DAE / IA - 2018

MATH - 233 APPLIED MATHEMATICS - II

PAPER 'B' PART - A (OBJECTIVE)

Time : 30 Minutes   Marks : 15

Q.1: *Encircle the correct answer.*

1. $\int \left(x^{n+1}\right) dx = ?$

[a] $\frac{x^{n+1}}{n+2}$ [b] $\frac{x^{n+2}}{n+2}$

[c] $(n+1)x^{n}$ [d] $\frac{x^2}{n}$

2. $\int \left(\frac{\cos x}{\sin x}\right) dx = ?$

[a] $\ell n \cos x$ [b] $\ell n \sin x$

[c] $\ell n \cot x$ [d] $\frac{\cos^2 x}{2}$

3. $\int (ax+b)\,dx = ?$

[a] $\frac{(ax+b)^2}{2a}$ [b] $\frac{(ax+b)^2}{2}$

[c] $\ell n(ax+b)$ [d] $a(ax+b)$

4. ~~$\int \left(\frac{-1}{\sqrt{1-x^2}}\right) dx = ?$~~

[a] $\sin^{-1} x$ [b] $\cos^{-1} x$

[c] $\sec^{-1} x$ [d] $\sqrt{1-x^2}$

5. $\int (\operatorname{cosec} x)\, dx = ?$

[a] $\ell n\,(\operatorname{cosec} x - \cot x)$ [b] $\ell n \sec x$

[c] $\ell n\,(\operatorname{cosec} x + \cot x)$ [d] $\cos x$

6. $\int_0^{\pi/2} (\cot x)\,dx = ?$

[a] -1 [b] 1 [c] 0 [d] ∞

7. $\int_0^1 \left(\frac{1}{x\sqrt{x^2-1}}\right) dx = ?$

[a] -1 [b] 1 [c] 0 [d] $\pi/2$

8. If a function $f(-x) = -f(x)$ then function is:

[a] Even [b] Odd

[c] Linear [d] Constant

9. Solution of differential equation $xdy + ydx = 0$ is;

[a] $y = cx$ [b] $y = x$

[c] $xy = c$ [d] $y^2 = x^2 + c$

10. Degree of differential equation $\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^3 = 0$ is:

[a] 3 [b] 2 [c] 0 [d] 1

11. The period of $\cos x$ is:

[a] π [b] 2π [c] $-\pi$ [d] -2π

12. ~~If an odd function, then Fourier coefficient~~ 'a_n' ~~is;~~

[a] 0 [b] 1 [c] -1 [d] 2

13. Laplace transform of the function $f(t) = t$ is:

[a] $\frac{1}{s}$ [b] $\frac{1}{s^2}$ [c] $\frac{1}{s^3}$ [d] $-\frac{1}{s}$

14. ~~$L^{-1}\left(\frac{1}{S^2+1}\right) = ?$~~

[a] $\sin t$ [b] $\cos t$ [c] $\sin\left(\frac{1}{t}\right)$ [d] $\cos\left(\frac{1}{t}\right)$

15. The Laplace transform of $f(t) = t^n$ is:

[a] $\frac{n}{s^{n+1}}$ [b] $\frac{n!}{s^{n+1}}$ [c] $\frac{1}{s^{n+1}}$ [d] $\frac{n!}{s^n}$

Answer Key

1	b	2	b	3	a	4	b	5	a
6	d	7	d	8	b	9	c	10	d
11	b	12	a	13	b	14	a	15	b

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DAE / IA - 2018

MATH - 233 APPLIED MATHEMATICS - II

PAPER 'B' PART - B (SUBJECTIVE)

