## DAE / IA - 2017

## MATH - 212 APPLIED MATHEMATICS - II

### PART - A (OBJECTIVE)

Time : 30 Minutes Marks : 20

**Q.1 :** *Encircle the correct answer.*

**1.** If $f(x)=3^x-1$, then $f(3)=?$

[a] $27$ [b] $8$

[c] $26$ [d] $16$

**2.** $\lim\limits_{x\to 2}(cx)=?$

[a] $2c$ [c] $c$

[c] $3$ [d] $4$

**3.** $\frac{d}{dx}(ax+b)^2=?$

[a] $2(ax+b)$ [b] $2a(ax+b)$

[c] $\frac{(ax+b)^3}{3}$ [d] $2(ax+b)b$

**4.** $\frac{d}{dx}\sqrt{1+x}=$

[a] $\frac{1}{\sqrt{1+x}}$ [b] $\frac{1}{2\sqrt{1+x}}$

[c] $(1+x)^{1/3}$ [d] $\frac{-1}{2\sqrt{1+x}}$

**5.** $\frac{d}{dx}(\operatorname{cosec}3x)=$

[a] $-\operatorname{cosec}3x\cot 3x$

[b] $-3\operatorname{cosec}3x\cot 3x$

[c] $\cot 3x$ [d] $\operatorname{cosec}3x$

**6.** $\cancel{\frac{d}{dx}\left(\sin^{-1}\sqrt{x}\right)=?}$

[a] $\frac{1}{\sqrt{1-x^2}}$ [b] $\frac{1}{\sqrt{1+x^2}}$

[c] $\frac{1}{2\sqrt{x}\sqrt{1-x}}$ [d] $\frac{1}{2\sqrt{x}(1-x)}$

**7.** $\cancel{\frac{d}{dx}\left(e^{3x}\right)=?}$

[a] $e^{3x-1}$ [b] $e^{x-1}$

[c] $3e^{3x}$ [d] $3xe^{3x}$

**8.** $\frac{d}{dx}(\ell n\sin x)=$

[a] $\cot x$ [b] $\frac{1}{\sin x}\ell n\sin x$

[c] $\ell n\cos x$ [d] $\tan x$

**9.** For a decreasing function $\frac{dy}{dx}$ is:

[a] $+ve$ [b] $-ve$

[c] zero [d] None of these

**10.** $\int\left(\sqrt{x}\right)dx=$

[a] $\frac{1}{2}x^{1/2}$ [b] $\frac{2x^{1/2}}{3}$

[c] $\frac{2}{3}x^{3/2}$ [d] $\frac{1}{2x^{1/2}}$

**11.** $\int(\sec x)\,dx=?$

**[a]** $\tan x$ **[b]** $\frac{\sec^2 x}{2}$

**[c]** $\ell n(\sec x+\tan x)$

**[d]** $\sec x\tan x$

**12.** $\cancel{\int\left(e^{2x}\right)dx=?}$

[a] $\frac{e^{2x}}{2}$ [b] $\frac{e^{x^2}}{2}$

[c] $2e^{2x}$ [d] $\frac{e^{2x+1}}{2}$

**13.** $\int(\cos x)\,dx=?$

[a] $\cos x$ [b] $\sin x$

[c] $\frac{\cos^2 x}{2}$ [d] $-\sin x$

**14.** $\int_0^{\pi/4}\left(\sec^2 x\right)dx=?$

[a] $1$ [b] $2$

[c] $0$ [d] $3$

**15.** When two lines are perpendicular:

[a] $m_1 = m_2$ [b] $m_1m_2 = -1$

[c] $m_1 = -m_2$ [d] $m_1m_2 = 1$

**16.** Equation of line in slope intercept form is:

[a] $\frac{x}{a} + \frac{y}{b} = 1$ [b] $y = mx + c$

[c] $y - y_1 = m(x - x_1)$

[d] $y + y_1 = m(x + x_1)$

**17.** Give three points are collinear if their slopes are:

[a] Equal [b] Unequal

[c] $m_1m_2 = -1$ [d] $m_1m_2 = 1$

**18.** $y - y_1 = m(x - x_1)$ is the:

[a] Slope intercept form

[b] Intercepts form

[c] Point - Slope form

[d] Two - Points form

**19.** Center of the circle $(x-1)^2 + (y-2)^2 = 16$ is:

[a] $(1, 2)$ [b] $(2, 1)$

[c] $(4, 0)$ [d] $(-1, -2)$

