DAE / IA - 2016

MATH - 233 APPLIED MATHEMATICS - II

PAPER 'A' PART - A (OBJECTIVE)

Time: 30 Minutes   Marks: 15

Q.1: *Encircle the correct answer.*

1. If $f(x) = 3^x - 1$, then $f(3) = ?$

[a] 27 [b] 8 [c] 26 [d] 16

2. Which one is the periodic function:

[a] $x^2 + 1$ [b] $2x$
[c] $\sin x$ [d] $x^3 + 1$

3. ~~$\lim_{x \to \frac{\pi}{3}} (\cos x) = ?$~~

[a] $\frac{\sqrt{3}}{2}$ [b] $\frac{1}{2}$ [c] 0 [d] $\frac{1}{\sqrt{2}}$

4. $\frac{d}{dx}\left(\frac{1}{x}\right) = ?$

[a] $\frac{1}{x^2}$ [b] $-\frac{1}{x^2}$ [c] $-\frac{1}{x^3}$ [d] $\frac{2}{x}$

5. $\frac{d}{dx}\left(\sqrt{1+x}\right) = ?$

[a] $\frac{1}{\sqrt{1+x}}$ [b] $\frac{1}{2\sqrt{1+x}}$
[c] $(1+x)^{1/3}$ [d] $\frac{-1}{2\sqrt{1+x}}$

6. $\frac{d}{dx}(2\cos 3x) = ?$

[a] $6\sin 3x$ [b] $-6\sin 3x$
[c] $-6\cos 3x$ [d] $6\cos 3x$

7. ~~$\frac{d}{dx}\left(\sin^{-1}\sqrt{x}\right) = ?$~~

[a] $\frac{1}{\sqrt{1-x^2}}$ [b] $\frac{1}{\sqrt{1+x^2}}$
[c] $\frac{1}{2\sqrt{x}\sqrt{1-x}}$ [d] $\frac{1}{2\sqrt{x}(1-x)}$

8. ~~$\frac{d}{dx}\left(e^{\sin x}\right) = ?$~~

[a] $e^{\cos x}$ [b] $\cos x e^{\sin x}$
[c] $\sin x e^{\sin x - 1}$ [d] $\sin x e^{\sin x}$

9. ~~$\frac{d}{dx}\left(x^a\right) = ?$~~

[a] ax^{a-1} [b] x^{n-1} [c] ax^a [d] x^a

10. If 2^{nd} derivative is $+ve$ at a point, then function is:

[a] Maximum [b] Minimum
[c] Point of inflection
[d] None of these

11. For a decreasing function $\frac{dy}{dx}$ is:

[a] $+ve$ [b] $-ve$
[c] zero [d] None of these

12. $\frac{d}{dx}\left(\tan x^2\right) = ?$

[a] $2x\sec^2 x^2$ [b] $\sec^2 x^2$
[c] $\sec x^2$ [d] $\tan x^2$

13. A value which occurs maximum number of times in the data is called;

[a] Mean
[b] Geometric Mean
[c] Harmonic Mean [d] Mode

14. The collection of all possible outcomes of an experiment is called;

[a] Population [b] An event
[c] Trail [d] Sample space

15. A quantity whose value remains fixed is called;

[a] Constant [b] Variable
[c] Parameter [d] Function

Answer Key

1	d	2	b	3	a	4	b	5	b
6	b	7	c	8	b	9	a	10	b
11	b	12	a	13	d	14	d	15	a

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DAE / IA - 2016

MATH - 233 APPLIED MATHEMATICS - II

PAPER 'A' PART - B(SUBJECTIVE)

Time : 2 : 30Hrs Marks : 60

Section - I

Q.1. Write short answers to any Eighteen (18) questions.

