DAE / IA - 2017

MATH - 233 APPLIED MATHEMATICS - II

PAPER 'B' PART - A (OBJECTIVE)

Time: 30 Minutes Marks: 15

Q.1: *Encircle the correct answer.*

1. $\int\left(x^{n+1}\right)dx = ?$

[a] $\frac{x^{n+1}}{n+2}$ [b] $\frac{x^{n+2}}{n+2}$

[c] $(n+1)x^n$ [d] $\frac{x^2}{n}$

2. $\int(\sec x)\,dx = ?$

[a] $\tan x$ [b] $\frac{\sec^2 x}{2}$

[c] $\ell n(\sec x + \tan x)$ [d] $\sec x \tan x$

3. ~~$\int\left(\frac{-1}{\sqrt{1-x^2}}\right)dx = ?$~~

[a] $\sin^{-1}x$ [b] $\cos^{-1}x$

[c] $\sec^{-1}x$ [d] $\sqrt{1-x^2}$

4. $\int\left(\sin^4 x\cos x\right)dx = ?$

[a] $\frac{\sin^5 x}{5}$ [b] $\frac{\sin^5 x\cos x}{5}$

[c] $\frac{\cos^2 x}{2}$ [d] $-\sin x\cos x$

5. ~~$\int\left(\frac{e^x}{1+e^x}\right)dx = ?$~~

[a] $1+e^x$ [b] $\ell n\left(1+e^x\right)$

[c] e^x [d] $\frac{\left(1+e^x\right)^2}{2}$

6. $\int_0^{\pi/4}\left(\sec^2 x\right)dx = ?$

[a] 1 [b] 2 [c] 0 [d] 3

7. $\int_0^1(1)\,dx =$

[a] -1 [b] 0 [c] 1 [d] 2

8. Order of differential equation $\left(\frac{d^3y}{dx^3}\right)^2 + \frac{dy}{dx} + y = 0$ is:

[a] 2 [b] 1 [c] 0 [d] 3

9. Degree of differential equation $\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^3 = 0$ is:

[a] 3 [b] 2 [c] 0 [d] 1

10. If a function $f(-x) = f(x)$ then function is:

[a] Even [b] Odd

[c] Linear [d] Constant

11. ~~If an even function, the Fourier coefficient~~ $'b_n'$ ~~is:~~

[a] 0 [b] 1 [c] -1 [d] 2

12. Laplace transform of the function $f(t) = 2$ is:

[a] $\frac{S}{2}$ [b] $\frac{2}{S^2}$ [c] $\frac{2}{S^3}$ [d] $\frac{2}{S}$

13. ~~The Inverse Laplace transform~~ $L^{-1}\left(\frac{1}{S^2}\right)$ ~~is equal to:~~

[a] 1 [b] t [c] t^2 [d] $\frac{t}{2}$

14. $\int\left(\sec^2 x\right)dx = ?$

[a] $\tan x$ [b] $\sec x$

[c] $-\tan x$ [d] $\cot x$

15. $\int\left(x^3\right)dx =$

[a] $\frac{x^4}{4}$ [b] $\frac{x^4}{3}$ [c] $3x^2$ [d] $4x^4$

Answer Key

1	b	2	c	3	a	4	b	5	b
6	a	7	c	8	b	9	d	10	a
11	a	12	d	13	b	14	a	15	a

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DAE / IA - 2017

MATH - 233 APPLIED MATHEMATICS - II

PAPER 'B' PART - B (SUBJECTIVE)

Time : 2 : 30Hrs Marks : 60

Section - I

Q.1. Write short answers to any Eighteen (18) questions.

1. Evaluate $\int\left(\sqrt{x}+\frac{1}{\sqrt{x}}\right)^2 dx$

Sol. $\int\left(\sqrt{x}+\frac{1}{\sqrt{x}}\right)^2 dx$

$$=\int\left[\left(\sqrt{x}\right)^2+2\left(\sqrt{x}\right)\left(\frac{1}{\sqrt{x}}\right)+\left(\frac{1}{\sqrt{x}}\right)^2\right]dx$$