**20.** For a point circle, the radius will be:

[a] 1 [b] $-1$

[c] 0 [d] Infinity

**Answer Key**

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1 | c | 2 | a | 3 | b | 4 | b | 5 | b |
| 6 | d | 7 | c | 8 | a | 9 | b | 10 | b |
| 11 | c | 12 | a | 13 | b | 14 | a | 15 | b |
| 16 | b | 17 | a | 18 | c | 19 | a | 20 | c |

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## DAE / IA - 2017

### MATH - 212 APPLIED MATHEMATICS - II

### PART - B (SUBJECTIVE)

Time : 2 : 30 Hrs Marks : 60

### Section - I

**Q.1 :** Write short answers to any Twenty Five (25) of the follwing questions. $\boxed{25 \times 2 = 50}$

**1.** if $f(x) = 3x^2 - 5x + 7$, find $f(4)$

**Sol.** $f(x) = 3x^2 - 5x + 7$

Put $x = 4$, we have:

$f(4) = 3(4)^2 - 5(4) + 7$

$f(4) = 48 - 20 + 7 = \boxed{35}$

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**2.** Is the following function even, odd or neither: $f(x) = 4x^3 - 2x + 6$

**Sol.** As, $f(x) = 4x^3 - 2x + 6$

Replace x by $-x$, we have:

$f(-x) = 4(-x)^3 - 2(-x) + 6$

$f(-x) = -4x^3 + 2x + 6$

$f(-x) = -(4x^3 - 2x - 6)$

$f(-x) \neq -f(x)$

Hence $f(x)$ is $\boxed{\text{neither}}$ even nor odd function.

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**3.** Evaluate $\lim\limits_{x \to 1} \frac{x^2 - 1}{x - 1}$

**Sol.** $\lim\limits_{x \to 1} \frac{x^2 - 1}{x - 1}$ $\left(\frac{0}{0}\right)$ form

$= \lim\limits_{x \to 1} \frac{(x)^2 - (1)^2}{x - 1}$

$= \lim\limits_{x \to 1} \frac{(x-1)(x+1)}{(x-1)}$

$= \lim\limits_{x \to 1} (x + 1) = 1 + 1 = \boxed{2}$

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**4.** Evaluate $\lim\limits_{\theta \to 0} \frac{1 - \cos\theta}{\theta \sin\theta}$

**Sol.** $\lim\limits_{\theta\to0}\dfrac{1-\cos\theta}{\theta\sin\theta}\ \left(\dfrac{0}{0}\right)$ form

$$=\lim_{\theta\to0}\frac{1-\cos\theta}{\theta\sin\theta}\times\frac{1+\cos\theta}{1+\cos\theta}$$

$$=\lim_{\theta\to0}\frac{(1)^2-(\cos\theta)^2}{\theta\sin\theta(1+\cos\theta)}$$

$$=\lim_{\theta\to0}\frac{1-\cos^2\theta}{\theta\sin\theta(1+\cos\theta)}$$

$$=\lim_{\theta\to0}\frac{\sin^2\theta}{\theta\sin\theta(1+\cos\theta)}$$

$$=\lim_{\theta\to0}\frac{\sin\theta}{\theta(1+\cos\theta)}$$

$$=\lim_{\theta\to0}\frac{\sin\theta}{\theta}\ .\ \lim_{\theta\to0}\frac{1}{(1+\cos\theta)}$$

$$=(1).\frac{1}{1+\cos0}=\frac{1}{1+1}=\boxed{\frac{1}{2}}$$

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**5.** **Differentiate $\frac{1}{5}x^{5/2}+\frac{1}{3}x^{3/2}$ w.r.t. 'x'.**