1. if $f(x) = 3x^2 - 5x + 7$, find $f(4)$

Sol. $f(x) = 3x^2 - 5x + 7$

Put $x = 4$, we have :

$f(4) = 3(4)^2 - 5(4) + 7$

$f(4) = 48 - 20 + 7 = \boxed{35}$

2. Is the function even, odd or neither? $f(x) = x\sqrt{x^2 - 1}$

Sol. As, $f(x) = x\sqrt{x^2 - 1}$

Replace x by $-x$, we have :

$f(-x) = -x\sqrt{(-x)^2 - 1}$

$f(-x) = -x\sqrt{x^2 - 1}$

$f(-x) = -f(x)$

Hence $f(x)$ is an $\boxed{\text{odd}}$ function.

3. Evaluate $\lim\limits_{x \to 1} \dfrac{x^2 - 1}{x - 1}$

Sol. $\lim\limits_{x \to 1} \dfrac{x^2 - 1}{x - 1} \left(\dfrac{0}{0}\right)$ form

$= \lim\limits_{x \to 1} \dfrac{(x)^2 - (1)^2}{x - 1}$

$= \lim\limits_{x \to 1} \dfrac{(x - 1)(x + 1)}{(x - 1)}$

$= \lim\limits_{x \to 1} (x + 1) = 1 + 1 = \boxed{2}$

4. ~~Evaluate: $\lim\limits_{x \to 0} \dfrac{1 - \cos x}{\sin^2 x}$~~

Sol. $\lim\limits_{x \to 0} \dfrac{1 - \cos x}{\sin^2 x} \left(\dfrac{0}{0}\right)$ form

$= \lim\limits_{x \to 0} \dfrac{1 - \cos x}{\sin^2 x} \times \dfrac{1 + \cos x}{1 + \cos x}$

$= \lim\limits_{x \to 0} \dfrac{(1)^2 - (\cos x)^2}{\sin^2 x(1 + \cos x)}$

$= \lim\limits_{x \to 0} \dfrac{1 - \cos^2 x}{\sin^2 x(1 + \cos x)}$

$= \lim\limits_{x \to 0} \dfrac{\sin^2 x}{\sin^2 x(1 + \cos x)}$

$= \lim\limits_{x \to 0} \dfrac{1}{1 + \cos x}$

$= \dfrac{1}{1 + \cos 0} = \dfrac{1}{1 + 1} = \boxed{\dfrac{1}{2}}$

5. Differentiate w.r.t. 'x' $-5 + 3x - \dfrac{3}{2}x^2 - 7x^3$

Sol. $\dfrac{d}{dx}\left(-5 + 3x - \dfrac{3}{2}x^2 - 7x^3\right)$

$= 0 + 3(1) - \dfrac{3}{2}(2x) - 7(3x^2)$

$= \boxed{3 - 3x - 21x^2}$

6. Find $\dfrac{dy}{dx}$ if $x^{2/3} + y^{2/3} = a^{2/3}$

Sol. Differentiate both sides w.r.t. 'x' :

$\dfrac{d}{dx}\left(x^{2/3} + y^{2/3}\right) = \dfrac{d}{dx}\left(a^{2/3}\right)$

$\dfrac{2}{3}x^{-1/3} + \dfrac{2}{3}y^{-1/3}\dfrac{dy}{dx} = 0$

$\dfrac{2}{3}y^{-1/3}\dfrac{dy}{dx} = -\dfrac{2}{3}x^{-1/3}$

$\dfrac{dy}{dx} = \left(-\dfrac{2}{3}x^{-1/3}\right) \cdot \left(\dfrac{3}{2y^{-1/3}}\right)$

$\dfrac{dy}{dx} = -\dfrac{x^{-1/3}}{y^{-1/3}} \Rightarrow \boxed{\dfrac{dy}{dx} = -\dfrac{y^{1/3}}{x^{1/3}}}$

7. ~~Find $\frac{dy}{dx}$ when $x = at^2$, $y = 2at$~~

Sol. As, $x = at^2$, $y = 2at$

$$\frac{d}{dt}(x) = \frac{d}{dt}(at^2) \quad\Big|\quad \frac{d}{dt}(y) = \frac{d}{dt}(2at)$$

$$\frac{dx}{dt} = a(2t) \quad\Big|\quad \frac{dy}{dt} = 2a(1)$$

$$\frac{dx}{dt} = 2at \quad\Big|\quad \frac{dy}{dt} = 2a$$

By using Chain's Rule :

$$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx} = 2a \times \frac{1}{2at} = \boxed{\frac{1}{t}}$$

