TAHIR MEHMOOD
M.Sc. Math
0345-6510779

CH # 6 (IInd Year)

Conic Section

(OBJECTIVE)

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Q. Write Standard Equation of Circle.

Ans. Let $C(h,k)$ be centre and r be radius of Circle then Standard Equation of Circle is defined as: $(x-h)^2+(y-k)^2=r^2$.

If $C(0,0)$ then Equation reduces to $x^2+y^2=r^2$.

Q. Find Centre and radius of General Equation of Circle.

Ans. The Equation $x^2+y^2+2gx+2fy+c=0$ is called General Equation of Circle. To find its centre and radius consider:

$$x^2+y^2+2gx+2fy+c=0$$

$$\Rightarrow x^2+2gx+y^2+2fy=-c$$

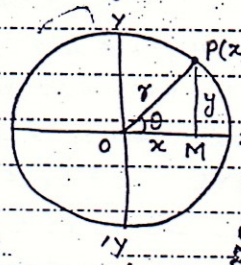
$$\Rightarrow x^2+2gx+g^2+y^2+2fy+f^2=f^2+g^2-c$$

$$\Rightarrow (x+g)^2+(y+f)^2=(\sqrt{f^2+g^2-c})^2$$

Thus centre $C(-g,-f)$ and radius $r=\sqrt{f^2+g^2-c}$

Q. Derive Parametric Equations of Circle.

Ans. Consider $O(0,0)$ be centre and "r" be radius of Circle. Let $P(x,y)$ be any point on it.



From diagram

$$\cos\theta=\frac{x}{r} \qquad \sin\theta=\frac{y}{r}$$

$$\Rightarrow x=r\cos\theta \quad \& \quad y=r\sin\theta \text{ for}$$

which are required Parametric Equations. $0\le\theta\le 2\pi$.

Q. Write down Charactristics of Circle Equation.

Ans. (i) It is Second degree Equation in two variables.

(ii) Coefficient of x^2 and Coefficient of y^2 are equal.

(iii) It is free from "xy" (product of x and y) term.

Q. Write Equation of Circle with centre $(-3,5)$ and radius 7.

Ans. Using Standard Equation $(x-h)^2+(y-k)^2=r^2$

$$\Rightarrow (x-(-3))^2+(y-5)^2=7^2 \Rightarrow x^2+9+6x+y^2+25-10y=49$$

$$\Rightarrow x^2+y^2+6x-10y-15=0 \text{ which is required.}$$

Q. Find Centre and radius of Circles: (i) $x^2+y^2+12x-10y=0$

(ii) $4x^2+4y^2-8x+12y-25=0$

Sol: (i) $x^2+y^2+12x-10y=0$

Comparing with $x^2+y^2+2gx+2fy+c=0$

$2g=12 \quad 2f=-10 \quad c=0$

$g=6 \quad f=-5 \quad c=0$

Centre $=C(-g,-f)=C(-6,5)$

Radius $=\sqrt{f^2+g^2-c}=\sqrt{25+36-0}$

$=\sqrt{61}$.

(ii) $4x^2+4y^2-8x+12y-25=0$

$\Rightarrow x^2+y^2-2x+3y-25/4=0$

Comparing with $x^2+y^2+2gx+2fy+c=0$

$2g=-2 \quad 2f=3 \quad c=-25/4$

$g=-1 \quad f=3/2 \quad c=-25/4$

Centre $=C(-g,-f)=C(1,-3/2)$

Radius $=\sqrt{f^2+g^2-c}=\sqrt{1+9/4+25/4}=\sqrt{38/4}$

$=\frac{\sqrt{38}}{2}$ or $\sqrt{19/2}$.

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Q. Show $3x-2y=0$ is tangent to Circle $x^2+y^2+6x-4y=0$.

Sol:- $\because x^2+y^2+6x-4y=0 \Rightarrow 2g=6 \quad 2f=-4 \quad c=0 \Rightarrow g=3, f=-2, c=0$

Centre $= C(-g,-f) = C(-3,2)$

Radius $= \sqrt{f^2+g^2-c} = \sqrt{9+4-0} = \sqrt{13}$

Now length of line from centre $(d) = \dfrac{|3(-3)-2(2)|}{\sqrt{9+4}} = \dfrac{|-13|}{\sqrt{13}} = \sqrt{13} = r$

So $3x-2y=0$ is tangent to Circle.

Q. Discuss the Position of $P(x_1,y_1)$ subject to $x^2+y^2+2gx+2fy+c=0$.

Sol:- If

(i) $x_1^2+y_1^2+2gx_1+2fy_1+c>0$ then $P(x_1,y_1)$ lies out Side of Circle.

(ii) $x_1^2+y_1^2+2gx_1+2fy_1+c=0$ then $P(x_1,y_1)$ lies on the Circle.

(iii) $x_1^2+y_1^2+2gx_1+2fy_1+c<0$ then $P(x_1,y_1)$ lies inside the Circle.

