DAE / IA - 2017

MATH - 233 APPLIED MATHEMATICS - II

PAPER 'A' PART - A (OBJECTIVE)

Time: 30 Minutes Marks: 15

Q.1: *Encircle the correct answer.*

1. Given $f(x) = \frac{1}{x} - 1$ then $f(2) = ?$

[a] 1 [b] 2 [c] $-\frac{1}{2}$ [d] 3

2. Which one is the periodic function:

[a] x^2+1 [b] $2x$
[c] $\sin x$ [d] x^3+1

3. $\lim_{x\to 2} (cx) = ?$

[a] $2c$ [c] c [c] 3 [d] 4

4. $\frac{d}{dx}(ax+b)^2 = ?$

[a] $2(ax+b)$ [b] $2a(ax+b)$
[c] $\frac{(ax+b)^3}{3}$ [d] $2(ax+b)b$

5. Second derivative of 'x^2' is:

[a] 2 [b] $2x$ [c] zero [d] $2x^2$

6. $\frac{d}{dx}(\operatorname{cosec} 3x) =$

[a] $-\operatorname{cosec} 3x \cot 3x$
[b] $-3\operatorname{cosec} 3x \cot 3x$
[c] $\cot 3x$ [d] $\operatorname{cosec} 3x$

7. ~~$\frac{d}{dx}(\sin^{-1} x) = ?$~~

[a] $\frac{1}{\sqrt{x^2-1}}$ [b] $\frac{-1}{\sqrt{1-x^2}}$
[c] $\frac{\sin^{-1}x}{\sqrt{1-x^2}}$ [d] $\frac{1}{\sqrt{1-x^2}}$

8. $\frac{d}{dx}(\ln \sin x) =$

[a] $\cot x$ [b] $\frac{1}{\sin x}\ln \sin x$
[c] $\ln \cos x$ [d] $\tan x$

9. $\frac{d}{dx}\left[\ln\left(x^2+1\right)\right] = ?$

[a] $\frac{x}{x^2+1}$ [b] $\frac{2x}{x^2+1}$
[c] $\ln(2x+1)$ [d] $2x\ln(2x+1)$

10. For a decreasing function $\frac{dy}{dx}$ is:

[a] $+ve$ [b] $-ve$
[c] zero [d] None of these

11. A function is maximum at a point if its 2^{nd} derivative is:

[a] $+ve$ [b] $-ve$
[c] zero [d] None of these

12. $\frac{d}{dx}(\cos x) = ?$

[a] $\sin x$ [b] $-\sin x$
[c] $-\cos x$ [d] $\cos x$

13. Firsthand information is called;

[a] Primary data
[b] Secondary data
[c] Raw data
[d] Continuous data

14. Mean, Median and Mode are the types of:

[a] Average [b] Function
[c] Variable [d] Constant

15. The sum of all variable divided by their number is called;

[a] Median
[b] Arithmetic Mean
[c] Mode
[d] Geometric Mean

Answer Key

1	c	2	c	3	a	4	b	5	a
6	b	7	a	8	a	9	b	10	b
11	b	12	b	13	a	14	a	15	b

****

DAE / IA - 2017

MATH - 233 APPLIED MATHEMATICS - II

PAPER 'B' PART - B (SUBJECTIVE)

Time : 2 : 30Hrs Marks : 60

Section - I

Q.1. Write short answers to any Eighteen (18) questions.

1. if $f(x)=\frac{x^2-3}{x+4}$, find $f(-3)$

Sol. $f(x)=\frac{x^2-3}{x+4}$

Put $x=-3$, we have:

$f(-3)=\frac{(-3)^2-3}{-3+4}=\frac{9-3}{1}=\boxed{6}$

2. Is the following function even, odd or neither: $f(x)=4x^3-2x+6$

Sol. As, $f(x)=4x^3-2x+6$

Replace x by $-x$, we have:

$f(-x)=4(-x)^3-2(-x)+6$

$f(-x)=-4x^3+2x+6$

$f(-x)=-(4x^3-2x-6)$

$f(-x)\neq-f(x)$

Hence $f(x)$ is **neither** even nor odd function.