8. **Differentiate $\frac{x}{x^2+1}$ w.r.t. 'x'**

Sol. Differentiate w.r.t. 'x':

$$\frac{d}{dx}\left(\frac{x}{x^2+1}\right) \quad \{\text{using Quotient Rule}\}$$

$$= \frac{(x^2+1)\left(\frac{d}{dx}(x)\right) - x\left(\frac{d}{dx}(x^2+1)\right)}{(x^2+1)^2}$$

$$= \frac{(x^2+1)(1) - x(2x+0)}{(x^2+1)^2}$$

$$= \frac{x^2+1-2x^2}{(x^2+1)^2} = \boxed{\frac{1-x^2}{(x^2+1)^2}}$$

9. Differentiate $\sin(\tan x)$ w.r.t. 'x'.

Sol. $\frac{d}{dx}[\sin(\tan x)]$

$$= \cos(\tan x).\left(\frac{d}{dx}(\tan x)\right)$$

$$= \boxed{\cos(\tan x)\sec^2 x}$$

10. Find the derivative of $x^2 \sec 4x$

Sol. $\frac{d}{dx}(x^2 \sec 4x)$ $\left\{\begin{matrix}\text{using}\\ \text{Product Rule}\end{matrix}\right\}$

$$= \left(\frac{d}{dx}(x^2)\right)\sec 4x + x^2\left(\frac{d}{dx}(\sec 4x)\right)$$

$$= 2x\sec 4x + x^2 \sec 4x \tan 4x \frac{d}{dx}(4x)$$

$$= 2x\sec 4x + x^2 \sec 4x \tan 4x(4)$$

$$= 2x\sec 4x + 4x^2 \sec 4x \tan 4x$$

$$= \boxed{2x\sec 4x(1 + 2x\tan 4x)}$$

11. ~~**Differentiate**~~ $\sin^{-1} x^2$ ~~**w.r.t. 'x'.**~~

Sol. $\frac{d}{dx}(\sin^{-1}x^2)$

$$= \frac{1}{\sqrt{1-(x^2)^2}} \frac{d}{dx}(x^2)$$

$$= \frac{1}{\sqrt{1-x^4}}(2x) = \boxed{\frac{2x}{\sqrt{1-x^4}}}$$

12. Differentiate $\frac{x}{\ell nx}$ w.r.t. 'x'.

Sol. $\frac{d}{dx}\left(\frac{x}{\ell n x}\right)$ {using Quotient Rule}

$$= \frac{\ell n x.\left(\frac{d}{dx}(x)\right) - x\left(\frac{d}{dx}(\ell n x)\right)}{(l n x)^2}$$

$$= \frac{\ell n x.(1) - x\left(\frac{1}{x}\right)}{(\ell n x)^2} = \boxed{\frac{\ell n x - 1}{(\ell n x)^2}}$$

13. ~~**Find $\frac{dy}{dx}$ for $e^{\sqrt{x+1}}$**~~

Sol. Let $y = e^{\sqrt{x+1}}$

Differentiate both sides w.r.t. 'x':

$$\frac{d}{dx}(y) = \frac{d}{dx}\left(e^{\sqrt{x+1}}\right)$$

$$\frac{dy}{dx} = e^{\sqrt{x+1}} . \left(\frac{d}{dx}\left(\sqrt{x+1}\right)\right)$$

$$\frac{dy}{dx} = e^{\sqrt{x+1}} \cdot \frac{1}{2}(x+1)^{-1/2}\left(\frac{d}{dx}(x+1)\right)$$