**Sol.** $\dfrac{d}{dx}\left(\dfrac{1}{5}x^{5/2}+\dfrac{1}{3}x^{3/2}\right)$

$$=\frac{1}{5}\left(\frac{5}{2}x^{3/2}\right)+\frac{1}{3}\left(\frac{3}{2}x^{1/2}\right)$$

$$=\boxed{\frac{1}{2}x^{3/2}+\frac{1}{2}x^{1/2}}$$

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**6.** **Differentiate $\sqrt{x^2+1}$ w.r.t. 'x'.**

**Sol.** $\dfrac{d}{dx}\left(\sqrt{x^2+1}\right)$

$$=\frac{1}{2}\left(x^2+1\right)^{\frac{1}{2}-1}\left(\frac{d}{dx}\left(x^2+1\right)\right)$$

$$=\frac{1}{2}\left(x^2+1\right)^{-1/2}(2x+0)$$

$$=\frac{1}{2\sqrt{x^2+1}}(2x)=\boxed{\frac{x}{\sqrt{x^2+1}}}$$

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**7.** **Differentiate $\sin(\tan x)$ w.r.t. 'x'.**

**Sol.** $\dfrac{d}{dx}\left[\sin(\tan x)\right]$

$$=\cos(\tan x).\left(\frac{d}{dx}(\tan x)\right)$$

$$=\boxed{\cos(\tan x)\sec^2 x}$$

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**8.** **Find $\frac{dy}{dx}$ if $x^{2/3}+y^{2/3}=a^{2/3}$**

**Sol.** Differentiate both sides w.r.t. 'x' :

$$\frac{d}{dx}\left(x^{2/3}+y^{2/3}\right)=\frac{d}{dx}\left(a^{2/3}\right)$$

$$\frac{2}{3}x^{-1/3}+\frac{2}{3}y^{-1/3}\frac{dy}{dx}=0$$

$$\frac{2}{3}y^{-1/3}\frac{dy}{dx}=-\frac{2}{3}x^{-1/3}$$

$$\frac{dy}{dx}=\left(-\frac{2}{3}x^{-1/3}\right).\left(\frac{3}{2y^{-1/3}}\right)$$

$$\frac{dy}{dx}=-\frac{x^{-1/3}}{y^{-1/3}}\ \Rightarrow\ \boxed{\frac{dy}{dx}=-\frac{y^{1/3}}{x^{1/3}}}$$

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**9.** **Differentiate**

$2x^2+x+1$ **w.r.t.** $x^2-x-1$

**Sol.** Let, $y=2x^2+x+1\ \&\ t=x^2-x-1$

Differentiate both sides w.r.t. 'x':

$$\frac{d}{dx}(y)=\frac{d}{dx}\left(2x^2+x+1\right)\ \Big|\ \frac{d}{dx}(t)=\frac{d}{dx}\left(x^2-x-1\right)$$

