DAE / IA - 2016

MATH - 233 APPLIED MATHEMATICS - II

PAPER 'B' PART - A (OBJECTIVE)

Time : 30 Minutes Marks : 15

Q.1: *Encircle the correct answer.*

1. $\int (x^3)\,dx = ?$

[a] $\frac{x^4}{4}$ [b] $\frac{x^4}{3}$ [c] $3x^2$ [d] $4x^4$

2. $\int \left(\frac{\cos x}{\sin x}\right) dx = ?$

[a] $\ell n \cos x$ [b] $\ell n \sin x$

[c] $\ell n \cot x$ [d] $\frac{\cos^2 x}{2}$

3. ~~$\int \left(\frac{-1}{\sqrt{1-x^2}}\right) dx = ?$~~

[a] $\sin^{-1} x$ [b] $\cos^{-1} x$

[c] $\sec^{-1} x$ [d] $\tan^{-1} x$

4. ~~$\int \left(\frac{e^x}{1+e^x}\right) dx = ?$~~

[a] $1+e^x$ [b] $\ell n\left(1+e^x\right)$

[c] e^x [d] $\frac{\left(1+e^x\right)^2}{2}$

5. $\int (\operatorname{cosec} x)\,dx = ?$

[a] $\ell n\left(\operatorname{cosec} x - \cot x\right)$ [b] $\ell n \sec x$

[c] $\ell n\left(\operatorname{cosec} x + \cot x\right)$ [d] $\cos x$

6. $\int_0^1 (1)\,dx = ?$

[a] -1 [b] 0 [c] 1 [d] 2

7. ~~$\int_1^3 \left(e^{2x}\right) dx = ?$~~

[a] $e^6 - e^2$ [b] $\frac{e^{2x}}{2}$

[c] $\frac{1}{2}\left(e^6 + e^2\right)$ [d] $\frac{1}{2}\left(e^6 - e^2\right)$

8. An equation involving one or more derivative of a function is called:

[a] Quadratic [b] Linear

[c] Differential [d] Cubic

9. Degree of differential equation

$\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^3 = 0$ is:

[a] 3 [b] 2 [c] 0 [d] 1

10. If a function $f(-x) = -f(x)$ then function is:

[a] Even [b] Odd

[c] Linear [d] Constant

11. ~~If an odd function, then Fourier coefficient~~ 'a_n' ~~is;~~

[a] 0 [b] 1 [c] -1 [d] 2

12. Laplace transform of the function $f(t) = 1$ is:

[a] $\frac{1}{S^3}$ [b] $\frac{1}{S^2}$ [c] $\frac{1}{S}$ [d] $-\frac{1}{S}$

13. ~~$L^{-1}\left(\frac{1}{S-1}\right)$~~ is equal to:

[a] e^{-t} [b] e^{2t} [c] $1/t$ [d] e^t

14. $\int (\sin x)\,dx = ?$

[a] $\cos x$ [b] $-\cos x$

[c] $\frac{\sin^2 x}{2}$ [d] $\operatorname{cosec} x$

15. $\int (x \sin x)\,dx = ?$

[a] $-x\cos x + \sin x$ [b] $\sin x$

[c] $x + \sin x$ [d] $\frac{x^2}{2}\cos x$

Answer Key

1	a	2	b	3	a	4	b	5	a
6	d	7	b	8	c	9	d	10	b
11	a	12	c	13	d	14	b	15	a

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DAE / IA - 2016

MATH - 233 APPLIED MATHEMATICS - II

PAPER 'B' PART - B (SUBJECTIVE)

Time: 2:30Hrs Marks: 60

Section - I

Q.1. Write short answersto any Eighteen (18) questions.

1. Evaluate $\int \sqrt{x}\, dx$

Sol. $\int \sqrt{x}\, dx$

$$= \frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1} + c$$

$$= \frac{x^{3/2}}{3/2} + c = \boxed{\frac{2}{3}x^{3/2} + c}$$

2. Evaluate $\int \cos^2 x dx$

Sol. $\int \cos^2 x dx$

$$= \int \frac{1+\cos 2x}{2} dx$$

$$= \frac{1}{2}\int (1+\cos 2x)\, dx$$

$$= \boxed{\frac{1}{2}\left[x + \frac{\sin 2x}{2}\right] + c}$$

3. Evaluate $\int \left(\frac{1+x}{x}\right) dx$

Sol. $\int \left(\frac{1+x}{x}\right) dx$

$$= \int \left(\frac{1}{x} + \frac{x}{x}\right) dx$$

$$= \int \left(\frac{1}{x} + 1\right) dx$$

$$= \boxed{\ell n x + x + c}$$

4. Evaluate $\int (\sin x - \cos x)^2\, dx$

Sol. $\int (\sin x - \cos x)^2\, dx$

$$= \int \left(\sin^2 x + \cos^2 x - 2\sin x \cos x\right) dx$$

$$= \int (1 - \sin 2x)\, dx \because \left\{\begin{matrix}\sin^2 x + \cos^2 x = 1 \\ \sin 2x = 2\sin x \cos x\end{matrix}\right\}$$

