right)$$

$$v = \cos 2t(2(1))$$

$$v = 2\cos 2t$$

Diff. again both sides w.r.t. 't':

$$a = \frac{dv}{dt} = \frac{d}{dt}(2\cos 2t)$$

$$a = 2(-\sin 2t)\left(\frac{d}{dt}(2t)\right)$$

$$a = 2(-\sin 2t)(2(1))$$

$$a = \boxed{-4\sin 2t}$$

16. **Find the turning point of the curve $y = x^2 - 3x + 3$.**

Sol. $y = x^2 - 3x + 3.$

Differentiate both sides w.r.t. 'x':

$$\frac{d}{dx}(y) = \frac{d}{dx}\left(x^2 - 3x + 3\right)$$

$$\frac{dy}{dx} = 2x - 3$$

For turning point Put $\frac{dy}{dx} = 0$

$$2x - 3 = 0$$

$$2x = 3$$

$$\boxed{x = 3/2}$$

17. **Find $\int (2x+9)^{-5/2}\,dx$**

Sol. $\int (2x+9)^{-5/2}\,dx$

$$= \frac{1}{2}\int (2x+9)^{-5/2}(2)\,dx$$

$$= \frac{1}{2}\left[\frac{(2x+9)^{-3/2}}{-3/2}\right] + c \quad \begin{Bmatrix}\text{using}\\ \text{Rule-I}\end{Bmatrix}$$

$$= \boxed{-\frac{1}{3}(2x+9)^{-3/2} + c}$$

18. **Evaluate $\int \left(\frac{x}{a+x}\right)dx$**

Sol. $\int \left(\frac{x}{a+x}\right)dx \quad \begin{Bmatrix}\text{Improper}\\ \text{Fraction}\end{Bmatrix}$

$$= \int \left(1 - \frac{a}{a+x}\right)dx \qquad \boxed{\begin{array}{r|l} & \;\;1 \\ \hline x+a & \;x \\ & \underline{\pm x \pm a} \\ & \;-a \end{array}}$$

$$= \int \left(1 - a(a+x)^{-1}\right)dx$$

$$= \boxed{x - a\,\ell n(a+x) + c} \quad \begin{Bmatrix}\text{using}\\ \text{Rule-II}\end{Bmatrix}$$

19. **Evaluate $\int \cos^2 x\,dx$**

Sol. $\int \cos^2 x\,dx$

$$= \int \frac{1+\cos 2x}{2}\,dx$$

$$= \frac{1}{2}\int (1+\cos 2x)\,dx$$

$$= \boxed{\frac{1}{2}\left[x + \frac{\sin 2x}{2}\right] + c}$$

20. **Evaluate $\int \frac{dx}{(1+x^2)\tan^{-1}x}$**

Sol. $\int \frac{dx}{(1+x^2)\tan^{-1}x}$

$$= \int \frac{1}{\tan^{-1}x}\cdot\frac{1}{(1+x^2)}\,dx$$

$$= \boxed{\ell n(\tan^{-1}x) + c}$$

21. **Find** $6\int x^2 e^{x^3} dx$

Sol. $6\int x^2 e^{x^3} dx$

$$= 6\int e^{x^3}\left(x^2\right)dx$$

$$= 6\int e^t \frac{dt}{3}$$

$$= \frac{6}{3}\int e^t dt$$

$$= 2e^t + c$$

$$= \boxed{2e^{x^3} + c}$$

$$\text{Put } x^3 = t$$

$$\frac{d}{dx}\left(x^3\right) = \frac{d}{dx}(t)$$

$$3x^2 = \frac{dt}{dx}$$

22. **Find the value of** $\int\left(\tan^{-1} x\right) dx$

Sol. $\int\left(\tan^{-1} x\right) dx = \int \tan^{-1} x.(1)dx$

Integrating by parts:

taking $u = \tan^{-1}x$ & $v = 1$

$$= \tan^{-1} x\int(1)\,dx - \int\left\{\frac{d}{dx}\left(\tan^{-1} x\right)\int(1)\,dx\right\}dx$$