Q. Determine the location of $P(5,6)$ w.r.t. Circle $x^2+y^2=81$.

Sol:- Consider $x^2+y^2=81 \Rightarrow x^2+y^2-81=0$

Now $(5)^2+(6)^2-81 = 25+36-81 = -20<0$

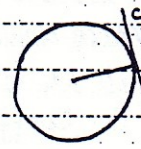
Thus $P(5,6)$ lies inside the Circle $x^2+y^2=81$.

Q. Under what Condition $y=mx+c$ will be tangent to $x^2+y^2=a^2$?

Sol:- $y=mx+c$ will be tangent to $x^2+y^2=a^2$

if $c^2=a^2(1+m^2)$ or $c=\pm a\sqrt{1+m^2}$

Then Equation of tangent will be $y=mx\pm a\sqrt{1+m^2}$.



Q. Find Equation of tangent to $x^2+y^2=25$ at $(4,3)$.

Sol:- Equation of tangent to $x^2+y^2=25$ is $xx_1+yy_1=25$

Thus $4x+3y=25$ is Eq of tangent.

Q. Find the length of tangent to $5x^2+5y^2+14x+12y-10=0$ from $P(-5,10)$.

Sol:- Length of tangent $|PT| = \sqrt{x_1^2+y_1^2+2gx_1+2fy_1+c}$

Now

$$5x^2+5y^2+14x+12y-10=0 \Rightarrow x^2+y^2+\frac{14}{5}x+\frac{12}{5}y-2=0$$

So $|PT| = \sqrt{(-5)^2+(10)^2+\frac{14}{5}(-5)+\frac{12}{5}(10)-2}$

$= \sqrt{25+100-14+24-2} = \sqrt{133}$.

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Q. Define "Parabola", "Focus", "Vertex" and Directrix.

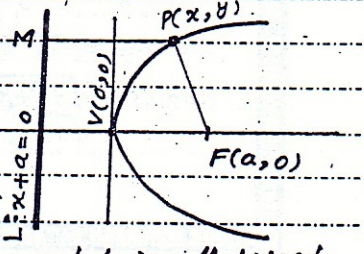
Parabola:- The set of all those points which are equidistant from a fixed point to a fixed line is called Parabola.

Focus:- The fixed point $F(a,0)$ is called focus.

Directrix:- The fixed line $x+a=0$ is called Directrix.

Vertex:- The point where parabola touches the axis of parabola is called vertex.

Axis:- The line through focus perpendicular to directrix is called Axis of Parabola.



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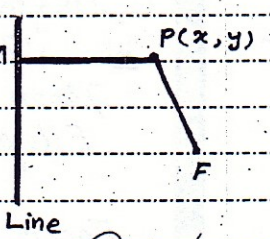
Q. Define "Eccentricity".

Ans. It is a ratio of the distance from a fixed point to a fixed line of a given point. It is denoted by e where $e = \frac{|PF|}{|PM|} > 0$



Q. How can you classify Conics by means of eccentricity?

Sol:-

If $e = 1$ then Conic is Parabola.

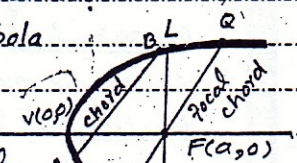
If $0 < e < 1$ then Conic is Ellipse.

If $e > 1$ then Conic is Hyperbola.

Q. What is chord, focal chord and Latus Rectum?

Ans. The line segment joining the two points of parabola is called Chord and the chord through the Focus is called Focal chord.

The focal chord perpendicular to Axis of parabola is called Latus Rectum. $\bar{LL'}$ is L.R.P.



Q. Discuss standard Parabolas.

Equations	Vertex	Focus	Axis	Directrix	Length of L.R.	Graph.
$y^2 = 4ax$	$V(0,0)$	$F(a,0)$	$y=0$	$x+a=0$	$4a$	
$y^2 = -4ax$	$V(0,0)$	$F(-a,0)$	$y=0$	$x-a=0$	$4a$	
$x^2 = 4ay$	$V(0,0)$	$F(0,a)$	$x=0$	$y+a=0$	$4a$	
$x^2 = -4ay$	$V(0,0)$	$F(0,-a)$	$x=0$	$y-a=0$	$4a$	

Q. Write Parametric Equations of Parabola.

Ans. Parametric Equations are $x = at^2$ and $y = 2at$ for $-\infty < t < \infty$.

Q. Find Vertex, Focus, Directrix and Length of L.R. of $y^2 = 8x$

Sol:- $y^2 = 8x$ can be written as: $(y-0)^2 = 8(x-0)$

Comparing with $(y-k)^2 = 4a(x-h)$

$\Rightarrow V(h,k) = V(0,0)$ and $4a = 8$ (L.R.) so $a = 2$

Focus $F(a,0) = F(2,0)$ Directrix: $x+a=0 \Rightarrow x+2=0$

Q. Find the length of Latus Rectum of: $x^2 - 4x - 8y + 4 = 0$

Sol:- $x^2 - 4x - 8y + 4 = 0 \Rightarrow x^2 - 4x + 4 = 8y$

$\Rightarrow (x-2)^2 = 8(y-0)$

Comparing with $(x-h)^2 = 4a(y-k)$

$\Rightarrow$ Vertex $= V(h,k) = V(2,0)$

Length of Latus Rectum $= 4a = 8$.