$$= x - \left(\frac{-\cos 2x}{2}\right) + c$$

$$= \boxed{x + \frac{1}{2}\cos 2x + c}$$

5. Evaluate $\int \left(\cos^4 x \sin x\right) dx$

Sol. $\int \left(\cos^4 x \sin x\right) dx$

$$= -\int \cos^4 x(-\sin x)\, dx$$

$$= -\frac{\cos^5 x}{5} + c = \boxed{-\frac{1}{5}\cos^5 x + c}$$

6. Evaluate $\int \frac{dx}{(1+x^2)\tan^{-1} x}$

Sol. $\int \frac{dx}{(1+x^2)\tan^{-1} x}$

$$= \int \frac{1}{\tan^{-1} x} \cdot \frac{1}{(1+x^2)}\, dx$$

$$= \boxed{\ell n\left(\tan^{-1} x\right) + c}$$

7. Evaluate $\int \left(\frac{1}{\sqrt{x}} \sin \sqrt{x}\right) dx$

Sol. $\int \left(\frac{1}{\sqrt{x}} \sin \sqrt{x}\right) dx$

$$\text{Put } \sqrt{x} = t \Rightarrow \frac{d}{dx}\left(\sqrt{x}\right) = \frac{d}{dx}(t)$$

$$\frac{1}{2}x^{-1/2} = \frac{dt}{dx}$$

$$\frac{1}{2\sqrt{x}} = \frac{dt}{dx} \Rightarrow \frac{1}{\sqrt{x}}dx = 2dt$$

$$= \int \sin\sqrt{x}.\left(\frac{1}{\sqrt{x}}\right)dx$$

$$= \int (\sin t)(2dt)$$

$$= 2\int (\sin t)\,dt$$

$$= 2(-\cos t) + c = \boxed{-2\cos\sqrt{x} + c}$$

8. Evaluate $\int \left(\frac{\ell nx}{x}\right)dx$

Sol. $\int \left(\frac{\ell nx}{x}\right)dx$

$$= \int \ell nx.\left(\frac{1}{x}\right)dx$$

$$= \boxed{\frac{1}{2}(\ell nx)^2 + c}$$

9. Evaluate $\int (x\cos 3x)dx$

Sol. $\int (x\cos 3x)dx$

Integrating by parts:

taking $u = x$ & $v = \cos 3x$

$$= x\int \cos 3x dx - \int \left[\frac{d}{dx}(x)\int \cos 3x dx\right]dx$$

$$= x\left(\frac{\sin 3x}{3}\right) - \int 1.\left(\frac{\sin 3x}{3}\right)dx$$

$$= \frac{x}{3}\sin 3x - \frac{1}{3}\int \sin 3x dx$$

$$= \frac{x}{3}\sin 3x - \frac{1}{3}\left(-\frac{\cos 3x}{3}\right) + c$$

$$= \boxed{\frac{x}{3}\sin 3x + \frac{1}{9}\cos 3x + c}$$

10. Evaluate $\int (x\ell nx)\,dx$

Sol. $\int (x\ell nx)\,dx$

$$= \int \ell nx.x\,dx$$

Integrating by parts:

taking $u = \ell nx$ & $v = x$

$$= \ell nx\int x\,dx - \int\left\{\frac{d}{dx}(\ell nx)\int x\,dx\right\}dx$$

$$= \ell nx.\frac{x^2}{2} - \int \frac{1}{x}.\frac{x^2}{2}dx$$

$$= \frac{x^2}{2}\ell nx - \frac{1}{2}\int x\,dx$$

$$= \frac{x^2}{2}\ell nx - \frac{1}{2}.\frac{x^2}{2} + c$$

$$= \boxed{\frac{x^2}{2}\ell nx - \frac{1}{4}x^2 + c}$$

11. Evaluate $\int_1^3 (x^2)dx$

Sol. $\int_1^3 (x^2)dx$

$$= \left[\frac{x^3}{3}\right]_1^3$$

$$= \frac{1}{3}\left[x^3\right]_1^3$$

$$= \frac{1}{3}\left[(3)^3 - (1)^3\right]$$

$$= \frac{1}{3}\left[27 - 1\right] = \boxed{\frac{26}{3}}$$

12. ~~**Find the area bounded by the line $3x - y - 3 = 0$ and $x = 1$ & $x = 5$.**~~