3. Evaluate: $\lim_{x\to3}\sqrt{25-x^2}$

Sol. $\lim_{x\to3}\sqrt{25-x^2}$

$=\sqrt{25-(3)^2}=\sqrt{25-9}=\sqrt{16}=\boxed{4}$

4. ~~Evaluate: $\lim_{x\to0}\frac{\tan x}{x}$~~

Sol. $\lim_{x\to0}\frac{\tan x}{x}\ \left(\frac{0}{0}\right)$ form

$=\lim_{x\to0}\frac{\sin x}{x.\cos x}\ \left\{\because \tan x=\frac{\sin x}{\cos x}\right\}$

$=\lim_{x\to0}\frac{\sin x}{x}.\lim_{x\to0}\frac{1}{\cos x}$

$=(1).\frac{1}{\cos 0}=\frac{1}{1}=\boxed{1}$

5. Differentiate $\frac{6}{x}+\frac{4}{x^2}-\frac{3}{x^3}$ w.r.t. 'x'.

Sol. $\frac{d}{dx}\left(\frac{6}{x}+\frac{4}{x^2}-\frac{3}{x^3}\right)$

$=\frac{d}{dx}\left(6x^{-1}+4x^{-2}-3x^{-3}\right)$

$=6(-1)x^{-2}+4(-2)x^{-3}-3(-3)x^{-4}$

$=\boxed{-\frac{6}{x^2}-\frac{8}{x^3}+\frac{9}{x^4}}$

6. Find $\frac{dy}{dx}$ if $x^3+y^3+4=0$

Sol. Differentiate both sides w.r.t. 'x':

$\frac{d}{dx}(x^3+y^3+4)=\frac{d}{dx}(0)$

$3x^2+3y^2\frac{dy}{dx}+0=0$

$3y^2\frac{dy}{dx}=-3x^2$

$\frac{dy}{dx}=\frac{-3x^2}{3y^2}\Rightarrow\boxed{\frac{dy}{dx}=-\frac{x^2}{y^2}}$

7. ~~Find $\frac{dy}{dx}$ if $x=\theta^2-\theta-1,\ y=2\theta^2+\theta+1$~~

Sol. Differentiate both sides w.r.t. 'θ':

$\frac{d}{d\theta}(x)=\frac{d}{d\theta}(\theta^2-\theta-1)$	$\frac{d}{d\theta}(y)=\frac{d}{d\theta}(2\theta^2+\theta+1)$
$\frac{dx}{d\theta}=2\theta-1-0$	$\frac{dy}{d\theta}=2(2\theta)+1+0$
$\frac{d\theta}{dx}=\frac{1}{2\theta-1}$	$\frac{dy}{d\theta}=4\theta+1$

using chain rule: $\frac{dy}{dx}=\frac{dy}{d\theta}\times\frac{d\theta}{dx}$

$\frac{dy}{dx}=(4\theta+1)\left(\frac{1}{2\theta-1}\right)=\boxed{\frac{4\theta+1}{2\theta-1}}$

8. Differentiate $\sqrt[3]{x^2+9x+8}$ w.r.t. 'x'.

Sol. $\dfrac{d}{dx}\left[\sqrt[3]{x^2+9x+8}\right]$

$=\dfrac{d}{dx}\left[\left(x^2+9x+8\right)^{1/3}\right]$

$=\dfrac{1}{3}\left(x^2+9x+8\right)^{\frac{1}{3}-1}\left(\dfrac{d}{dx}\left(x^2+9x+8\right)\right)$

$=\dfrac{1}{3}\left(x^2+9x+8\right)^{-2/3}\left(2x+9(1)+0\right)$

$=\boxed{\dfrac{1}{3}\left(x^2+9x+8\right)^{-2/3}(2x+9)}$

9. IF $y=\dfrac{x}{1+x}$, Find $\dfrac{dy}{dx}$

Sol. Differentiate both sides w.r.t. 'x':

$\dfrac{d}{dx}(y)=\dfrac{d}{dx}\left(\dfrac{x}{1+x}\right)$

$\dfrac{dy}{dx}=\dfrac{\left(\dfrac{d}{dx}(x)\right)(1+x)-x\left(\dfrac{d}{dx}(1+x)\right)}{(1+x)^2}$