$$\frac{dy}{dx}=2(2x)+1+0$$

$$\frac{dy}{dx}=4x+1$$

$$\frac{dt}{dx}=2x-1-0$$

$$\frac{dt}{dx}=2x-1$$

$$\frac{dx}{dt}=\frac{1}{2x-1}$$

using chain rule : $\dfrac{dy}{dt}=\dfrac{dy}{dx}\times\dfrac{dx}{dt}$

$$\frac{dy}{dt}=(4x+1)\left(\frac{1}{2x-1}\right)=\boxed{\frac{4x+1}{2x-1}}$$

**10.** **Differentiate $\tan^{-1}\sqrt{x}$ w.r.t. 'x'.**

**Sol.** $\frac{d}{dx}\left(\tan^{-1}\sqrt{x}\right)$

$$= \frac{1}{1+(\sqrt{x})^2}\cdot\frac{d}{dx}\left(\sqrt{x}\right)$$

$$= \frac{1}{1+x}\cdot\frac{1}{2}\left(x^{-1/2}\right)$$

$$= \boxed{\frac{1}{2\sqrt{x}(1+x)}}$$

**11.** **Differentiate $x\ell n x - x$ w.r.t. 'x'.**

**Sol.** $\frac{d}{dx}(x\ell n x - x)$

$$= \left[\left(\frac{d}{dx}(x)\right)\ell n x + x\left(\frac{d}{dx}(\ell n x)\right)\right] - \frac{d}{dx}(x)$$

$$= (1)\ell n x + x\left(\frac{1}{x}\right) - 1$$

$$= \ell n x + \cancel{1} - \cancel{1} = \boxed{\ell n x}$$

**12.** **Differentiate $e^{\sin 2x}$ w.r.t. 'x'.**

**Sol.** Let $y = e^{\sin 2x}$

Differentiate both sides w.r.t. 'x':

$$\frac{d}{dx}(y) = \frac{d}{dx}\left(e^{\sin 2x}\right)$$

$$\frac{dy}{dx} = e^{\sin 2x}\left(\frac{d}{dx}(\sin 2x)\right)$$

$$\frac{dy}{dx} = e^{\sin 2x}(\cos 2x)\left(\frac{d}{dx}(2x)\right)$$

$$\frac{dy}{dx} = e^{\sin 2x}\cos 2x(2)$$

$$\boxed{\frac{dy}{dx} = 2e^{\sin 2x}\cos 2x}$$

**13.** **Differentiate**

**$\sin x$ w.r.t. $\tan x$**

**Sol.** Let, $y = \sin x$ & $t = \tan x$

Differentiate both equations

both sides w.r.t. 'x':

$$\frac{d}{dx}(y) = \frac{d}{dx}(\sin x)$$

$$\frac{dy}{dx} = \cos x$$

$$\frac{d}{dx}(t) = \frac{d}{dx}(\tan x)$$

$$\frac{dt}{dx} = \sec^2 x$$

$$\frac{dx}{dt} = \frac{1}{\sec^2 x}$$

By using Chain's Rule: $\frac{dy}{dt} = \frac{dy}{dx}\cdot\frac{dx}{dt}$

$$\frac{dy}{dt} = (\cos x)\left(\frac{1}{\sec^2 x}\right)$$

$$\frac{dy}{dt} = (\cos x)(\cos^2 x) \Rightarrow \boxed{\frac{dy}{dt} = \cos^3 x}$$

**14.** **If ~~$x = \sin 2t,\ y = 2\cos t,$~~**

**~~then find $\frac{dy}{dx}$.~~**

**Sol.** As, $x = \sin 2t$ & $y = 2\cos t$

Differentiate both equations

both sides w.r.t. 't':

$$\frac{d}{dt}(t) = \frac{d}{dt}(\sin 2t)$$

$$\frac{dt}{dx} = \cos 2t\,\frac{d}{dt}(2t)$$

$$\frac{dt}{dx} = \cos 2t\,(2)$$

$$\frac{dx}{dt} = \frac{1}{2\cos 2t}$$

$$\frac{d}{dt}(y) = \frac{d}{dt}(2\cos t)$$

$$\frac{dy}{dt} = 2(-\sin t)$$

$$\frac{dy}{dt} = -2\sin t$$

By using Chain's Rule: $\frac{dy}{dx} = \frac{dy}{dt}\times\frac{dt}{dx}$

$$\frac{dy}{dx} = (-2\sin t)\left(\frac{1}{2\cos 2t}\right) = \boxed{-\frac{\sin t}{\cos 2t}}$$