$$\frac{dy}{dx} = e^{\sqrt{x+1}} \cdot \frac{1}{2\sqrt{x+1}}(1+0)$$

$$\boxed{\frac{dy}{dx} = \frac{e^{\sqrt{x+1}}}{2\sqrt{x+1}}}$$

14. **Find the derivative of $x\cot x$ w.r.t. 'x'.**

Sol. $\frac{d}{dx}(x\cot x)$ {using Product Rule}

$$= \left(\frac{d}{dx}(x)\right)\cot x + x\left(\frac{d}{dx}(\cot x)\right)$$

$$= 1.\cot x + x\left(-\csc^2 x\right)$$

$$= \boxed{\cot x - x\csc^2 x}$$

15. **Differentiate $x\ln 3x$ w.r.t. 'x'.**

Sol. $\frac{d}{dx}(x\ln 3x)$ {using Product Rule}

$$= \left(\frac{d}{dx}(x)\right)\ln 3x + x\left(\frac{d}{dx}(\ln 3x)\right)$$

$$= 1.\ln 3x + x\left(\frac{1}{3x}.\frac{d}{dx}(3x)\right)$$

$$= \ln 3x + x\left(\frac{1}{3x}(3(1))\right)$$

$$= \boxed{\ln 3x + 1}$$

16. ~~**Differentiate $\cos x$ w.r.t. $\tan x$.**~~

Sol. Let, $y = \cos x$ & $t = \tan x$

Differentiate both equations both sides w.r.t. 'x':

$$\frac{d}{dx}(y) = \frac{d}{dx}(\cos x) \qquad \frac{d}{dx}(t) = \frac{d}{dx}(\tan x)$$

$$\frac{dy}{dx} = -\sin x \qquad \frac{dt}{dx} = \sec^2 x$$

$$\frac{dx}{dt} = \frac{1}{\sec^2 x}$$

By using Chain's Rule: $\frac{dy}{dt} = \frac{dy}{dx} \times \frac{dx}{dt}$

$$\frac{dy}{dt} = (-\sin x)\left(\frac{1}{\sec^2 x}\right) = \boxed{-\sin x\cos^2 x}$$

17. **Find the critical values (a turning point) for x of the function $2x^4 - x^2$**

Sol. Let $y = 2x^4 - x^2$

Differentiate both sides w.r.t. 'x':

$$\frac{d}{dx}(y) = \frac{d}{dx}\left(2x^4 - x^2\right)$$

$$\frac{dy}{dx} = 2\left(4x^3\right) - 2x$$

$$\frac{dy}{dx} = 8x^3 - 2x$$

For critical values, put $\frac{dy}{dx} = 0$

$$8x^3 - 2x = 0$$

$$2x\left(4x^2 - 1\right) = 0$$

$$2x\left[(2x)^2 - (1)^2\right] = 0$$

$$2x(2x-1)(2x+1) = 0$$

$2x = 0$	$2x - 1 = 0$	$2x + 1 = 0$
$x = \frac{0}{2}$	$2x = 1$	$2x = -1$
$\boxed{x = 0}$	$\boxed{x = \frac{1}{2}}$	$\boxed{x = -\frac{1}{2}}$

18. **Find the maximum and minimum (extreme) values of the function $x^2(x-3)$.**

Sol. Let $y = x^2(x-3)$

$$y = x^3 - 3x^2 \quad \to (i)$$

Differentiate both sides w.r.t. 'x':

$$\frac{d}{dx}(y) = \frac{d}{dx}\left(x^3 - 3x^2\right)$$

$$\frac{dy}{dx} = 3x^2 - 3(2x)$$

$$\frac{dy}{dx} = 3x^2 - 6x \rightarrow (ii)$$

For critical values, put $\frac{dy}{dx} = 0$

$$3x^2 - 6x = 0$$
$$3x(x-2) = 0$$
$$3x = 0 \quad | \quad x - 2 = 0$$
$$\boxed{x = 0} \quad | \quad \boxed{x = 2}$$

Differentiate eq.(ii) both sides w.r.t. 'x':

$$\frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dx}(3x^2 - 6x)$$
$$\frac{d^2y}{dx^2} = 3(2x) - 6(1)$$
$$\frac{d^2y}{dx^2} = 6x - 6 \rightarrow (iii)$$