$$=\int\left[x+2+\frac{1}{x}\right]dx=\boxed{\frac{x^2}{2}+2x+\ell nx+c}$$

2. Evaluate $\int(\sin x-\cos x)dx$

Sol. $\int(\sin x-\cos x)dx$

$$=\boxed{-\cos x-\sin x+c}$$

3. Evaluate $\int\left(3x^2+2x+1\right)dx$

Sol. $\int\left(3x^2+2x+1\right)dx$

$\int\left(3x^2+2x+1\right)dx$

$$=3\frac{x^3}{3}+2\frac{x^2}{2}+x+c$$

$$=\boxed{x^3+x^2+x+c}$$

4. Evaluate $\int\left(\frac{\sin^2 x-\cos^2 x}{\sin^2 x\cos^2 x}\right)dx$

Sol. $\int\left(\frac{\sin^2 x-\cos^2 x}{\sin^2 x\cos^2 x}\right)dx$

$$=\int\left[\frac{\sin^2 x}{\sin^2 x\cos^2 x}-\frac{\cos^2 x}{\sin^2 x\cos^2 x}\right]dx$$

$$=\int\left[\frac{1}{\cos^2 x}-\frac{1}{\sin^2 x}\right]dx$$

$$=\int\left(\sec^2 x-\operatorname{cosec}^2 x\right)dx$$

$$=\tan x-(-\cot x)+c \quad \left\{\begin{array}{l}\text{Using formula \# 13 \&}\\ \text{14 from page \# 282}\end{array}\right\}$$

$$=\boxed{\tan x+\cot x+c}$$

5. Evaluate $\int\left(\sqrt{\sin x}\cos x\right)dx$

Sol. $\int\left(\sqrt{\sin x}\cos x\right)dx$

$$=\int(\sin x)^{1/2}\cos x\,dx$$

$$=\frac{(\sin x)^{3/2}}{3/2}+c=\boxed{\frac{2}{3}(\sin x)^{3/2}+c}$$

6. Evaluate $\int\left(\frac{e^x+e^{-x}}{e^x-e^{-x}}\right)dx$

Sol. $\int\left(\frac{e^x+e^{-x}}{e^x-e^{-x}}\right)dx$

$$=\int\frac{dt}{t}$$

$$=\int\left(\frac{1}{t}\right)dt$$

$$=\ell n(t)+c$$

$$=\boxed{\ell n\left(e^x-e^{-x}\right)+c}$$

Put $e^x-e^{-x}=t$

$\frac{d}{dx}\left(e^x-e^{-x}\right)=\frac{d}{dx}(t)$

$e^x-e^{-x}(-1)=\frac{dt}{dx}$

$\left(e^x+e^{-x}\right)dx=dt$

7. Evaluate $\int\frac{dx}{x(1+\ell nx)}$

Sol. $\int\frac{dx}{x(1+\ell nx)}$

$$=\int\frac{1}{(1+\ell nx)}.\frac{1}{x}.dx$$

$$=\boxed{\ell n(1+\ell nx)+c}$$

8. Evaluate $\int\left(x^2+3x+4\right)^3(2x+3)\,dx$

Sol. $\int\left(x^2+3x+4\right)^3(2x+3)dx$

$$=\boxed{\frac{\left(x^2+3x+4\right)^4}{4}+c}$$

9. Evaluate $\int(x\cos x)dx$

Sol. $\int(x\cos x)dx$

Integrating by parts:

taking $u = x$ & $v = \cos x$

$= x\int\cos x dx - \int\left[\frac{d}{dx}(x)\int\cos x dx\right]dx$

$= x(\sin x) - \int 1.(\sin x)dx$

$= x\sin x - \int\sin x dx$

$= x\sin x - (-\cos x) + c$

$= \boxed{x\sin x + \cos x + c}$

10. Evaluate $\int(\ell n\, x)dx$

Sol. $\int(\ell n\, x)dx$

$= \int(\ell n\, x \cdot 1)dx$

Integrating by parts:

taking $u = \ell n\, x$ & $v = 1$

$= \ell n\, x\int(1)dx - \int\left[\frac{d}{dx}(\ell n\, x)\int(1)dx\right]dx$

$= \ell n\, x(x) - \int\frac{1}{x}.(x)dx$

$= x\ell n\, x - \int(1)dx$

$= x\ell n\, x - (x) + c = \boxed{x(\ell n\, x - 1) + c}$

11. Evaluate $\int_1^3(x^2)dx$

Sol. $\int_1^3(x^2)dx = \left[\frac{x^3}{3}\right]_1^3 = \frac{1}{3}\left[x^3\right]_1^3$