Time: 2:30Hrs Marks: 60

Section - I

Q.1. **Write short answers to any Eighteen (18) questions.**

1. Evaluate $\int\left(12-\frac{2}{t^2}+\frac{4}{t^3}\right)dt$

Sol. $\int\left(12-\frac{2}{t^2}+\frac{4}{t^3}\right)dt$

$$=\int\left(12-2t^{-2}+4t^{-3}\right)dt$$

$$=12t-2\frac{t^{-1}}{-1}+4\frac{t^{-2}}{-2}+c$$

$$=\boxed{12t+\frac{2}{t}-\frac{2}{t^2}+c}$$

2. Find $\int\left(\frac{x^4}{x+1}\right)dx$

Sol. $\int\left(\frac{x^4}{x+1}\right)dx$ {Improper Fraction}

$$\begin{array}{r|l} & x^3-x^2+x-1 \\ x+1 & x^4 \\ & \pm x^4 \pm x^3 \\ \hline & -x^3 \\ & \mp x^3 \mp x^2 \\ \hline & x^2 \\ & \pm x^2 \pm x \\ \hline & -x \\ & \mp x \mp 1 \\ \hline & 1 \end{array}$$

$$=\int\left(x^3-x^2+x-1+\frac{1}{x+1}\right)dx$$

$$=\int\left(x^3-x^2+x-1+(x+1)^{-1}\right)dx$$

$$=\boxed{\frac{x^4}{4}-\frac{x^3}{3}+\frac{x^2}{2}-x+\ell n(x+1)+c}$$

3. Evaluate $\int\left(\sec^4 x\right)dx$

Sol. $\int\left(\sec^4 x\right)dx$

$$=\int\left(\sec^2 x.\sec^2 x\right)dx$$

$$=\int\left(1+\tan^2 x\right)\sec^2 x\,dx \quad \because \left\{\sec^2 x = 1+\tan^2 x\right\}$$

$$=\int\left(\sec^2 x+\tan^2 x.\sec^2 x\right)dx$$

$$=\boxed{\tan x+\frac{\tan^3 x}{3}+c}$$

4. Evaluate $\int\left(\sqrt{\sin x}\cos x\right)dx$

Sol. $\int\left(\sqrt{\sin x}\cos x\right)dx$

$$=\int(\sin x)^{1/2}\cos x\,dx$$

$$=\frac{(\sin x)^{3/2}}{3/2}+c=\boxed{\frac{2}{3}(\sin x)^{3/2}+c}$$

5. Evaluate $\int\left(\frac{e^{2x}}{1+e^{2x}}\right)dx$

Sol. $\int\left(\frac{e^{2x}}{1+e^{2x}}\right)dx$

Put $1+e^{2x}=t$

$$\frac{d}{dx}\left(1+e^{2x}\right)=\frac{d}{dx}(t)$$

$$o+e^{2x}(2)=\frac{dt}{dx}$$

$$e^{2x}dx=\frac{1}{2}dt$$

$$=\int\frac{\frac{1}{2}dt}{t}=\frac{1}{2}\int\left(\frac{1}{t}\right)dt$$

$$=\frac{1}{2}\ell n(t)+c=\boxed{\frac{1}{2}\ell n\left(1+e^{2x}\right)+c}$$

6. Evaluate $\int\left(\frac{x^3+x^2+x+1}{\sqrt{x}}\right)dx$

Sol. $\int\left(\frac{x^3+x^2+x+1}{\sqrt{x}}\right)dx$

$$=\int x^{-\frac{1}{2}}\left(x^3+x^2+x+1\right)dx$$

$$=\int\left(x^{3-\frac{1}{2}}+x^{2-\frac{1}{2}}+x^{1-\frac{1}{2}}+x^{-\frac{1}{2}}\right)dx$$

$$=\int\left(x^{5/2}+x^{3/2}+x^{1/2}+x^{-1/2}\right)dx$$

$$=\frac{x^{7/2}}{7/2}+\frac{x^{5/2}}{5/2}+\frac{x^{3/2}}{3/2}+\frac{x^{1/2}}{1/2}+c\ \left\{\text{using Rule-I}\right\}$$

$$=\boxed{\frac{2}{7}x^{7/2}+\frac{2}{5}x^{5/2}+\frac{2}{3}x^{3/2}+2\sqrt{x}+c}$$