$$= \tan^{-1} x.(x) - \int \frac{1}{1+x^2}(x)\,dx$$

$$= x\tan^{-1} x - \frac{1}{2}\int \frac{2x}{1+x^2}dx$$

$$= \boxed{x\tan^{-1} x - \frac{1}{2}\,\ell n\left(1+x^2\right) + c}$$

23. **Evaluate** $\int(\ell n\, x)dx$

Sol. $\int(\ell n\, x)dx$

$$= \int(\ell n\, x \cdot 1)dx$$

Integrating by parts:

taking $u = \ell n\, x$ & $v = 1$

$$= \ell n\, x\int(1)\,dx - \int\left[\frac{d}{dx}(\ell n\, x)\int(1)dx\right]dx$$

$$= \ell n\, x(x) - \int \frac{1}{x}.(x)\,dx$$

$$= x\ell n\, x - \int(1)\,dx$$

$$= x\ell n\, x - (x) + c = \boxed{x(\ell n\, x - 1) + c}$$

24. **Evaluate** $\int \frac{\cot x}{\ell n \sin x}dx$

Sol. $\int \frac{\cot x}{\ell n \sin x}dx$

$$= \int \frac{1}{\ell n x \sin x}.\cot x\ dx$$

$$\text{Put} \quad \ell n \sin x = t$$

$$\frac{d}{dx}(\ell n \sin x) = \frac{d}{dx}(t)$$

$$\frac{1}{\sin x}\ \frac{d}{dx}(\sin x) = \frac{dt}{dx}$$

$$\frac{1}{\sin x}\ \cos x = \frac{dt}{dx}$$

$$\cot x\ dx = dt$$

$$= \int\left(\frac{1}{t}\right)dt$$

$$= \ell n(t) + c$$

$$= \boxed{\ell n(\ell n \sin x) + c}$$

25. **Evaluate** $\int_1^8 \frac{dx}{\sqrt[3]{x}}$

Sol. $\int_1^8 \frac{dx}{\sqrt[3]{x}} = \int_1^8 x^{-1/3}dx$

$$= \left[\frac{x^{2/3}}{2/3}\right]_1^8 = \frac{3}{2}\left[x^{2/3}\right]_1^8$$

$$= \frac{3}{2}\left[(8)^{2/3} - (1)^{2/3}\right]$$

$$= \frac{3}{2}\left[\left(2^3\right)^{2/3} - (1)^{2/3}\right]$$

$$= \frac{3}{2}(4-1) = \frac{3}{2}(3) = \boxed{9/2}$$

26. **Evaluate** $\int_0^{\pi/4} \frac{dx}{\cos^2 x}$

Sol. $\int_0^{\pi/4} \frac{dx}{\cos^2 x} = \int_0^{\pi/4}\left(\sec^2 x\right)dx$

$$=\left[\tan x\right]_0^{\pi/4} \quad \begin{Bmatrix}\text{Using formula \# 13}\\ \text{from page \# 282}\end{Bmatrix}$$

$$=\tan\left(\frac{\pi}{4}\right)-\tan(0)$$

$$=\tan(45^\circ)-\tan(0^\circ)\begin{Bmatrix}\frac{\pi}{4}\times\frac{180}{\pi}=45^\circ\\ 0\times\frac{180}{\pi}=0^\circ\end{Bmatrix}$$

$$=1-0=\boxed{1}\begin{Bmatrix}\text{using calculator}\\ \tan(45^\circ)=1\ \&\ \tan(0^\circ)=0\end{Bmatrix}$$

27. **Evaluate** $\int_0^3 \sqrt[3]{(3x-1)^2}\,dx$

Sol. $\int_0^3 \sqrt[3]{(3x-1)^2}\,dx$

$$=\int_0^3 (3x-1)^{2/3}\ dx$$

$$=\frac{1}{3}\int_0^3 (3x-1)^{2/3}(3)\,dx$$

$$=\frac{1}{3}\left[\frac{(3x-1)^{5/3}}{5/3}\right]_0^3 \begin{Bmatrix}\text{using}\\ \text{Rule-1}\end{Bmatrix}$$