$= \frac{1}{3}\left[(3)^3 - (1)^3\right] = \frac{1}{3}[27-1] = \boxed{\frac{26}{3}}$

12. **Find the area bounded by the line $3x - y - 3 = 0$ and $x = 1$ & $x = 5$.**

Sol. $\text{Area} = \int_a^b y\, dx$

$A = \int_1^5(3x-3)dx$

$A = 3\int_1^5(x-1)dx$

$A = 3\left[\frac{x^2}{2} - x\right]_1^5$

As
$3x - y - 3 = 0$
$-y = -3x + 3$
$y = 3x - 3$

$A = 3\left[\left(\frac{(5)^2}{2} - (5)\right) - \left(\frac{(1)^2}{2} - (1)\right)\right]$

$A = 3\left[\frac{25}{2} - 5 - \frac{1}{2} + 1\right] = 3\left[\frac{25 - 10 - 1 + 2}{2}\right]$

$A = 3\left(\frac{16}{2}\right) = 3(8) = \boxed{24\text{ sq.unit}}$

13. Evaluate $\int_0^{\pi/4}\frac{dx}{\cos^2 x}$

Sol. $\int_0^{\pi/4}\frac{dx}{\cos^2 x} = \int_0^{\pi/4}(\sec^2 x)dx$

$= [\tan x]_0^{\pi/4}$ {Using formula # 13 from page # 282}

$= \tan\left(\frac{\pi}{4}\right) - \tan(0)$

$= \tan(45^\circ) - \tan(0^\circ)$ $\left\{\frac{\pi}{4}\times\frac{180}{\pi} = 45^\circ,\ 0\times\frac{180}{\pi} = 0^\circ\right\}$

$= 1 - 0 = \boxed{1}$ {using calculator $\tan(45^\circ) = 1$ & $\tan(0^\circ) = 0$}

14. Find the solution of $\frac{dy}{dx} = \frac{y}{4+x^2}$

Sol. $\frac{dy}{dx} = \frac{y}{4+x^2}$

$\frac{dy}{y} = \frac{dx}{4+x^2}$

Integrating both sides, we have:

$\int\left(\frac{1}{y}\right)dy = \int\left(\frac{1}{(2)^2 + (x)^2}\right)dx$

$\boxed{\ell n\, y = \frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right) + c}$

15. Evaluate $\int_0^{\pi/4}\left(1+\sec^2 x\right) dx$

Sol. $\int_0^{\pi/4}\left(1+\sec^2 x\right) dx$

$=\left[x+\tan x\right]_0^{\pi/4}$ {Using formula # 01 & 13 from page # 282}

$=\left[\frac{\pi}{4}+\tan\left(\frac{\pi}{4}\right)\right]-\left[0+\tan(0)\right]$

$=\left[\frac{\pi}{4}+\tan(45^\circ)\right]-\left[0+\tan(0^\circ)\right]$

$=\frac{\pi}{4}+1-0-0=\boxed{\frac{\pi+4}{4}}$

16. Find the general solution

$\left(e^x+e^{-x}\right)\frac{dy}{dx}=\left(e^x-e^{-x}\right)$

Sol. $\left(e^x+e^{-x}\right)\frac{dy}{dx}=\left(e^x-e^{-x}\right)$

$dy=\left(\frac{e^x-e^{-x}}{e^x+e^{-x}}\right)dx$

Integrating both sides, we have:

$\int 1dy=\int\left(\frac{e^x-e^{-x}}{e^x+e^{-x}}\right)dx$

$\boxed{y=\ell n\left(e^x+e^{-x}\right)+c}$

17. What are Fourier coefficients.

Sol. Constants $a_{0,}$ a_n and b_n present in the Fourier series are called Fourier coefficients.