$\dfrac{dy}{dx}=\dfrac{(1)(1+x)-x(0+1)}{(1+x)^2}$

$\dfrac{dy}{dx}=\dfrac{1+x-x}{(1+x)^2}=\boxed{\dfrac{1}{(1+x)^2}}$

10. Differentiate $\cos^2(ax+b)$ w.r.t. 'x'.

Sol. $\dfrac{d}{dx}\left(\cos^2(ax+b)\right)$

$=2\cos(ax+b)\left(\dfrac{d}{dx}\cos(ax+b)\right)$

$=2\cos(ax+b).\left(-\sin(ax+b)\right)\left(\dfrac{d}{dx}(ax+b)\right)$

$=-2\sin(ax+b)\cos(ax+b).\left(a(1)+0\right)$

$=\boxed{-a\sin 2(ax+b)}$

11. Find the derivative of $x^2\tan x$.

Sol. $\dfrac{d}{dx}\left(x^2\tan x\right)$ {using Product Rule}

$=\left(\dfrac{d}{dx}\left(x^2\right)\right)\tan x+x^2\left(\dfrac{d}{dx}(\tan x)\right)$

$=\boxed{2x\tan x+x^2\sec^2 x}$

12. Differentiate $x\ell n x-x$ w.r.t. 'x'.

Sol. $\dfrac{d}{dx}(x\ell n x-x)$

$=\left[\left(\dfrac{d}{dx}(x)\right)\ell n x+x\left(\dfrac{d}{dx}(\ell n x)\right)\right]-\dfrac{d}{dx}(x)$

$=(1)\ell n x+x\left(\dfrac{1}{x}\right)-1=\ell n x+1-1=\boxed{\ell n x}$

13. Find $\dfrac{dy}{dx}$ of $e^x\ell n x$

Sol. Let, $y=e^x\ell n x$

Differentiate both sides w.r.t. 'x':

$\dfrac{d}{dx}(y)=\dfrac{d}{dx}\left(e^x\ell n x\right)$

$\dfrac{dy}{dx}=\dfrac{d}{dx}\left(e^x\right)\ell n x+e^x\dfrac{d}{dx}(\ell n x)$

$\dfrac{dy}{dx}=e^x\ell n x+e^x\left(\dfrac{1}{x}\right)$

$\boxed{\dfrac{dy}{dx}=e^x\left(\ell n x+\dfrac{1}{x}\right)}$

14. ~~If $y=\tan\left(p\tan^{-1}x\right)$, show that: $\left(1+x^2\right)\dfrac{dy}{dx}=p\left(1+y^2\right)$~~

Sol. As, $y=\tan\left(p\tan^{-1}x\right)$

$\Rightarrow\quad \tan^{-1}y=p\tan^{-1}x$

Differentiate both sides w.r.t. 'x':

$\dfrac{d}{dx}\left(\tan^{-1}y\right)=\dfrac{d}{dx}\left(p\tan^{-1}x\right)$

$\dfrac{1}{1+y^2}\dfrac{dy}{dx}=p\dfrac{1}{1+x^2}$

$\boxed{\left(1+x^2\right)\dfrac{dy}{dx}=p\left(1+y^2\right)}$ **Proved.**

15. ~~Differentiate $\sin x$ w.r.t. $\tan x$~~

Sol. Let, $y = \sin x$ & $t = \tan x$

Differentiate both equations both sides w.r.t. 'x':

$$\frac{d}{dx}(y) = \frac{d}{dx}(\sin x)$$
$$\frac{dy}{dx} = \cos x$$

$$\frac{d}{dx}(t) = \frac{d}{dx}(\tan x)$$
$$\frac{dt}{dx} = \sec^2 x$$
$$\frac{dx}{dt} = \frac{1}{\sec^2 x}$$

By using Chain's Rule:

$$\frac{dy}{dt} = \frac{dy}{dx} \cdot \frac{dx}{dt} = (\cos x)\left(\frac{1}{\sec^2 x}\right)$$

$$\frac{dy}{dt} = (\cos x)(\cos^2 x) \Rightarrow \boxed{\frac{dy}{dt} = \cos^3 x}$$