Put $x = 0$ in eq.(iii) & eq.(i)

$$\frac{d^2y}{dx^2} = 6(0) - 6 = 0 - 6 = -6 < 0$$
$$y_{max} = (0)^2(0-3)$$
$$\boxed{y_{max} = 0}$$

Put $x = 2$ in eq.(iii) & eq.(i)

$$\frac{d^2y}{dx^2} = 6(2) - 6 = 12 - 6 = 6 > 0$$
$$y_{min} = (2)^2(2-3)$$
$$\boxed{y_{min} = -4}$$

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19. **Find the mean of the scores 0, 1, 4, 5, 9, 9.**

Sol. $\text{Mean} = \frac{\sum x}{n} = \frac{0+1+4+5+9+9}{6}$

$$\bar{x} = \frac{28}{6} = \frac{14}{3} = \boxed{4\frac{2}{3}}$$

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20. **Define median.**

Sol. The value which divides an arrange data into two equal parts is called Median.

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21. ~~**Find standard deviation of the values:**~~ **2, 4, 6, 8, 10.**

Sol.

x	x^2
2	4
4	16
6	36
8	64
10	100
$\sum x = 30$	$\sum x^2 = 220$

$$S.D. = \sqrt{\frac{\sum x^2}{n} - \left(\frac{\sum x}{n}\right)^2}$$
$$\sigma = \sqrt{\left(\frac{220}{5}\right) - \left(\frac{30}{5}\right)^2}$$
$$\sigma = \sqrt{44 - 36}$$
$$\sigma = \sqrt{8} = \boxed{2.83}$$

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22. **If a die is rolled once, what is the probability of getting a 4?**

Sol. $S = \{1, 2, 3, 4, 5, 6\}$, $n(S) = 6$

Let A be event that getting number is 4. $A = \{4\}$

$$n(A) = 1 \therefore P(A) = \frac{n(A)}{n(S)} = \boxed{\frac{1}{6}}$$

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23. **If $f(x) = 3x^3 + 2x^2 - x + 4$, prove that: $2f(3) = 25f(1)$**

Sol. As, $f(x) = 3(x)^3 + 2(x)^2 - x + 4 \rightarrow (i)$

Put $x = 3$, in eq.(i):

$$f(3) = 3(3)^3 + 2(3)^2 - 3 + 4$$
$$f(3) = 81 + 18 - 3 + 4 = 100$$

Put $x = 1$, in eq.(i):

$$f(1) = 3(1)^3 + 2(1)^2 - 1 + 4$$
$$f(1) = 3 + 2 - 1 + 4 = 8$$
$$2f(3) = 25f(1)$$
$$2(100) = 25(8)$$
$$200 = 200$$

L.H.S. = R.H.S. **Proved.**

24. If $y=\sqrt{\tan x+\sqrt{\tan x+\sqrt{\tan x+\ldots\infty}}}$,

prove that: $(2y-1)\frac{dy}{dx}=\sec^2 x$

Sol.

$$\text{As, } y=\sqrt{\tan x+\sqrt{\tan x+\sqrt{\tan x+\ldots\infty}}}$$

Squaring both sides:

$$y^2=\left[\sqrt{\tan x+\sqrt{\tan x+\sqrt{\tan x+\ldots\infty}}}\right]^2$$

$$y^2=\tan x+\sqrt{\tan x+\sqrt{\tan x+\ldots\infty}}$$

$$y^2=\tan x+y\left\{\because \text{ Given that } y=\sqrt{\tan x+\sqrt{\tan x+\sqrt{\tan x+\ldots\infty}}}\right\}$$

$$y^2-y=\tan x$$

Differentiate both sides w.r.t. 'x':

$$\frac{d}{dx}(y^2-y)=\frac{d}{dx}(\tan x)$$

$$2y\frac{dy}{dx}-\frac{dy}{dx}=\sec^2 x$$

$$\boxed{(2y-1)\frac{dy}{dx}=\sec^2 x}$$ **Proved.**

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25. Differentiate $\left(x^2+3x+9\right)^{3/2}$ w.r.t. 'x'.