**15.** **Find the derivative w.r.t. 'x' of**

**$x\sqrt{x+1}$**

**Sol.** $\frac{d}{dx}\left(x\sqrt{x+1}\right)$ {using Product Rule}

$$= \left(\frac{d}{dx}(x)\right)\sqrt{x+1} + x\left(\frac{d}{dx}\left(\sqrt{x+1}\right)\right)$$

$$=1.\sqrt{x+1}+x.\frac{1}{2}(x+1)^{-1/2}\left(\frac{d}{dx}(x+1)\right)$$

$$=\sqrt{x+1}+\frac{x}{2\sqrt{x+1}}(1+0)$$

$$=\sqrt{x+1}+\frac{x}{2\sqrt{x+1}}=\frac{2\left(\sqrt{x+1}\right)^2+x}{2\sqrt{x+1}}$$

$$=\frac{2(x+1)+x}{2\sqrt{x+1}}=\frac{2x+2+x}{2\sqrt{x+1}}=\boxed{\frac{3x+2}{2\sqrt{x+1}}}$$

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**16.** **Find the critical values (turning points) for x of the function $x^2-4x-1$**

**Sol.** Let $y=x^2-4x-1$

Differentiate both sides w.r.t. 'x':

$$\frac{d}{dx}(y)=\frac{d}{dx}\left(x^2-4x-1\right)$$

$$\frac{dy}{dx}=2x-4(1)-0$$

$$\frac{dy}{dx}=2x-4$$

For critical values, put $\frac{dy}{dx}=0$

$$2x-4=0$$

$$2x=4\Rightarrow x=\frac{4}{2}\quad\Rightarrow\quad\boxed{x=2}$$

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**17.** **Evaluate $\int\left(\sqrt{x}+\frac{1}{\sqrt{x}}\right)dx$**

**Sol.** $\int\left(\sqrt{x}+\frac{1}{\sqrt{x}}\right)dx$

$$=\int\left(x^{1/2}+x^{-1/2}\right)dx$$

$$=\frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1}+\frac{x^{-\frac{1}{2}+1}}{-\frac{1}{2}+1}+c$$

$$=\frac{x^{3/2}}{3/2}+\frac{x^{1/2}}{1/2}+c=\boxed{\frac{2}{3}x^{3/2}+2x^{1/2}+c}$$

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**18.** **Evaluate $\int\frac{1}{2}\left(e^{\frac{1}{2}x}-e^{-\frac{1}{2}x}\right)dx$**

**Sol.** $\int\frac{1}{2}\left(e^{\frac{1}{2}x}-e^{-\frac{1}{2}x}\right)dx$

$$=\frac{1}{2}\left(\frac{e^{\frac{1}{2}x}}{1/2}-\frac{e^{-\frac{1}{2}x}}{-1/2}\right)+c=\boxed{e^{\frac{1}{2}x}+e^{-\frac{1}{2}x}+c}$$

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**19.** **Find $\int\left(\cot^2 x\right)dx$**

**Sol.** $\int\left(\cot^2 x\right)dx$

$$=\int\left(\operatorname{cosec}^2 x-1\right)dx$$

$$=-\cot x-x+c=\boxed{-\cot x-x+c}$$

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**20.** **Find $\int\left(\frac{x^2+1}{x+1}\right)dx$**

**Sol.** $\int\left(\frac{x^2+1}{x+1}\right)dx$

$$=\int\left(x-1+\frac{2}{x+1}\right)dx\qquad\boxed{\begin{array}{r} x-1 \\ x+1\,\big)\overline{\,x^2+1\,} \\ \underline{\pm x^2\pm x} \\ -x+1 \\ \underline{\mp x\mp 1} \\ 2 \end{array}}$$

$$=\boxed{\frac{x^2}{2}-x+2\,\ell n(x+1)+c}$$

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**21.** **Evaluate $\int\left(\sqrt{\sin x}\cos x\right)dx$**