18. Find Laplace transform of a constant 'k'.

Sol. $L\{K\}=KL\{1\}=K\left(\frac{1}{s}\right)=\boxed{\frac{K}{s}}$

19. If $L\left\{e^{at}\right\}=\frac{1}{s-a}$ then what will be the Laplace transformation of $e^{t/2}$.

Sol. As, $L\left\{e^{at}\right\}=\frac{1}{s-a}$

Put $a=\frac{1}{2}$, we have:

$L\left\{e^{\frac{1}{2}t}\right\}=\frac{1}{s-\frac{1}{2}}$

$=\frac{1}{\frac{2s-1}{2}}=\boxed{\frac{2}{2s-1}}$

20. Find the solution of $dy=e^{x+y}dx$

Sol. $dy=e^{x+y}dx$

$dy=e^x.e^y dx \quad \because e^{x+y}=e^x.e^y$

$\frac{1}{e^y}dy=e^x dx$

Integrating both sides, we have:

$\int e^{-y}dy=\int e^x dx$

$\frac{e^{-y}}{-1}=e^x+c$

$-e^{-y}=e^x+c$

$\boxed{e^x+e^{-y}+c=0}$

21. Evaluate $\int 8(2x+1)^3 dx$

Sol. $\int 8(2x+1)^3 dx$

$=4\int(2x+1)^3(2)\,dx$

$=4\frac{(2x+1)^4}{4}+c$ {using Rule-I}

$=\boxed{(2x+1)^4+c}$

22. Find $\int\left(\frac{1}{t^3}+\frac{1}{t^2}-2\right)dt$

Sol. $\int\left(\frac{1}{t^3}+\frac{1}{t^2}-2\right)dt$

$= \int \left(t^{-3} + t^{-2} - 2\right) dt$

$= \frac{t^{-2}}{-2} + \frac{t^{-1}}{-1} - 2t + c$

$= \boxed{\frac{-1}{2t^2} - \frac{1}{t} - 2t + c}$

23. Find $\int \frac{1}{25 + x^2}\, dx$

Sol. $\int \frac{1}{25 + x^2}\, dx = \int \frac{1}{(5)^2 + (x)^2}\, dx$

$= \boxed{\frac{1}{5}\tan^{-1}\left(\frac{x}{5}\right) + c}$ {Using formula # 17 from page # 282}

24. ~~What is the inverse transformation of $\frac{1}{s+a}$?~~

Sol. $L^{-1}\left\{\frac{1}{s+a}\right\}$

$= L^{-1}\left\{\frac{1}{s-(-a)}\right\} = \boxed{e^{-at}}$

25. Evaluate $\int \frac{dx}{2x+3}$

Sol. $\int \frac{dx}{2x+3}$

$= \frac{1}{2}\int \frac{2dx}{2x+3} = \boxed{\frac{1}{2}\ell n(2x+3) + c}$

26. Evaluate $\int \left(ax^n + bx^m\right) dx$

Sol. $\int \left(ax^n + bx^m\right) dx$

$= \boxed{\frac{ax^{n+1}}{n+1} + \frac{bx^{m+1}}{m+1} + c}$

27. Evaluate $\int \left(\sin^6 x \cos x\right) dx$

Sol. $\int \left(\sin^6 x \cos x\right) dx$

$= \boxed{\frac{\sin^7 x}{7} + c}$

Section - II

Note: Attemp any three (3) questions $3 \times 8 = 24$

Q.2.[a] Evaluate $\int \left(\frac{x^3 - 8}{x+2}\right) dx$

Sol. See Q.13 of Ex # 7.1 (Page # 287)

[b] Evaluate $\int \frac{dx}{1 - \cos x}$

Sol. See Q.6 of Ex # 7.2 (Page # 292)

Q.3.[a] Evaluate $\int \frac{x+2}{\sqrt{2x^2 + 8x + 9}}\, dx$

Sol. See Q.1(iv) of Ex # 8.1 (Page # 318)

[b] ~~Evaluate $\int \left(x^2 \tan^{-1} x\right) dx$.~~

Sol. See Q.2(ii) of Ex # 8.3 (Page # 341)

Q.4.[a] Evaluate $\int_0^{\pi/4} \left(\tan^2 x\right) dx$

Sol. See Q.2(ix) of Ex # 9.1 (Page # 385)

[b] ~~Compute the area of the region bounded by the curve $y = x^4$ and line $y = 8x$.~~

Sol. See Q.10 of Ex # 9.2 (Page # 393)

Q.5.[a] Find the general solution of

$$(x+1)\frac{dy}{dx} = x\left(y^2 + 1\right)$$

Sol. See Q.11 of Ex # 10 (Page # 416)

[b] Integrate $\int \left(\frac{\cot x}{\ell n \sin x}\right) dx$

Sol. See Q.2(iii) of Ex # 8.1 (Page # 321)

Q.6. Find $L\{\cos \omega t\}$.

Sol. See proof of Formula 06 of Chapter 12.

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