7. Integrate $\int\left(e^{\tan x}\sec^2 x\right)dx$

Sol. $\int\left(e^{\tan x}\sec^2 x\right)dx$

Put $\tan x = t$

$\frac{d}{dx}(\tan x)=\frac{d}{dx}(t)$

$\sec^2 x=\frac{dt}{dx}$

$\sec^2 x\,dx = dt$

$$=\int\left(e^t\right)dt$$

$$=e^t+c$$

$$=\boxed{e^{\tan x}+c}$$

8. Evaluate $\int\left(\sin^3 x\right)dx$

Sol. $\int\left(\sin^3 x\right)dx=\int\left(\sin^2 x.\ \sin x\right)dx$

$$=\int\left(1-\cos^2 x\right)\sin x\ dx$$

$$=\int\left(\sin x-\cos^2 x\sin x\right)dx$$

$$=\int\left[\sin x+\cos^2 x\left(-\sin x\right)\right]dx$$

$$=\boxed{-\cos x+\frac{\cos^3 x}{3}+c}$$

9. ~~Evaluate $\int\left(e^x\sin x\right)dx$~~

Sol. Let $I=\int e^x\sin x\,dx$

Integrating by parts :

taking $u=e^x$ & $v=\sin x$

$$I=e^x\int\sin x\ dx-\int\left\{\frac{d}{dx}\left(e^x\right)\int\sin x\ dx\right\}dx$$

$$I=e^x\left(-\cos x\right)-\int e^x\left(-\cos x\right)dx$$

$$I=-e^x\cos x+\int e^x\cos x\ dx$$

Integrating by parts again :

taking $u=e^x$ & $v=\cos x$

$$I=-e^x\cos x+e^x\int\cos x\ dx-\int\left\{\frac{d}{dx}\left(e^x\right)\int\cos x\,dx\right\}dx$$

$$I=-e^x\cos x+e^x\sin x-\int e^x\sin x\ dx$$

$$I=-e^x\cos x+e^x\sin x-I+c$$

$$I+I=-e^x\cos x+e^x\sin x+c$$

$$2I=e^x\left(-\cos x+\sin x\right)+c$$

$$I=\boxed{\frac{e^x}{2}\left(\sin x-\cos x\right)+c}$$

10. ~~Evaluate $\int\frac{1}{a^2+9x^2}dx$~~

Sol. $\int\frac{1}{a^2+9x^2}dx$

$$=\int\frac{1}{9\left(\frac{a^2}{9}+x^2\right)}dx=\frac{1}{9}\int\frac{1}{\left(\frac{a}{3}\right)^2+(x)^2}dx$$

$$=\frac{1}{9}\ \frac{1}{\left(\frac{a}{3}\right)}\tan^{-1}\left(\frac{x}{a/3}\right)+c$$

$$=\boxed{\frac{1}{3a}\tan^{-1}\left(\frac{3x}{a}\right)+c}$$

11. Evaluate $\int_1^8 \frac{dx}{\sqrt[3]{x}}$

Sol. $\int_1^8 \frac{dx}{\sqrt[3]{x}} = \int_1^8 x^{-1/3}\,dx$

$= \left[\frac{x^{2/3}}{2/3}\right]_1^8 = \frac{3}{2}\left[x^{2/3}\right]_1^8$

$= \frac{3}{2}\left[(8)^{2/3} - (1)^{2/3}\right]$

$= \frac{3}{2}\left[\left(2^3\right)^{2/3} - (1)^{2/3}\right]$

$= \frac{3}{2}(4-1) = \frac{3}{2}(3) = \boxed{9/2}$

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12. Evaluate $\int_0^{\pi/4} \left(\tan^2 x\right) dx$

Sol. $\int_0^{\pi/4} \left(\tan^2 x\right) dx$

$= \int_0^{\pi/4} \left(\sec^2 x - 1\right) dx$ $\left\{\because \tan^2 x = \sec^2 x - 1\right\}$

$= \left[\tan x - x\right]_0^{\pi/4}$ {Using formula # 13 & 01 from page # 282}

$= \left[\tan\left(\frac{\pi}{4}\right) - \frac{\pi}{4}\right] - \left[\tan(0) - 0\right]$