Sol. Area $= \int_a^b y\,dx$

$$A = \int_1^5 (3x - 3)dx$$

$$A = 3\int_1^5 (x - 1)dx$$

As
$3x - y - 3 = 0$
$-y = -3x + 3$
$y = 3x - 3$

$$A = 3\left[\frac{x^2}{2} - x\right]_1^5$$

$$A = 3\left[\left(\frac{(5)^2}{2} - (5)\right) - \left(\frac{(1)^2}{2} - (1)\right)\right]$$

$$A = 3\left[\frac{25}{2} - 5 - \frac{1}{2} + 1\right]$$

$$A = 3\left[\frac{25 - 10 - 1 + 2}{2}\right]$$

$$A = 3\left(\frac{16}{2}\right) = 3(8) = \boxed{24\,\text{sq.unit}}$$

13. **Find the general solution of**

$$\mathbf{xdy = 3ydx}$$

Sol. $xdy = 3ydx$

$$\frac{1}{y}dy = \frac{3}{x}dx$$

Integrating both sides, we have :

$$\int \frac{1}{y}dy = \int \frac{3}{x}dx$$

$$\ell n\, y = 3\ell n\, x + \ell n\, c$$

$$\ell n\, y = \ell n\, x^3 + \ell n\, c$$

$$\ell n\, y = \ell n\left(cx^3\right) \Rightarrow \boxed{y = cx^3}$$

14. **Evaluate** $\int_{-\pi/2}^{\pi/2} (\cos x)\, dx$

Sol. $\int_{-\pi/2}^{\pi/2} (\cos x)\, dx$

$$= \left[\sin x\right]_{-\pi/2}^{\pi/2}$$

$$= \sin\left(\frac{\pi}{2}\right) - \sin\left(-\frac{\pi}{2}\right)$$

$$= \sin(90^\circ) - \sin(-90^\circ) \left\{\begin{array}{l} \frac{\pi}{2} \times \frac{180}{\pi} = 90^\circ \\ -\frac{\pi}{2} \times \frac{180}{\pi} = -90^\circ \end{array}\right\}$$

$$= 1 - (-1) \left\{\begin{array}{c} \text{using calculator} \\ \sin(90^\circ) = 1 \;\&\; \sin(-90^\circ) = -1 \end{array}\right\}$$

$$= 1 + 1 = \boxed{2}$$

15. **Evaluate** $\int_0^{\pi/4} \left(1 + \sec^2 x\right) dx$

Sol. $\int_0^{\pi/4} \left(1 + \sec^2 x\right) dx$

$$= \left[x + \tan x\right]_0^{\pi/4} \left\{\begin{array}{c} \text{Using formula \# 01 \& 13} \\ \text{from page \# 282} \end{array}\right\}$$

$$= \left[\frac{\pi}{4} + \tan\left(\frac{\pi}{4}\right)\right] - \left[0 + \tan(0)\right]$$

$$= \left[\frac{\pi}{4} + \tan(45^\circ)\right] - \left[0 + \tan(0^\circ)\right]$$

$$= \frac{\pi}{4} + 1 - 0 - 0 \left\{\begin{array}{c} \text{using calculator} \\ \tan 45^\circ = 1 \;\&\; \tan 0^\circ = 0 \end{array}\right\}$$

$$= \boxed{\frac{\pi + 4}{4}}$$

16. **Solve the differential equation**

$$\mathbf{(1 - x)dy = (1 + y)dx}$$

Sol. $(1 - x)dy = (1 + y)dx$

$$\frac{1}{(1 + y)}dy = \frac{1}{(1 - x)}dx$$

Integrating both sides, we have :

$$\int \frac{1}{1 + y}dy = -\int \frac{-1}{1 - x}dx$$

$$\ell n(1 + y) = -\ell n(1 - x) + \ell n\, c$$

$$\ell n(1 + y) = \ell n\left(\frac{c}{1 - x}\right)$$

$$(1 + y) = \frac{c}{1 - x}$$

$$\boxed{(1 + y)(1 - x) = c}$$

17. **If a function is even integrable on** $[-\pi, \pi]$ **then which co-efficient exist.**

Sol. a_0 and a_n exists and $b_n = 0$.

18. **Find the Laplace transforms of 1.**

Sol. $L\{1\} = \boxed{\frac{1}{s}}$

19. **If** $L\{t^n\} = \frac{n!}{s^{n+1}}$ **then what will be** $L\{t^7\}$**.**

Sol. As, $L\{t^n\}=\dfrac{n!}{s^{n+1}}$

Put $n=7$, we have:

$L\{t^7\}=\dfrac{7!}{s^{7+1}}=\boxed{\dfrac{5040}{s^8}}$

20. **Find the solution of**

$$\frac{dy}{dx}=-\sin x+3x^2$$

Sol. $\dfrac{dy}{dx}=-\sin x+3x^2$

$dy=\left(-\sin x+3x^2\right)dx$

Integrating both sides, we have:

$\int 1\,dy=\int\left(-\sin x+3x^2\right)dx$

$y=-(-\cos x)+3\left(\dfrac{x^3}{3}\right)+c$

$\boxed{y=\cos x+x^3+c}$

21. **Find the value of**

$$\int 10\left(x^2-3x+4\right)^9(2x-3)\,dx$$

Sol. $\int 10\left(x^2-3x+4\right)^9(2x-3)\,dx$

$=10\dfrac{\left(x^2-3x+4\right)^{10}}{10}+c$ {using Rule-I}

$=\boxed{\left(x^2-3x+4\right)^{10}+c}$

22. **Find** $\int\left(x+\dfrac{1}{x}\right)^2dx$

Sol. $\int\left(x+\dfrac{1}{x}\right)^2dx$

$=\int\left(x^2+\dfrac{1}{x^2}+2\right)dx$

$=\int\left(x^2+x^{-2}+2\right)dx$

$=\dfrac{x^3}{3}+\dfrac{x^{-1}}{-1}+2x+c$

$=\boxed{\dfrac{x^3}{3}-\dfrac{1}{x}+2x+c}$

23. **Find** $\int\dfrac{(\ell nx)^3}{x}dx$

Sol. $\int\dfrac{(\ell nx)^3}{x}dx$

$=\int(\ell nx)^3.\left(\dfrac{1}{x}\right)dx$

$=\boxed{\dfrac{(\ell nx)^4}{4}+c}$

24. **Evaluate** $\int\left(x\sec^2x\right)dx$

Sol. $\int\left(x\sec^2x\right)dx$

Integrating by parts:

taking $u=x$ & $v=\sec^2x$

$=x\int\sec^2x dx-\int\left[\dfrac{d}{dx}(x)\int\sec^2x dx\right]dx$

$=x\tan x-\int(1\cdot\tan x)dx$

$=x\tan x-\int(\tan x)dx$

$=\boxed{x\tan x-\ell n\sec x+c}$

25. ~~**Find** $L^{-1}\left\{\dfrac{1}{s-a}-\dfrac{1}{s+a}\right\}$~~

Sol. $L^{-1}\left\{\dfrac{1}{s-a}-\dfrac{1}{s+a}\right\}$

$=L^{-1}\left\{\dfrac{1}{s-a}\right\}-L^{-1}\left\{\dfrac{1}{s+a}\right\}$

$=\boxed{e^{at}-e^{-at}}$

26. **Evaluate** $\int\left(\dfrac{x^3+1}{x^5}\right)dx$

Sol. $\int\left(\frac{x^3+1}{x^5}\right)dx$

$= \int x^{-5}\left(x^3+1\right)dx$

$= \int\left(x^{-2}+x^{-5}\right)dx$

$= \frac{x^{-2+1}}{-2+1}+\frac{x^{-5+1}}{-5+1}+c$

$= \frac{x^{-1}}{-1}+\frac{x^{-4}}{-4}+c$

$= \boxed{-\frac{1}{x}-\frac{1}{4x^4}+c}$

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27. Find $\int\left(e^x+e^{2x}+e^{3x}\right)dx$

Sol. $= \int e^x dx+\int e^{2x}dx+\int e^{3x}dx$

$= \boxed{e^x+\frac{e^{2x}}{2}+\frac{e^{3x}}{3}+c}$

Section - II

Note: Attemp any three (3) questions $3\times 8 = 24$

Q.2.[a] Evaluate $\int\frac{1}{\sqrt{x+a}+\sqrt{x+b}}dx$

Sol. See Q.15 of Ex# 7.1 (Page # 287)

[b] Evaluate $\int(\tan x+\cot x)^2 dx$

Sol. See Q.10 of Ex# 7.2 (Page # 294)

Q.3.[a] Evaluate $\int\left(x\sqrt{x-a}\right)dx$

Sol. See Q.1(iii) of Ex# 8.1 (Page # 318)

[b] Evaluate $\int \ell n\left(x^2+1\right)dx$

Sol. See Q.3(iii) of Ex# 8.3 (Page # 346)

Q.4.[a] Evaluate $\int_2^3\left(\frac{x}{1+x^2}\right)dx$

Sol. See Q.1(iii) of Ex# 9.1 (Page # 375)

[b] ~~Find area bounded by y = 3x, y = x² between x = 1 and x = 3.~~

Sol. See Q.5 of Ex# 9.2 (Page # 390)

Q.5.[a] Find the general solution of

$$ydx = 2(xy+x)dy$$

Sol. See Q.6 of Ex# 10 (Page # 413)

[b] Evaluate $\int\left(\sin^3 x\right)dx$

Sol. See Q.4(i) of Ex# 8.1 (Page # 323)

Q.6. Find $L\{\sin wt\}$

Sol. See example # 06 of Chapter 12.

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