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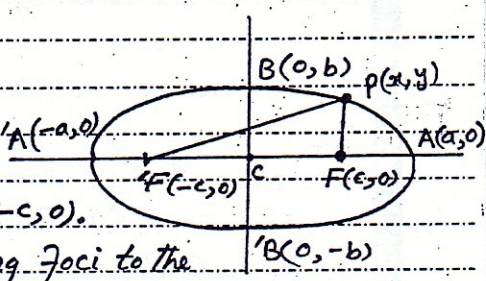
Q: Define "Ellipse" with its particulars:

Sol:- The set of all those points whose sum of the distances from the two fixed points remains constant is called Ellipse.

$$|PF| + |P'F| = \text{Constant } (2a)$$



Foci:- Fixed points from where distances of point P are taken are known as Foci $F(c,0)$, $'F(-c,0)$.

Major Axis:- The line segment joining Foci to the ends of Ellipse is called Major Axis having length "2a".

Minor Axis:- The line segment through centre and perpendicular to major axis is called Minor Axis having Length "2b"

Vertices:- The points on major axis where ellipse touches it are called vertices $A(a,0)$, $'A(-a,0)$.

Co-vertices:- The points on minor axis where ellipse touches it are called Co-vertices $B(0,b)$, $'B(0,-b)$.

Centre:- The mid point of Foci (vertices or Co-vertices) is called Centre $C(0,0)$.

Equation:- $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ for $a > b$ and $c^2 = a^2 - b^2$

Directrices:- $x = \pm \frac{a}{e} = \pm \frac{a^2}{c}$

Eccentricity:- $e = c/a$ where $0 < e < 1$

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Q: Find centre, vertices, Co-vertices, Length of major and minor axis, Foci, Directrices, eccentricity of $\frac{x^2}{16} + \frac{y^2}{9} = 1$.

Sol:- Consider $\frac{x^2}{4^2} + \frac{y^2}{3^2} = 1$

Here $a^2 = 4^2 \Rightarrow a = 4$ $\quad b^2 = 3^2 \Rightarrow b = 3$

$c^2 = a^2 - b^2 = 16 - 9 = 7 \Rightarrow c = \sqrt{7}$

Centre: $C(0,0)$ $\quad$ Vertices: $A(\pm a, 0) = A(\pm 4, 0)$

Co-vertices: $B(0, \pm b) = B(0, \pm 3)$

Length of major axis $= 2a = 2(4) = 8$

Length of minor axis $= 2b = 2(3) = 6$

Foci: $F(\pm c, 0) = F(\pm\sqrt{7}, 0)$

Eccentricity: $e = c/a = \frac{\sqrt{7}}{4}$

Directrices: $x = \pm \frac{a}{e} = \pm \frac{4}{(\sqrt{7}/4)} = \pm \frac{16}{\sqrt{7}}$.

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Q. Find Centre, Vertices, Co-vertices, Length of major and minor axis, Foci, Directrices, eccentricity of: $9x^2+4y^2-18x+8y-23=0$.

Sol:- $9x^2-18x+4y^2+8y-23=0$

$\Rightarrow 9x^2-18x+9+4y^2+8y+4 = 23+9+4$

$\Rightarrow 9(x^2-2x+1)+4(y^2+2y+1)=36$

$\Rightarrow \frac{(x-1)^2}{4}+\frac{(y+1)^2}{9}=1 \Rightarrow \frac{(y+1)^2}{3^2}+\frac{(x-1)^2}{2^2}=1.$

Centre: $C(1,-1)$ $a=3$ $b=2$ $c^2=a^2-b^2=9-4=5$

Vertices: $A(1,-1\pm 3)=A(1,-1\pm 3)$ $c=\sqrt{5}$

Co-vertices: $B(1\pm 2,-1)$

Length of Major axis $=2a=2(3)=6$

Length of Minor axis $=2b=2(2)=4$

Foci: $F(1,-1\pm\sqrt{5})$

Eccentricity $= e = c/a = \frac{\sqrt{5}}{3}$

Directrices: $y+1=\pm\frac{a}{e} \Rightarrow y+1=\pm\frac{3}{(\sqrt{5}/3)}=\pm\frac{9}{\sqrt{5}}$

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Q. Write parametric Equations of Ellipse.

Ans. $x=a\cos\theta$, $y=b\sin\theta$. for $0\le\theta\le 2\pi$.

Q. Find the length of Latus Rectum of $\frac{x^2}{25}+\frac{y^2}{16}=1$

Sol:- $\because a^2=25$ $b^2=16$ so $a=5$ $b=4$

Length of Latus Rectum $|L'L| = \frac{2b^2}{a} = \frac{2(16)}{5}=\frac{32}{5}$.

Q. What do you mean by Latus Rectum?

Sol:- The focal chord perpendicular to major axis is called Latus Rectum $L'L$ having length $\frac{2b^2}{a}$.