$= \left[\tan(45^\circ) - \frac{\pi}{4}\right] - \left[\tan(0^\circ) - 0\right]$

$= 1 - \frac{\pi}{4} - 0 - 0$ {using calculator tan45° = 1 & tan0° = 0}

$= \boxed{\frac{4-\pi}{4}}$

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13. **Find the area bounded by the line 3x – y – 3 = 0 and x = 1 & x = 5.**

Sol. $\text{Area} = \int_a^b y\,dx$

$A = \int_1^5 (3x-3)\,dx$

$A = 3\int_1^5 (x-1)\,dx$

As
$3x - y - 3 = 0$
$-y = -3x + 3$
$y = 3x - 3$

$A = 3\left[\frac{x^2}{2} - x\right]_1^5$

$A = 3\left[\left(\frac{(5)^2}{2} - (5)\right) - \left(\frac{(1)^2}{2} - (1)\right)\right]$

$A = 3\left[\frac{25}{2} - 5 - \frac{1}{2} + 1\right]$

$A = 3\left[\frac{25 - 10 - 1 + 2}{2}\right]$

$A = 3\left(\frac{16}{2}\right) = 3(8) = \boxed{24\,\text{sq.unit}}$

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14. Evaluate $\int_0^1 \frac{1}{\sqrt{1-x^2}}\,dx$

Sol. $\int_0^1 \frac{1}{\sqrt{1-x^2}}\,dx$

$= \left[\sin^{-1} x\right]_0^1$

$= \sin^{-1}(1) - \sin^{-1}(0)$

$= 90^\circ - 0^\circ = \frac{\pi}{2} - 0 = \boxed{\frac{\pi}{2}}$

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15. Evaluate $\int_1^3 \frac{2x-1}{x^2 - x + 1}\,dx$

Sol. $\int_1^3 \frac{2x-1}{x^2 - x + 1}\,dx$

$= \left[\ell n\left(x^2 - x + 1\right)\right]_1^3$

$= \ell n\left((3)^2 - 3 + 1\right) - \ell n\left((1)^2 - 1 + 1\right)$

$= \ell n(9 - 3 + 1) - \ell n(1 - 1 + 1)$

$= \ell n(7) - \ell n(1)$

$= \ell n(7) - 0 = \boxed{\ell n(7)}$

16. Find $\int\left(\frac{\sin^2 x}{\cos^4 x}\right)dx$

Sol. $\int\left(\frac{\sin^2 x}{\cos^4 x}\right)dx$

$$= \int\frac{\sin^2 x}{\cos^2 x.\cos^2 x}dx$$

$$= \int\left(\frac{\sin^2 x}{\cos^2 x}.\frac{1}{\cos^2 x}\right)dx$$

$$= \int\tan^2 x.\sec^2 x\,dx \quad \because \left\{\frac{d}{dx}(\tan^2 x) = \sec^2 x\right\}$$

$$= \boxed{\frac{\tan^3 x}{3}+c} \quad \{\text{using Rule-I}\}$$

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17. Evaluate $\int\frac{dx}{(1+x^2)\tan^{-1}x}$

Sol. $\int\frac{dx}{(1+x^2)\tan^{-1}x}$

$$= \int\frac{1}{\tan^{-1}x}\cdot\frac{1}{(1+x^2)}dx$$

$$= \boxed{\ell n(\tan^{-1}x)+c}$$

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18. Evaluate $\int\frac{dx}{x(\ell n x)^4}$

Sol. $\int\frac{dx}{x(\ell n x)^4}$

$$= \int(\ell n x)^{-4}.\frac{1}{x}dx$$

$$= \frac{(\ell nx)^{-4+1}}{-3}+c \quad \{\text{using Rule-I}\}$$

$$= \frac{(\ell nx)^{-3}}{-3}+c = \boxed{-\frac{1}{3(\ell nx)^3}+c}$$

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19. Find the solution of

$$\frac{dy}{dx} = -\sin x + 3x^2$$

Sol. $\frac{dy}{dx} = -\sin x + 3x^2$

$$dy = \left(-\sin x + 3x^2\right)dx$$

Integrating both sides, we have:

$$\int 1\,dy = \int\left(-\sin x + 3x^2\right)dx$$

$$y = -(-\cos x) + 3\left(\frac{x^3}{3}\right) + c$$

$$\boxed{y = \cos x + x^3 + c}$$

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20. Find the solution of

$$dy = e^{x+y}dx$$

Sol. $dy = e^{x+y}dx$

$$dy = e^x.e^y dx \quad \because e^{x+y} = e^x.e^y$$

$$\frac{1}{e^y}dy = e^x dx$$

Integrating both sides, we have:

$$\int e^{-y}dy = \int e^x dx$$

$$\frac{e^{-y}}{-1} = e^x + c$$

$$-e^{-y} = e^x + c$$

$$\boxed{e^x + e^{-y} + c = 0}$$

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21. Find the order and degree of differential equation

$$\left[\frac{d^2y}{dx^2}\right]^3 - \left[\frac{d^3y}{dx^3}\right]^2 = y$$

Sol. $\boxed{\text{Oder} = 3 \ \& \ \text{Degree} = 2}$

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22. What are Fourier coefficients.

Sol. Constants a_0, a_n and b_n present in the Fourier series are called Fourier coefficients.

23. If a function is even integrable on $[-\pi, \pi]$ then which co-efficient exist.

Sol. a_0 and a_n exists and $b_n = 0$.

24. Let $f(t) = \cos 3t$, Find $L\{f(t)\}$.

Sol. $L\{\cos 3t\}$

$$= \frac{s}{(s)^2 + (3)^2} = \boxed{\frac{s}{s^2 + 9}}$$

25. Find the Laplace transform of $t^2 + at + b$

Sol. $L\{t^2 + at + b\}$

$$= L\{t^2\} + aL\{t\} + bL\{1\}$$

$$= \frac{2!}{s^3} + a\left(\frac{1}{s^2}\right) + b\left(\frac{1}{s}\right)$$

$$= \boxed{\frac{2}{s^3} + \frac{a}{s^2} + \frac{b}{s}}$$

26. Write the formula for $L\{u''(t)\}$.

Sol. $L\{u''(t)\}$

$$= s^2(L\{u(t)\}) - su(0) - u'(0)$$

27. ~~Find the inverse Laplace transformation of $\frac{5}{s-3}$.~~

Sol. $L^{-1}\left(\frac{5}{s-3}\right)$

$$= 5L^{-1}\left\{\frac{1}{s-3}\right\} = \boxed{5e^{3t}}$$

Section - II

Note: Attemp any three (3) questions $3 \times 8 = 24$

Q.2.[a] Evaluate $\int (\cos^4 2x)\, dx$

Sol. See Q.3 of Ex # 7.2 (Page # 291)

[b] Evaluate $\int \left(\frac{ax + bx^{-3} + cx^{-7}}{x^{-2}}\right) dx$

Sol. See Q.5 of Ex # 7.1 (Page # 284)

Q.3.[a] ~~Evaluate~~ $\int \frac{dx}{\sqrt{a^2 + x^2}}$

Sol. See Q.1(iv) of Ex # 8.2 (Page # 330)

[b] Evaluate $\left(\int \sec^2 x \,\ell n \tan x\right) dx$

Sol. See Q.3(vii) of Ex # 8.3 (Page # 349)

Q.4.[a] Evaluate $\int_0^a \frac{dx}{\sqrt{x+a} + \sqrt{x}}$

Sol. See Q.1(viii) of Ex # 9.1 (Page # 377)

[b] ~~Compute the area of the region bounded by the curve $y = x^4$ and line $y = 8x$.~~

Sol. See Q.10 of Ex # 9.2 (Page # 393)

Q.5.[a] A particle is moving in a straight line and its acceleration is given by a = 4t + 9

(i) Find the v(velocity) in terms of t if v = 15m/sec, when t = 0

(ii) Find s(distance) in terms of t if s = 0 , when t = 0

Sol. See Q.18 of Ex # 10 (Page # 420)

Q.6.[a] Find $L\{t^3\}$.

Sol. See example # 05 of Chapter 12.

[b] ~~Find $L^{-1}\left\{\frac{1}{(s+1)(s-2)}\right\}$.~~

Sol. See Q.6(vi) of Ex # 12 (Page # 473)

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