16. ~~Differentiate $\sin^{-1}\sqrt{x}$ w.r.t. 'x'.~~

Sol. $\frac{d}{dx}\left(\sin^{-1}\sqrt{x}\right)$

$$= \frac{1}{\sqrt{1-(\sqrt{x})^2}} \frac{d}{dx}(\sqrt{x})$$

$$= \frac{1}{\sqrt{1-x}} \cdot \frac{1}{2}(x)^{\frac{1}{2}-1} \frac{d}{dx}(x)$$

$$= \frac{1}{\sqrt{1-x}} \cdot \frac{1}{2}(x)^{-\frac{1}{2}}(1)$$

$$= \frac{1}{\sqrt{1-x}} \cdot \frac{1}{2\sqrt{x}} = \boxed{\frac{1}{2\sqrt{x}\sqrt{1-x}}}$$

17. **Find the critical values (or turning points) for x of the function $x^2 - 4x - 1$**

Sol. Let $y = x^2 - 4x - 1$

Differentiate both sides w.r.t. 'x':

$$\frac{d}{dx}(y) = \frac{d}{dx}(x^2 - 4x - 1)$$

$$\frac{dy}{dx} = 2x - 4(1) - 0$$

$$\frac{dy}{dx} = 2x - 4$$

For critical values, put $\frac{dy}{dx} = 0$

$$2x - 4 = 0$$

$$2x = 4 \Rightarrow \boxed{x = 2}$$

18. **Find the extreme values of the function $x^2 - 4x - 6$**

Sol. Let, $y = x^2 - 4x - 6 \rightarrow (i)$

Differentiate both sides w.r.t. 'x':

$$\frac{d}{dx}(y) = \frac{d}{dx}(x^2 - 4x - 6)$$

$$\frac{dy}{dx} = 2x - 4(1) - 0$$

$$\frac{dy}{dx} = 2x - 4 \quad \rightarrow (ii)$$

For critical values, put $\frac{dy}{dx} = 0$

$$2x - 4 = 0$$

$$2x = 4$$

$$x = \frac{4}{2} \Rightarrow \boxed{x = 2}$$

Diff. eq.(ii) both sides w.r.t. 'x':

$$\frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dx}(2x - 4)$$

$$\frac{d^2y}{dx^2} = 2(1) - 0$$

$$\frac{d^2y}{dx^2} = 2 \rightarrow (iii)$$

Put $x = 2$ in eq.(iii) & eq.(i)

$$\frac{d^2y}{dx^2} = 2 > 0$$

$$y_{min} = (2)^2 - 4(2) - 6$$

$$y_{min} = 4 - 8 - 6 \Rightarrow \boxed{y_{min} = -10}$$

19. Find the mean of the following scores $4, 0, 2, 9, 0.$

Sol. $\text{Mean} = \frac{\sum x}{n} = \frac{4+0+2+9+0}{5} = \frac{15}{3} = \boxed{3}$

20. ~~Find standard deviation of the values:~~ $2, 4, 6, 8, 10.$

Sol.

x	x^2
2	4
4	16
6	36
8	64
10	100
$\sum x = 30$	$\sum x^2 = 220$

$$\text{S.D.} = \sqrt{\frac{\sum x^2}{n} - \left(\frac{\sum x}{n}\right)^2}$$

$$\sigma = \sqrt{\left(\frac{220}{5}\right) - \left(\frac{30}{5}\right)^2}$$

$$\sigma = \sqrt{44-36}$$

$$\sigma = \sqrt{8} = \boxed{2.83}$$

21. Define median.

Sol. The value which divides an arrange data into two equal parts is called Median.

22. If $\mathbf{f(x) = a^x}$, show that $\mathbf{f(-p) = \frac{1}{f(p)}}$

Sol. $\text{L.H.S.} = f(-p) = a^{-p}$

$= \frac{1}{a^p} = \frac{1}{f(p)} = \text{R.H.S.}$ **Proved.**

23. ~~Find $\frac{dy}{dx}$ if $y = x\sin^{-1}x$~~

Sol. $y = x\sin^{-1}x$

Differentiate both sides w.r.t. 'x':

$$\frac{d}{dx}(y) = \frac{d}{dx}\left(x\sin^{-1}x\right)$$

$$\frac{dy}{dx} = \left(\frac{d}{dx}(x)\right)\sin^{-1}x + x\left(\frac{d}{dx}\left(\sin^{-1}x\right)\right)$$