Q. Show that Circle is a Special Case of Ellipse.

Sol:- Let $a=b$ then $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

can be written as:

$$\frac{x^2}{a^2}+\frac{y^2}{a^2}=1$$

$$x^2+y^2=a^2$$

which is a Circle.

Thus if $a=b$ then Ellipse reduces to Circle.

Q. Find the equation of Ellipse whose centre $(0,0)$ and vertices $(\pm 5,0)$ and Foci $F(\pm 3,0)$

Sol:- $\because a=5$ and $c=3$ so $b^2=a^2-c^2=25-9=16 \Rightarrow b=4$.

So equation is $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1 \Rightarrow \frac{x^2}{25}+\frac{y^2}{16}=1$.

CH #6 (IInd Year).

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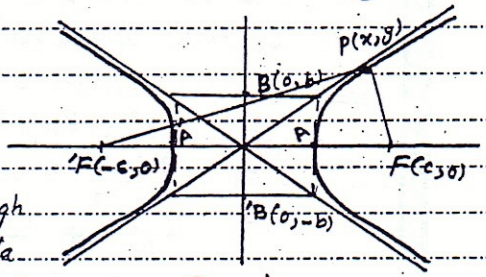
M.Sc. Math
0345-6510779

Q. Define "Hyperbola" with its particulars.

Ans. The set of All those points in a plane whose difference of the distances from the 2 fixed points remain Constant is called Hyperbola.

$|PF| - |PF'| = 2a$ (Constant).



Foci:- The fixed points from where the distances are taken are called Foci.

$F(c,0)$, $'F(-c,0)$

Transverse Axis:- The line segment through Foci that joins the two points of hyperbola is called Transverse Axis having length "2a" also called Focal Axis.

Conjugate Axis:-

The line segment joining two points of Hyperbola imaginarly and through centre perpendicular to Transverse Axis is called Conjugate Axis having Length, "2b".

Vertices:- The points on Transverse axis where hyperbola meets are called vertices $A(a,0)$, $'A(-a,0)$.

Co-Vertices:- The points on Conjugate Axis where hyperbola meets imaginarly are called Co-vertices. $B(0,b)$, $'B(0,-b)$.

Centre:- The mid point of Foci (vertices/Covertices) is called Centre $C(0,0)$.

Equation:- $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ where $c^2 = a^2 + b^2$

Eccentricity:- $e = \frac{c}{a}$ where $e > 1$

Directrices:- $x = \pm\frac{a}{e} = \pm\frac{a^2}{c}$.

Equations of Asymptotes:- $y = \pm\frac{b}{a}x$

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If $a = b$ then hyperbola is called "Rectangular Hyperbola".

Parametric Equations:- $x = a\sec\theta$ $y = b\tan\theta$.

where $0 \le \theta \le 2\pi$.

Length of Latus Rectum $LL' = \frac{2b^2}{a}$.

Q. What are Central Conics?

Ans. The Conics which have Centre are called Central Conics such as Ellipse and Hyperbola, Circle.

Q. Find Foci interms of eccentricity.

Ans: $\because e = \frac{c}{a}$ So $c = ae$ Thus $F(ae,0)$, $'F(-ae,0)$.

CH #6 (IInd Year)

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Q. Find Centre, Vertices, Co-vertices, Length of Transverse and Conjugate Axis, Foci, Eccentricity, Directrices, Equations of Asymptote and Length of Latus Rectum of $\frac{x^2}{25} - \frac{y^2}{16} = 1$.

Sol:-

$\because \frac{(x-0)^2}{5^2} - \frac{(y-0)^2}{4^2} = 1$

Here $a^2 = 5^2 \Rightarrow a = 5$ $\quad b^2 = 4^2 \Rightarrow b = 4$

$c^2 = a^2 + b^2 = 25 + 16 = 41 \Rightarrow c = \sqrt{41}$.

Centre: $C(0,0)$

Vertices: $A(\pm a, 0) = A(\pm 5, 0)$

Co-Vertices: $B(0, \pm b) = B(0, \pm 4)$

Length of Transverse axis $(2a) = 2(5) = 10$

Length of Conjugate axis $(2b) = 2(4) = 8$

Foci: $F(\pm c, 0) = F(\pm \sqrt{41}, 0)$.

Eccentricity $(e) = \frac{c}{a} = \frac{\sqrt{41}}{5}$

Directrices: $x = \pm \frac{a}{e} \Rightarrow x = \pm \frac{5}{(\sqrt{41}/5)} = \pm \frac{25}{\sqrt{41}}$.

Equations of asymptotes: $y = \pm \frac{b}{a} x$

$y = \pm \frac{4}{5} x$

$y = \frac{4}{5} x \Rightarrow 4x - 5y = 0$ $\quad y = -\frac{4}{5} x \Rightarrow 4x + 5y = 0$

Length of Latus rectum $= \frac{2b^2}{a} = \frac{2(16)}{5} = \frac{32}{5}$.