**Sol.** $\int\left(\sqrt{\sin x}\cos x\right)dx$

$$=\int(\sin x)^{1/2}\cos x\,dx$$

$$=\frac{(\sin x)^{3/2}}{3/2}+c=\boxed{\frac{2}{3}(\sin x)^{3/2}+c}$$

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**22.** **Evaluate $\int\frac{dx}{\left(1+x^2\right)\tan^{-1}x}$**

**Sol.** $\int \frac{dx}{(1+x^2)\tan^{-1}x}$

$$= \int \frac{1}{\tan^{-1}x} \cdot \frac{1}{(1+x^2)}\, dx$$

$$= \boxed{\ell n\left(\tan^{-1}x\right)+c}$$

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**23.** **Evaluate** $\int\left(\frac{\ell nx}{x}\right)dx$

**Sol.** $\int\left(\frac{\ell nx}{x}\right)dx$

$$= \int \ell nx.\left(\frac{1}{x}\right)dx = \boxed{\frac{1}{2}(\ell nx)^2+c}$$

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**24.** **Evaluate** $\int(x\cos 3x)dx$

**Sol.** $\int(x\cos 3x)dx$

Integrating by parts:

taking $u = x$ & $v = \cos 3x$

$$= x\int \cos 3x dx - \int\left[\frac{d}{dx}(x)\int \cos 3x dx\right]dx$$

$$= x\left(\frac{\sin 3x}{3}\right) - \int 1.\left(\frac{\sin 3x}{3}\right)dx$$

$$= \frac{x}{3}\sin 3x - \frac{1}{3}\int \sin 3x dx$$

$$= \frac{x}{3}\sin 3x - \frac{1}{3}\left(-\frac{\cos 3x}{3}\right)+c$$

$$= \boxed{\frac{x}{3}\sin 3x + \frac{1}{9}\cos 3x + c}$$

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**25.** **Evaluate** $\int_1^8 \frac{dx}{\sqrt[3]{x}}$

**Sol.** $\int_1^8 \frac{dx}{\sqrt[3]{x}} = \int_1^8 x^{-1/3}\, dx$

$$= \left[\frac{x^{2/3}}{2/3}\right]_1^8 = \frac{3}{2}\left[x^{2/3}\right]_1^8$$

$$= \frac{3}{2}\left[(8)^{2/3} - (1)^{2/3}\right]$$

$$= \frac{3}{2}\left[\left(2^3\right)^{2/3} - (1)^{2/3}\right]$$

$$= \frac{3}{2}(4-1) = \frac{3}{2}(3) = \boxed{9/2}$$

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**26.** **Evaluate** $\int_0^{\pi/6}\left(\sec^2 x\right)dx$

**Sol.** $\int_0^{\pi/6}\left(\sec^2 x\right)dx$

$$= [\tan x]_0^{\pi/6} = \tan\left(\frac{\pi}{6}\right) - \tan(0)$$

$$= \tan(30°) - \tan(0°)$$

$$= \frac{1}{\sqrt{3}} - 0 = \boxed{\frac{1}{\sqrt{3}}}$$

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**27.** **Evaluate** $\int\left(\frac{x^3+1}{x^5}\right)dx$

**Sol.** $\int\left(\frac{x^3+1}{x^5}\right)dx$

$$= \int x^{-5}\left(x^3+1\right)dx$$

$$= \int\left(x^{-2}+x^{-5}\right)dx$$

$$= \frac{x^{-2+1}}{-2+1} + \frac{x^{-5+1}}{-5+1} + c$$

$$= \frac{x^{-1}}{-1} + \frac{x^{-4}}{-4} + c = \boxed{-\frac{1}{x} - \frac{1}{4x^4} + c}$$