$$=\frac{1}{3}\times\frac{3}{5}\left[(3x-1)^{5/3}\right]_0^3$$

$$=\frac{1}{5}\left[(3(3)-1)^{5/3}-(3(0)-1)^{5/3}\right]$$

$$=\frac{1}{5}\left[(8)^{5/3}-(-1)^{5/3}\right]$$

$$=\frac{1}{5}\left[32-(-1)\right]=\boxed{\frac{33}{5}}$$

28. **Evaluate** $\int\left(\frac{1}{\sqrt{x}}\sin\sqrt{x}\right)dx$

Sol. $\int\left(\frac{1}{\sqrt{x}}\sin\sqrt{x}\right)dx$

$$\boxed{\begin{array}{l}\text{Put } \sqrt{x}=t \ \Rightarrow\ \frac{d}{dx}\left(\sqrt{x}\right)=\frac{d}{dx}(t)\\ \frac{1}{2}x^{-1/2}=\frac{dt}{dx}\\ \frac{1}{2\sqrt{x}}=\frac{dt}{dx}\ \Rightarrow\ \frac{1}{\sqrt{x}}dx=2dt\end{array}}$$

$$=\int\sin\sqrt{x}.\left(\frac{1}{\sqrt{x}}\right)dx$$

$$=\int(\sin t)(2dt)$$

$$=2\int(\sin t)\,dt$$

$$=2(-\cos t)+c$$

$$=\boxed{-2\cos\sqrt{x}+c}$$

29. **Find the equation of a line through the point $(3,-2)$ with slope $m=\frac{3}{4}$.**

Sol. Equation of line in point - slope form :

$$y-y_1=m(x-x_1)$$

$$y-(-2)=\frac{3}{4}(x-3)$$

$$4(y+2)=3(x-3)$$

$$4y+8=3x-9$$

$$4y+8-3x+9=0$$

$$-3x+4y+17=0$$

$$\boxed{3x-4y-17=0}$$

30. **Find the point of intersection of the lines:** $\begin{cases}x+2y-3=0\\ 2x-3y+8=0\end{cases}$

Sol. Let $\begin{cases}x+2y-3=0 \quad \rightarrow(i)\\ 2x-3y+8=0\rightarrow(ii)\end{cases}$

Multipling eq.(i) by 2 and subtraction it from eq.(ii), we have :

$$2x+4y-6=0$$

$$\underline{\pm 2x\mp 3y\pm 8=0}$$

$$7y-14=0$$

$$7y=14$$

$$y=\frac{14}{7}\ \Rightarrow\ \boxed{y=2}$$

Put $y = 2$ in eq.(i), we have:

$$x + 2(2) - 3 = 0$$

$$x + 4 - 3 = 0$$

$$x + 1 = 0 \Rightarrow \boxed{x = -1}$$

Hence require point of intersection: $\boxed{(-1, 2)}$

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31. ~~**Find the distance from the point $(-2, 1)$ to the line $3x + 4y - 12 = 0$**~~

Sol. Distance between point & line

$$D = \frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$$

$$D = \frac{|3(-2) + 4(1) - 12|}{\sqrt{(3)^2 + (4)^2}}$$

$$D = \frac{|-6 + 4 - 12|}{\sqrt{9 + 16}}$$

$$D = \frac{|-14|}{\sqrt{25}} = \boxed{\frac{14}{5}}$$

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32. ~~**Find the angle between the lines having slopes -3 and 2.**~~

Sol. Let, $m_1 = -3$ and $m_2 = 2$

$$\theta = \tan^{-1}\left(\frac{m_2 - m_1}{1 + m_2 m_1}\right)$$

$$\theta = \tan^{-1}\left(\frac{2 - (-3)}{1 + (2)(-3)}\right)$$

$$\theta = \tan^{-1}\left(\frac{2 + 3}{1 - 6}\right)$$

$$\theta = \tan^{-1}\left(\frac{5}{-5}\right)$$

$$\theta = \tan^{-1}(-1) = \boxed{135^\circ}$$

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33. **Find the coordinates of the mid-point of the segment $P_1(3, 7)$ and $P_2(-2, 3)$.**

Sol. $\text{Mid-point} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

$$= \left(\frac{3 + (-2)}{2}, \frac{7 + 3}{2}\right)$$

$$= \left(\frac{3 - 2}{2}, \frac{10}{2}\right)$$

$$= \boxed{\left(\frac{1}{2}, 5\right)}$$

---

34. **Define imaginary circle.**

Sol. A circle is called imaginary circle if $r < 0$.