Q. What is Equation of tangent to $y^2 = 4ax$, $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ at $P(x_1, y_1)$

Sol:-

$y^2 = 4ax$ $\quad yy_1 = 2a(x + x_1)$

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ $\quad \frac{xx_1}{a^2} + \frac{yy_1}{b^2} = 1$

$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ $\quad \frac{xx_1}{a^2} - \frac{yy_1}{b^2} = 1$

Q. Find Equation tangent to $y^2 = 4ax$ at $P(at^2, 2at)$

Sol:- $\because yy_1 = 2a(x + x_1)$

$\Rightarrow y(2at) = 2a(x + at^2) \Rightarrow yt = x + at^2$

$x - yt + at^2 = 0$

Q. Find Eq of tangent to $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ at $P(a\cos\theta, b\sin\theta)$.

Sol:- $\because \frac{xx_1}{a^2} + \frac{yy_1}{b^2} = 1$

$\Rightarrow \frac{x}{a^2}(a\cos\theta) + \frac{y}{b^2}(b\sin\theta) = 1 \Rightarrow \frac{x}{a}\cos\theta + \frac{y}{b}\sin\theta = 1$.

CH # 6 (IInd Year).

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Q. Find Eq of tangent to $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ at $P(a\sec\theta, b\tan\theta)$.

Sol:-

$\because \quad \frac{xx_1}{a^2} - \frac{yy_1}{b^2} = 1$

$\frac{x}{a^2}(a\sec\theta) - \frac{y}{b^2}(b\tan\theta) = 1 \Rightarrow \frac{x}{a}\sec\theta - \frac{y}{b}\tan\theta = 1.$

Q. Under what Conditions $y = mx + c$ is tangent to (i) $y^2 = 4ax$

(ii) $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ (iii) $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

Sol:-

$y = mx + c$ will be tangent to

(i) $y^2 = 4ax$ if $c = a/m$ Eq of tangent: $y = mx + a/m$ $m \neq 0$

(ii) $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ if $c^2 = a^2m^2 + b^2$ Eq of tangent: $y = mx \pm \sqrt{a^2m^2 + b^2}$

(iii) $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ if $c^2 = a^2m^2 - b^2$ Eq of tangent: $y = mx \pm \sqrt{a^2m^2 - b^2}$.

Q. What is the general Equation of Conics without Rotation?

Ans:- The Eq $Ax^2 + By^2 + Gx + Fy + C = 0$ is called general Equation of Conics without rotation.

It will represent a

(i) Circle if $A = B$ but $A \neq 0, B \neq 0$

(ii) Parabola if $A = 0$ or $B = 0$ but not Both zero at a time.

(iii) Ellipse if $A \neq B$ having Same Sign.

(iv) Hyperbola if $A \neq B$ having opposite Sign.

Q. Discuss the nature of Conic: $5x^2 + 5y^2 + 4x + 8y + 7 = 0$

Ans:- $\because A = 5 \quad B = 5$ so Conic is Circle as $A = B$.

Q, Discuss the nature of Conic: $5x^2 + 15x + 14y - 21 = 0$

Ans:- $\because A = 5$ and $B = 0$ so it is parabola.

Q. Discuss the nature of Conic: $5x^2 + 3y^2 - 7x + 15y - 13 = 0$

Ans: $\because A = 5$ and $B = 3$ and have same Sign so it is Ellipse.

Q. Discuss the nature of Conic: $5x^2 - 3y^2 - 7x + 15y - 13 = 0$

Ans:- $\because A = 5$ and $B = -3$ and have opposite Sign so it is Hyperbola.

Q. What is the general Equation of Conics with rotation?

Ans: The Equation $ax^2 + by^2 + 2hxy + 2gx + 2fy + c = 0$ is called general Equation of Conics with rotation. xy is called rotation variable.

Q. What is discriminant of Conic Equation?

Ans. The quantity $h^2 - ab$ is called discriminant of the Equation of Conic.

CH #6 (IInd Year).

TAHIR MEHMOOD
M.Sc. Math
0345-6510779

Q. How can you identify a Conic by discriminant?

Ans: $ax^2+by^2+2gx+2fy+2hxy+c=0$ will be

(i) Circle or Ellipse if : $h^2-ab<0$ or $h^2<ab$

(ii) Parabola if $h^2-ab=0$ or $h^2=ab$

(iii) Hyperbola if $h^2-ab>0$ or $h^2>ab$.

Q. How can you measure angle of rotation?

Ans: Angle of rotation can be measere as

$$\tan 2\theta = \frac{2h}{a-b} \quad \text{where } 0<\theta<\pi/2.$$

If $a=b$ then $\theta = 45^\circ$.