---

**28.** **Find the distance between** $(-4, 2)$ & $(0, 5)$.

**Sol.** $D = \sqrt{(x_1-x_2)^2+(y_1-y_2)^2}$

$$D = \sqrt{(-4-0)^2+(2-5)^2}$$

$$D = \sqrt{(-4)^2+(-3)^2}$$

$$D = \sqrt{16+9} = \sqrt{25} = \boxed{5}$$

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**29.** **Show that the points (2, 6), (-8, 1) and (-2, 4) are collinear.**

**Sol.** $\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = \begin{vmatrix} 2 & 6 & 1 \\ -8 & 1 & 1 \\ -2 & 4 & 1 \end{vmatrix}$

$$= 2\begin{vmatrix} 1 & 1 \\ 4 & 1 \end{vmatrix} - 6\begin{vmatrix} -8 & 1 \\ -2 & 1 \end{vmatrix} + 1\begin{vmatrix} -8 & 1 \\ -2 & 4 \end{vmatrix}$$

$$= 2(1-4) - 6(-8+2) + 1(-32+2)$$

$$= 2(-3) - 6(-6) + 1(-30)$$

$$= -6 + 36 - 30 = \boxed{0}$$

Hence given points are collinear.

**Proved.**

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**30.** **Find the equation of a line through the point $(3, -2)$ with slope $m = \frac{3}{4}$.**

**Sol.** Equation of line in point - slope form :

$$y - y_1 = m(x - x_1)$$

$$y - (-2) = \frac{3}{4}(x - 3)$$

$$4(y+2) = 3(x-3)$$

$$4y + 8 = 3x - 9$$

$$4y + 8 - 3x + 9 = 0$$

$$-3x + 4y + 17 = 0$$

$$\boxed{3x - 4y - 17 = 0}$$

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**31.** **Reduce the equation $3x + 4y - 2 = 0$ into intercept form.**

**Sol.** $3x + 4y - 2 = 0$

$3x + 4y = 2$

Dividing both sides by 2, we have :

$$\frac{3x}{2} + \frac{4y}{2} = \frac{2}{2}$$

$$\frac{x}{2/3} + \frac{y}{2/4} = 1 \Rightarrow \boxed{\frac{x}{2/3} + \frac{y}{1/2} = 1}$$

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**32.** **Find the slope of a line which is perpendicular to the line joining $P_1(2, 4)$, $P_2(-2, 1)$.**

**Sol.** Slope of line joining given point :

$$= m_1 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1-4}{-2-2} = \frac{-3}{-4} = \frac{3}{4}$$

Slope of require line $= m_2 = ?$

As, both lines are perpendicular,

So, $m_1 m_2 = -1 \Rightarrow \left(\frac{3}{4}\right)m_2 = -1$

$$\Rightarrow m_2 = -1 \times \frac{4}{3} \quad \Rightarrow \boxed{m_2 = -\frac{4}{3}}$$

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**33.** **Is the point $(0, 4)$ inside or outside the circle of radius $4$ with center at $(-3, 1)$.**

**Sol.** $|\overline{CP}|$ = Distance between Center $(-3, 1)$ and point $(0, 4)$.

$$|\overline{CP}| = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

$$|\overline{CP}| = \sqrt{(0+3)^2 + (4-1)^2}$$

$$|\overline{CP}| = \sqrt{(3)^2 + (3)^2} = \sqrt{9+9}$$

$$|\overline{CP}| = \sqrt{18} = \boxed{4.4} > 4$$

As distance between both points is greater than 4 so the point $(0, 4)$ lie $\boxed{\text{outside}}$ the circle.