Sol.

$$\frac{d}{dx}\left(\left(x^2+3x+9\right)^{3/2}\right)$$

$$=\frac{3}{2}\left(x^2+3x+9\right)^{\frac{3}{2}-1}\left(\frac{d}{dx}\left(x^2+3x+9\right)\right)$$

$$=\frac{3}{2}\left(x^2+3x+9\right)^{1/2}\left(2x+3(1)+0\right)$$

$$=\boxed{\frac{3}{2}\left(x^2+3x+9\right)^{1/2}(2x+3)}$$

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26. ~~Find $\frac{dy}{dx}$ of $e^x \ell n x$~~

Sol. Let, $y=e^x\ell n x$

Differentiate both sides w.r.t. 'x':

$$\frac{d}{dx}(y)=\frac{d}{dx}\left(e^x\ell n x\right)$$

$$\frac{dy}{dx}=\frac{d}{dx}\left(e^x\right)\ell n x+e^x\frac{d}{dx}(\ell n x)$$

$$\frac{dy}{dx}=e^x\ell n x+e^x\left(\frac{1}{x}\right)$$

$$\boxed{\frac{dy}{dx}=e^x\left(\ell n x+\frac{1}{x}\right)}$$

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27. **The distance x meters moved by a point in t seconds is given by $x=t^3+3t^2+4$. Find the velocity and acceleration after 3 seconds.**

Sol. As, $x=t^3+3t^2+4$

$$v=\frac{dx}{dt}=\frac{d}{dt}\left(t^3+3t^2+4\right)$$

$$v=3t^2+3(2t)+0$$

$$v=3t^2+6t\rightarrow(i)$$

Differentiate eq.(i) both sides w.r.t. 'x':

$$a=\frac{dv}{dt}=\frac{d}{dt}\left(3t^2+6t\right)$$

$$a=3(2t)+6(1)$$

$$a=6t+6\rightarrow(ii)$$

Velocity after 3 seconds:

Put $t=3$ in eq.(i)

$$v=3(3)^2+6(3)$$

$$v=27+18$$

$$\boxed{v=45\text{ m/s}}$$

Acceleration after 3 seconds:

Put $t=3$ in eq.(ii)

$$v=6(3)+6$$

$$v=18+6$$

$$\boxed{a=24\text{ m/s}^2}$$

Section - II

Note: Attemp any three (3) questions $3\times 8 = 24$

Q.2.(a) Prove that: $f\left[f(x)\right] = x$, for the function $f(x) = \dfrac{x+1}{x-1}$

Sol. See Q.10 of Ex # 1.1 (Page # 9)

(b) Evaluate : $\lim\limits_{x\to 0} \dfrac{\sqrt{4+x}-2}{x}$

Sol. See Q.2(ii) of Ex # 1.2 (Page # 15)

Q.3.(a) Differentiate $\left(\dfrac{x+1}{x-1}\right)^2$ w.r.t. 'x'.

Sol. See Q.4(ix) of Ex # 2.2 (Page # 60)

(b) ~~Find $\dfrac{dy}{dx}$ for $e^{ax}\sin bx$.~~

Sol. See Q.2(vi) of Ex # 3.3 (Page # 146)

Q.4.(a) If $\sin y = x\sin(a+y)$, prove that: $\dfrac{dy}{dx} = \dfrac{\sin^2(a+y)}{\sin a}$

Sol. See Q.4 of Ex # 3.1 (Page # 119)

(b) ~~Differentiate~~ ~~$\tan^{-1}(\sec x + \tan x)$~~ ~~w.r.t.~~ 'x'.

Sol. See Q.2(iii) of Ex # 3.2 (Page # 130)

Q.5. Find the maximum and minimum (extreme) values of the following function. $(x-2)^2(x-1)$.

Sol. See Q.2(vi) of Ex # 4.2 (Page # 192)

Q.6. Compute mean from the data given:

Class Interval	Frequency
0 – 5	4
5 – 10	6
10 – 15	10
15 – 20	16
20 – 25	12
25 – 30	8
30 – 35	4

Sol. See Q.6 of Ex # 5.1 (Page # 233)

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