Q. Under what Condition $ax^2+by^2+2gx+2fy+2hxy+c=0$ is a pair of Straight Line?

Ans. If $\begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix} = 0$ then it represents a pair of Straight Lines having angle θ where

$$\tan\theta = \frac{2\sqrt{h^2-ab}}{a+b}$$

Q. Show $10xy+8x-15y-12=0$ is pair of St. Lines.

Ans: $\because a=0=b \quad 2h=10 \quad 2g=8 \quad 2f=-15 \quad c=-12$

$\Rightarrow a=0=b \quad h=5 \quad g=4 \quad f=-15/2 \quad c=-12$

Consider

$$\begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix} = \begin{vmatrix} 0 & 5 & 4 \\ 5 & 0 & -15/2 \\ 4 & -15/2 & -12 \end{vmatrix}$$

$= 0-5\left(-60+30\right)+4\left(\frac{-75}{2}-0\right) = 0-5(-30)-75(2)$

$= 150-150 = 0$

So it is a pair of St. Lines.

Q. Find Equation of tangent to $3x^2-7y^2+2x-y-48=0$ at $P(4,1)$.

Sol:- Eq of tangent is $3xx_1-7yy_1+(x+x_1)-\frac{1}{2}(y+y_1)-48=0$

$3x(4)-7y(1)+(x+4)-\frac{1}{2}(y+1)-48=0$

$\Rightarrow 24x-14y+2x+8-y-1-96=0$ Multiplying by 2

$\Rightarrow 26x-15y-89=0$

Q. Discuss the nature of $25x^2+9y^2+50x-36y+24xy-164=0$

Ans: $\because a=25 \quad b=9 \quad 2h=24 \Rightarrow h=12$

So $h^2-ab = (12)^2-(25)(9) = 144-225 = -81 < 0$

So Conic is Ellipse or Circle.

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Encircle the Correct Answer.

(i) $(x-h)^2+(y-k)^2=r^2$ is called Standard Eq. of _____.

a- ✓ Circle b- Parabola c- Ellipse d- Hyperbola.

(ii) Centre of $x^2+y^2+12x+13y+21=0$ is _____.

a- $(0,0)$ b- $(12,13)$ c- $(-12,-13)$ d- ✓ $(-6,-13/2)$

(iii) Parametric Equations of Circle are _____.

a- ✓ $x=a\cos\theta$, $y=a\sin\theta$ b- $x=a\sin\theta$, $y=b\cos\theta$

c- $x=a\cos\theta$, $y=b\sin\theta$ d- $x=a\sec\theta$ $y=b\tan\theta$.

(iv) Radius of $x^2+y^2+2x-4y+3=0$ is _____

a- $2\sqrt{2}$ b- 2 ✓ c- $\sqrt{2}$ d- $\sqrt{17}$.

(v) The _____ Equation of Circle is $x^2+y^2+2gx+2fy+c=0$

a- Standard ✓ b- General c- Parametric d- None of them

(vi) Centre of $x^2+y^2+2gx+2fy+c=0$ is _____.

a- (g,f) b- (f,g) ✓ c- $(-g,-f)$ d- $(-f,-g)$.

(vii) Radius of $x^2+y^2+2gx+2fy+c=0$ is _____.

✓ a- $\sqrt{f^2+g^2-c}$ b- $\sqrt{f^2+g^2+c}$ c- $\sqrt{f^2+g^2+c^2}$ d- $\sqrt{f^2+g^2-c^2}$

(viii) Equation Circle with centre $(0,0)$ and radius "a" is _____.

✓ a- $x^2+y^2=a^2$ b- $(x-h)^2+(y-k)^2=a^2$ c- $x^2+y^2=r^2$ d- $ax^2+ay^2=a^2$.

(ix) Equation of point Circle is _____.

a- $x^2+y^2=a^2$ b- $x^2+y^2=r^2$ ✓ c- $x^2+y^2=0$ d- $x^2+y^2=a$

(x) Equation of _____ has not term "xy".

a- ✓ Circle b- Parabola c- Hyperbola d- Ellipse.

(xi) $S=S(C,r)=\{P(x,y) \mid |CP|=r\}$ is the set notation of _____.

a- Parabola ✓ b- Circle c- Ellipse d- Hyperbola.

(xii) Distance of tangent from Centre of Circle is equal to _____.

a- ✓ Radius b- Diameter c- Chord d- Tangent.

(xiii) $P(x_1,y_1)$ lies out side the Circle if $x_1^2+y_1^2+2gx_1+2fy_1+c$ _____

✓ a- >0 b- $=0$ c- <0 d ≤ 0

(xiv) Point _____ lies inside the Circle $x^2+y^2=25$

a- $(-3,5)$ ✓ b- $(-3,2)$ c- $(-4,5)$ d- $(-4,3)$.

(xv) $y=mx+c$ will be tangent to $x^2+y^2=a^2$ if _____.

a- $c=\sqrt{1+m^2}$ ✓ b- $c^2=a^2(1+m^2)$ c- $c^2=(1+m^2a^2)$ d- $a^2=c^2$

(xvi) $y=mx+c$ can intersect the Circle at the most at _____ Point/Points.

a- 1 ✓ b- 2 c- 3 d- 4

(xvii) Horizontal tangent on $x^2+y^2=a^2$ will be _____.

a- $y=a$ b- $y=-a$ c- $x=a$ ✓ d- a,b both

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(xviii) Length of diameter to $x^2+y^2=a^2$ is _____
a- a b- a^2 c- $2a^2$ ✓d- 2a

(xix) Angle inscribed in semicircle is _____
a- Acute ✓b- Right c- Abtuse d- Reflex.