$$\frac{dy}{dx} = (1)\sin^{-1}x + x\left(\frac{1}{\sqrt{1-x^2}}\right)$$

$$\boxed{\frac{dy}{dx} = \sin^{-1}x + \frac{x}{\sqrt{1-x^2}}}$$

24. Differentiate $\boldsymbol{\ell n\sqrt{x}}$ **w.r.t. 'x'.**

Sol. $\frac{d}{dx}\left(\ell n\sqrt{x}\right)$

$$= \frac{1}{\sqrt{x}}\frac{d}{dx}\left(\sqrt{x}\right)$$

$$= \frac{1}{\sqrt{x}}\cdot\frac{1}{2}(x)^{\frac{1}{2}-1}\frac{d}{dx}(x)$$

$$= \frac{1}{\sqrt{x}}\cdot\frac{1}{2}(x)^{-\frac{1}{2}}(1)$$

$$= \frac{1}{\sqrt{x}}\cdot\frac{1}{2\sqrt{x}} = \frac{1}{2\left(\sqrt{x}\right)^2} = \boxed{\frac{1}{2x}}$$

25. The velocity V m/S of a point moving in a straight line is given after t second by $V = 3t^2 + 4t$ find the acceleration after 2 second.

Sol. As, $v = 3t^2 + 4t$

$$a = \frac{dv}{dt} = \frac{d}{dt}\left(3t^2 + 4t\right)$$

$$a = 3(2t) + 4(1)$$

$$a = 6t + 4$$

Acceleration after 2 seconds:

$$a = 6(2) + 4$$

$$a = 12 + 4$$

$$\boxed{a = 16\ m/s^2}$$

26. Find $\frac{dy}{dx}$ if $y = x^3 + x^2 + 2x + 3$

Sol. Differentiate both sides w.r.t. 'x':

$$\frac{d}{dx}(y) = \frac{d}{dx}\left(x^3 + x^2 + 2x + 3\right)$$

$$\frac{dy}{dx} = 3x^2 + 2x + 2(1) + 0$$

$$\boxed{\frac{dy}{dx} = 3x^2 + 2x + 2}$$

27. ~~Evaluate $\lim_{x\to\infty}\left(1-\frac{1}{x}\right)^x$.~~

Sol. $\lim_{x\to\infty}\left(1-\frac{1}{x}\right)^x$

$$= \lim_{x\to\infty}\left(\left(1-\frac{1}{x}\right)^{-x}\right)^{-1}$$

$$= e^{-1} = \boxed{1/e}$$

Section - II

Note: Attemp any three (3) questions $3\times 8 = 24$

Q.2.(a) Show that $\frac{e^x+1}{e^x-1}$ is an odd function of x.

Sol. See Q.12(i) of Ex # 1.1 (Page # 10)

(b) Evaluate $\lim_{\theta\to 0}\frac{1-\cos\theta}{\theta\sin\theta}$

Sol. See Q.1(v) of Ex # 1.3 (Page # 26)

Q.3.(a) Differentiate $\sqrt{\frac{a+x}{a-x}}$ w.r.t. 'x'.

Sol. See Q.4(v) of Ex # 2.2 (Page # 58)

(b) Differentiate $\ell n\left(\frac{x}{\sqrt{1+x^2}}\right)$ w.r.t. 'x'.

Sol. See Q.4(v) of Ex # 2.2 (Page # 58)

Q.4.(a) If $xy = \cos(x+y)$, show that

$$\frac{dy}{dx}+\frac{y+\sin(x+y)}{x+\sin(x+y)} = 0$$

Sol. See Q.5[a] of Ex # 3.1 (Page # 120)

(b) Find the derivative of $x^2\sec 4x$

Sol. See Q.3(ii) of Ex # 3.1 (Page # 114)

Q.5. Find the maximum and minimum (extreme) values of the following function:

$$\frac{x^3}{3}-\frac{3x^2}{2}+2x+5$$

Sol. See Q.2(iv) of Ex # 4.2 (Page # 189)

Q.6. ~~Find the standard deviation for the following data:~~

Marks	Frequency
10 – 20	6
20 – 30	12
30 – 40	20
40 – 50	24
50 – 60	10
60 – 70	4
70 – 80	4

Sol. See example # 11 of Chapter 05.

****