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**34.** **For the triangle $A(1, 3)$, $B(-2, 1)$, $C(0, -4)$, find slope of a line parallel to $\overline{AC}$.**

**Sol.** Slope of line $\overline{AC} = \frac{y_2 - y_1}{x_2 - x_1}$

$$= \frac{-4-3}{0-1} = \frac{-7}{-1} = 7$$

Hence Slope of line parallel to $\overline{AC} = \boxed{7}$

**35.** **Find the equation of circle with center $(-1, 2)$ and radius $r=\sqrt{2}$.**

**Sol.** Standard form of equation of circle:

$$(x-h)^2+(y-k)^2=r^2$$

Put $h=-1,\ k=2\ \&\ r=\sqrt{2}$

$$(x-(-1))^2+(y-2)^2=(\sqrt{2})^2$$

$$(x+1)^2+(y-2)^2=2$$

$$x^2+2x+1+y^2-4y+4-2=0$$

$$\boxed{x^2+y^2+2x-4y+3=0}$$

**36.** **Find the center and radius of the circle: $x^2+y^2-4x+6y-12=0$**

**Sol.** $x^2+y^2-4x+6y-12=0$

Comparing with General form of equation of circle:

$$x^2+y^2+2gx+2fy+c=0$$

$$2g=-4 \quad \Big| \quad 2f=6 \quad \Big| \quad c=-12$$

$$g=-\frac{4}{2}=-2 \quad \Big| \quad f=\frac{6}{2}=3$$

$$\text{Center}=(-g,\,-f)$$

$$=(-(-2),\,-3)=\boxed{(2,\,-3)}$$

$$\text{Radius}=r=\sqrt{g^2+f^2-c}$$

$$r=\sqrt{(-2)^2+(3)^2-(-12)}$$

$$r=\sqrt{4+9+12}=\sqrt{25}=\boxed{5}$$

**37.** **Write the general form of the circle, also represent the center and radius in this form.**

**Sol.** $x^2+y^2+2gx+2fy+c=0$

$$\text{Center}=\boxed{(-g,\,-f)}$$

$$\&\ \text{Radius}=\boxed{\sqrt{g^2+f^2-c}}$$

## Section - II

**Note:** Attemp any three (3) questions $\boxed{3\times 10=30}$

**Q.2.[a]** If $f(x)=\dfrac{x-1}{x+1}$, show that;

$$\frac{f(x)-f(y)}{1+f(x)f(y)}=\frac{x-y}{1+xy}$$

**Sol.** See Q.9 of Ex # 1.1 (Page # 7)

**[b]** Differentiate $\left(\dfrac{x+1}{x-1}\right)^2$ w.r.t. 'x'.

**Sol.** See Q.4(ix) of Ex # 2.2 (Page # 58)

**Q.3.[a]** Find the derivative of $\dfrac{\sin x}{1-\cos x}$ w.r.t. 'x'.

**Sol.** See Q.3(ix) of Ex # 3.1 (Page # 116)

**[b]** Find the maximum and minimum (extreme) values of the function $(x-2)^2(x-1)$.

**Sol.** See Q.2(vi) of Ex # 4.2 (Page # 190)

**Q.4.[a]** Evaluate $\displaystyle\int \frac{dx}{\sqrt{x+a}+\sqrt{x+b}}$

**Sol.** See Q.15 of Ex # 5.1 (Page # 231)

**[b]** Evaluate $\int(\sin^3 x)dx$

**Sol.** See Q.4(i) of Ex # 6.1 (Page # 267)

**Q.5.[a]** ~~Integrate~~ $\xcancel{\int (x^2\tan^{-1}x)dx}$

**Sol.** See Q.2(ii) of Ex # 6.3 (Page # 285)

**[b]** Find the point which is two third of the way from the point (5, 1) to the point (-2, 9).

**Sol.** See Q.6 of Ex # 8.2 (Page # 366)

**Q.6.[a]** Show that the given points are the vertices of a parallelogram (-3, 1), (-1, 7), (2, 8) and (0, 2).

**Sol.** See Q.5[a] of Ex # 8.3 (Page # 376)

**[b]** Find the equation of circle having (-2, 5) and (3, 4) as the end points of its diameter. Find also its center and radius.

**Sol.** See Q.8[a] of Ex # 9 (Page # 447)

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