(xx) If $0<e<1$ then Conic is _____
a- Circle b- Parabola ✓c- Ellipse d- Hyperbola.

(xx) The eccentricity e of parabola is _____
a- $e<1$ ✓b- $e=1$ c- $e>1$ d- $e<0$

(xxi) Focus of $y^2=-4ax$ is _____.
a- F(a,0) ✓b- F(-a,0) c- F(0,a) d- F(0,-a)

(xxii) Directrix of $x^2=4ay$ is _____.
a- $x=a$ b- $x=-a$ c- $y=a$ ✓d $y=-a$.

(xxiii) Length of Latus Rectum of $(y-2)^2=16x-8$ is _____.
a- ✓16 b- 8 c- 4 d- 1/2

(xxiv) Focus of $y^2=-8(x-2)$ is _____
a- ✓(0,0) b- (0,2) c- (2,0) d- (-2,0).

(xxv) Parametric Equations of Parabola are _____
a- $x=at$ $y=2at^2$ b- $x=2at^2$ $y=at$ c- $x=at^2$ $y=2at^2$ d- ✓$x=at^2, y=2at$

(xxvi) If $0<e<1$ then Conic will be _____.
a- Circle b- Parabola c- Hyperbola ✓d- Ellipse

(xxvii) In Ellipse ____ $=a^2(1-e^2)$
a- a^2 b- ✓b^2 c- c^2 d- b^2+c^2

(xxviii) Length of major axis in $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is _____
a- 2b b- ✓2a c- 2c d- 4a

(xxix) Ellipse and Hyperbola are _____ Conics.
a- ✓Central b- Radial c- Space d- Linear.

(xxx) Length of major axis in $\frac{x^2}{25}+\frac{y^2}{16}=1$ is _____
a- 25 b- 16 ✓c- 10 d- 8

(xxxi) Directrices of Ellipse are $x=$ _____
a- $\pm\frac{c}{a}$ b- $\pm a/c$ c- $\pm a/e^2$ ✓d- $\pm\frac{a^2}{c}$

(xxxii) Centre of $\frac{2x+1}{16}+\frac{y-3}{9}=1$ is _____
a- (-1, 3) b- (2, 3) c- (-2, 3) ✓d- (-1/2, 3)

(xxxiii) Length of Latus rectum of $\frac{x^2}{25}+\frac{y^2}{9}=1$ is _____
a- $\frac{9}{5}$ b- ✓$\frac{18}{5}$ c- $5/9$ d- $\frac{25}{4}$

(xxxiv) Locus of points $P(x,y)$ such that $|PF|+|PF'|=2a$ is _____.
a- Circle b- Parabola ✓c- Ellipse d- Hyperbola.

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(xxxv) The Equation of tangent to Circle $x^2+y^2=a^2$ at $P(x_1,y_1)$ is _____
a- $xx_1+yy_1=a$ b- $x_1^2+y_1^2=a^2$ ✓c- $xx_1+yy_1=a^2$ d- $x^2x_1^2+y^2y_1^2=a^2$

(xxxvi) In hyperbola _____
a- $c^2=a^2-b^2$ b- ✓ $c^2=a^2+b^2$ c- $b^2=a^2+c^2$ d- $c^2+a^2+b^2=0$

(xxxvii) $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ is the Equation of _____
a- Ellipse b- Circle c- Parabola ✓d- Hyperbola

(xxxviii) Hyperbola has _____ branch / branches.
a- One ✓b- two c- Three d- Four.

(xxxix) Length of Latus rectum of Hyperbola is _____.
a- $\frac{2b^2}{a^2}$ b- $\frac{2b}{a^2}$ ✓c- $\frac{2b^2}{a}$ d- $\frac{2b}{a}$

(xxxx) Equations of Asymptotes of $\frac{x^2}{16}-\frac{y^2}{25}=1$ are _____
a- $y=\pm\frac{4}{5}x$ b- ✓ $y=\pm\frac{5}{4}x$ c- $y=\pm\frac{3}{4}x$ d- $y=\pm 3/5\,x$

(xxxxi) Equation of directrix of $x^2=-4ay$ is _____
a- $x+a=0$ b- $x-a=0$ c- $y+a=0$ ✓d- $y-a=0$

(xxxxii) $y=mx+c$ will be tangent to $y^2=4ax$ if _____
a- $c=ma$ b- $\frac{c}{m}=a$ c- $a=mc^2$ ✓d- $c=a/m$

(xxxxiii) $y=mx+c$ will be tangent to $\frac{x^2}{a^2}+\frac{y^2}{a^2}=1$ if _____
a- $c^2=a^2+b^2m^2$ b- $c^2=a^2+b^2$ ✓c- $c^2=a^2m^2+b^2$ d- $a^2=b^2+m^2c^2$