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35. **Find the equation of circle with center on origin and radius is $\frac{1}{2}$.**

Sol. Standard form of equation of circle:

$$(x - h)^2 + (y - k)^2 = r^2$$

Put $h = 0$, $k = 0$ & $r = \frac{1}{2}$

$$(x - 0)^2 + (y - 0)^2 = \left(\frac{1}{2}\right)^2$$

$$\boxed{x^2 + y^2 - \frac{1}{4} = 0}$$

---

36. **Find the center and radius of the circle $6x^2 + 6y^2 - 18y = 0$**

Sol. $6x^2 + 6y^2 - 18y = 0$

Dividing each term by 6, we get:

$$x^2 + y^2 - 3y = 0$$

Comparing with general form:

$$x^2 + y^2 + 2gx + 2fy + c = 0$$

$2g = 0$ $g = 0$	$2f = -3$ $f = -\frac{3}{2}$	$c = 0$

$$\text{Center}=(-g,-f)$$

$$\text{Center}=\left(0,-\left(-\frac{3}{2}\right)\right)$$

$$\text{Center}=\boxed{\left(0\,,{}^{3}\!/_{2}\right)}$$

$$\text{Radius}=r=\sqrt{g^2+f^2-c}$$

$$r=\sqrt{(0)^2+\left(-\frac{3}{2}\right)^2-0}$$

$$r=\sqrt{\frac{9}{2}}=\boxed{\frac{3}{2}}$$

---

37. **Write the general form of the circle, also represent the center and radius in this form.**

Sol. $x^2+y^2+2gx+2fy+c=0$

$$\text{Center}=\boxed{(-g\,,-f)}$$

$$\&\ \text{Radius}=\boxed{\sqrt{g^2+f^2-c}}$$

Section - II

Note : Attemp any three (3) questions $\boxed{3\times10=30}$

Q.2.[a] If $f(x)=\frac{x-1}{x+1}$, show that;

$$\frac{f(x)-f(y)}{1+f(x)f(y)}=\frac{x-y}{1+xy}$$

Sol. See Q.9 of Ex # 1.1 (Page # 7)

[b] If $y=x^4+2x^2$, then prove that : $\frac{dy}{dx}=4x\sqrt{y+1}$

Sol. See Q.4(ii) of Ex # 2.3 (Page # 78)

Q.3.[a] If $\xcancel{x=a(\cos t+\sin t)}$,

$\xcancel{y=a(\sin t-t\cos t), \text{ find } \frac{dy}{dx}}$.

Sol. See Q.8(ii) of Ex # 3.1 (Page # 122)

[b] Show that $\frac{\ell nx}{x}$ has a maximum value at $x=e$.

Sol. See Q.15 of Ex # 5.1 (Page # 231)

Q.4.[a] Evaluate $\int\frac{dx}{\sqrt{x+a}+\sqrt{x+b}}$

Sol. See Q.15 of Ex # 5.1 (Page # 231)

[b] ~~Evaluate~~ $\xcancel{\int\frac{dx}{\sqrt{a^2-x^2}}}$

Sol. See example # 9 of Chapter 06.

Q.5.[a] ~~Evaluate~~ $\xcancel{\int(e^x\sin x)dx}$

Sol. See Q.5(i) of Ex # 6.3 (Page # 297)

[b] Find the value of 'y' so that the distance between $(1,y)$ and $(-1,4)$ is 2.

Sol. See Q.8 of Ex # 8.1 (Page # 359)

Q.6.[a] Show that the points $(2,6)$, $(-8,1)$ and $(-2\,,\ 4)$ are collinear by using slope.

Sol. See Q.7(i) of Ex # 8.3 (Page # 379)

[b] Find the equation of the circle which is tangent to the positive x-axis and y-axis and radius 5 units.

Sol. See Q.6 [a] of Ex # 9 (Page # 444)

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