(xxxxiv) Length of Conjugate axis of $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ is _____.
a- $2c$ b- $2a$ c- $2ab$ ✓d- $2b$

(xxxxv) Hyperbola has _____ Foci.
a- One ✓b- Two c- Three d- None

(xxxxvi) $Ax^2+By^2+Gx+Fy+C=0$ is Circle if _____
a- ✓ $A=B$ b- $G=F$ c- $A=C$ d- $B=C$

(xxxxvii) $Ax^2+By^2+Gx+Fy+C=0$ is Parabola if _____
a- $A=B$ b- $A=0=B$ c- $A\neq 0\neq B$ ✓d- $A=0$ or $B=0$

(xxxxviii) $5x^2+5y^2+10x+15y+21=0$ is a/an _____ Equation.
a- Parabola b- Ellipse c- ✓ Circle d- Hyperbola

(xxxxix) $ax^2+by^2+2hxy+2gx+2fy+c=0$ is Parabola if _____
a- $h^2-ab<0$ ✓b- $h^2-ab=0$ c- $h^2-ab>0$ d- $h^2-gf=0$

(xxxxx) Angle of rotation θ is determine as: $\tan 2\theta=$ _____.
a- $\frac{2\sqrt{h^2-ab}}{a+b}$ b- ✓ $\frac{2h}{a-b}$ c- $\frac{h^2}{a-b}$ d- $\frac{\sqrt{h^2-ab}}{a+b}$

(xxxxxi) If $a=b$ then angle of rotation is _____.
a- $30°$ ✓b- $45°$ c- $60°$ d- $90°$

(xxxxxii) $h^2-ab>0$ then Conic is _____.
a- Circle b- Ellipse c- Parabola ✓d- Hyperbola

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(53) $h^2-ab=0$ Then Conic is _____.

a- Parabola ✓ b- Circle c- Ellipse d- Hyperbola.

(54) Discriminant of $ax^2+by^2+2gx+2fy+2hxy+c=0$ is _____.

a- h^2-2ab b- h^2+ab c- h^2+2ab d- ✓ h^2-ab

(55) $ax^2+by^2+2hxy+2gx+2fy+c=0$ reduces to pair of Lines if _____.

a- $\begin{vmatrix} a & h & g \\ h & g & f \\ h & g & f \end{vmatrix}=0$ b- $\begin{vmatrix} a & f & g \\ f & b & h \\ g & h & c \end{vmatrix}=0$ ✓ c- $\begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix}=0$ d- $\begin{vmatrix} c & f & g \\ f & a & h \\ g & h & b \end{vmatrix}=0$

(56) Angle of rotation θ where _____.

a- $0<\theta<\pi$ ✓ b- $0<\theta<\pi/2$ c- $\pi/2<\theta<\pi$ d- $-\pi<\theta<\pi$

(57) $5x^2+7y^2+10x+7y+5=0$ is _____.

a- Circle b- Parabola ✓ c- Ellipse d- Hyperbola.

(58) Radius of $x^2+y^2+4x-6y+15=0$ is _____.

a- $(-2,-3)$ ✓ b- $(-2,3)$ c- $(-3,-2)$ d- $(-3,2)$

(59) $(x-x_1)(x-x_2)+(y-y_1)(y-y_2)=0$ is the Equation of _____.

a- Prabola b- Ellipse c- ✓ Circle d- Hyperbola

(60) $2x^2+3y^2-10xy+4x-6y+31=0$ is _____.

a- Circle b- Ellipse c- Parabola ✓ d- Hyperbola

(61) $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ has vertices _____ for $a>b$.

a- $A(0,\pm a)$ b- ✓ $A(\pm a,0)$ c- $A(a,-a)$ d- $A(-a,a)$

(62) A Circle is called unit Circle if _____.

a- $r=0$ ✓ b- $r=1$ c- $r<0$ d- $r>0$

(63) _____ is/are fixed point/points of Ellipse.

a- Centre b- Vertices c- Co-vertices ✓ d- Foci.

(64) Centre of $\frac{(x-h)^2}{a^2}-\frac{(y-k)^2}{b^2}=1$ is _____.

a- $(0,0)$ ✓ b- (h,k) c- (k,h) d- $(0,k)$

(65) _____ Tangents can be drawn on circle from any point outside.

a- One ✓ b- Two c- Three d- Infinite.

(66) $x=a\cos\theta$, $y=b\sin\theta$ will be Circle if _____.

a- $\sin\theta=\cos\theta$ ✓ b- $a=b$ c- $a+b=0$ d- $a-2b=0$

(67) _____ is the closest point to Focus in Parabola.

a- ✓ Vertex. b- Centre c- Any Tangent point d- Every point.

(68) At the most _____ tangents can be drawn from a point on Hyperbola.

a- one b- Two c- Three ✓ d- Four

(69) $y=1-x+x^2$ is the Equation of _____.

a- Circle ✓ b- Parabola c- Ellipse d- Hyperbola

(70) For Parabola $\frac{|PF|}{|PM|}=$ _____.

a- ✓ 1 b- 2 c- -1 d- 0