TAHIR MEHMOOD
M.Sc Math
0345-6510779

2nd Year

CH # 3

معروضی (ریاضی)

Integration:-

The reverse process of differentiation is called integral or anti-derivation or anti-differentiation.

* Some Useful Formulae *

$\int 0\,dx = c$ (constant)

$\int 1\,dx = x + c$ $\qquad \int x^0\,dx = x + c$

$\int \lim_{x\to 0} \frac{\sin x}{x}\,dx = x + c$ $\qquad \int \lim_{x\to 0} (1+x)^{\frac{1}{x}}\,dx = xe + c$ where $e \approx 2.7183$

Simple Form	General Form
i) $\int x^n dx = \frac{x^{n+1}}{n+1} + c$	$\int (ax+b)^n dx = \frac{(ax+b)^{n+1}}{a(n+1)} + c$ for $n \neq -1$
ii) $\int \sin x\,dx = -\cos x + c$	$\int \sin(ax+b)\,dx = -\frac{\cos(ax+b)}{a} + c$
iii) $\int \cos x\,dx = \sin x + c$	$\int \cos(ax+b)\,dx = \frac{1}{a}\sin(ax+b) + c$
iv) $\int \sec^2 x\,dx = \tan x + c$	$\int \sec^2(ax+b)\,dx = \frac{1}{a}\tan(ax+b) + c$
v) $\int \operatorname{cosec}^2 x\,dx = -\cot x + c$	$\int \operatorname{cosec}^2(ax+b)\,dx = \frac{-1}{a}\cot(ax+b) + c$
vi) $\int \tan x \sec x\,dx = \sec x + c$	$\int \tan(ax+b)\cdot\sec(ax+b)\,dx = \frac{1}{a}\sec(ax+b) + c$
vii) $\int \cot x \operatorname{cosec} x\,dx = -\operatorname{cosec} x + c$	$\int \cot(ax+b)\cdot\operatorname{cosec}(ax+b)\,dx = \frac{-1}{a}\operatorname{cosec}(ax+b) + c$
viii) $\int e^x dx = e^x + c$	$\int e^{ax} dx = \frac{1}{a}e^{ax} + c$, $\int e^{\lambda x+\mu} dx = \frac{1}{\lambda}e^{\lambda x+\mu} + c$
ix) $\int a^x dx = \frac{a^x}{\ln a} + c$	$\int a^{\lambda x+\mu} dx = \frac{1}{\lambda \ln a} a^{\lambda x+\mu} + c$
x) $\int \frac{1}{x} dx = \ln x + c$	$\int \frac{1}{ax+b} dx = \frac{1}{a}\ln\lvert ax+b\rvert + c$
xi) $\int \tan x\,dx = \ln\lvert\sec x\rvert + c$	$\int \tan(ax+b)\,dx = \frac{1}{a}\ln\lvert\sec(ax+b)\rvert + c$
xii) $\int \cot x\,dx = \ln\lvert\sin x\rvert + c$	$\int \cot(ax+b)\,dx = \frac{1}{a}\ln\lvert\sin(ax+b)\rvert + c$
xiii) $\int \sec x\,dx = \ln\lvert\sec x + \tan x\rvert + c$	$\int \sec(ax+b)\,dx = \frac{1}{a}\ln\lvert\sec(ax+b)+\tan(ax+b)\rvert + c$
xiv) $\int \operatorname{cosec} x\,dx = \ln\lvert\operatorname{cosec} x - \cot x\rvert + c$	$\int \operatorname{cosec}(ax+b)\,dx = \frac{1}{a}\ln\lvert\operatorname{cosec}(ax+b)-\cot(ax+b)\rvert + c$
xv) $\int \frac{1}{\sqrt{1-x^2}} dx = \sin^{-1} x + c$	$\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\left(\frac{x}{a}\right) + c$
xvi) $\int \frac{1}{1+x^2} dx = \tan^{-1} x + c$	$\int \frac{dx}{a^2+x^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + c$.
xvii) $\int \frac{1}{x\sqrt{x^2-1}} dx = \sec^{-1} x + c$	$\int \frac{dx}{x\sqrt{x^2-a^2}} = \frac{1}{a}\sec^{-1}\left(\frac{x}{a}\right) + c$
xviii) $\int \frac{1}{1-x^2} dx = \frac{1}{2}\ln\left\lvert\frac{1+x}{1-x}\right\rvert + c$	$\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert$
xix) $\int \frac{1}{x^2-1} dx = \frac{1}{2}\ln\left\lvert\frac{x-1}{x+1}\right\rvert + c$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + c$.

xx) $\int \sinh x\,dx = \cosh x + c$ $\qquad$ xxi) $\int \cosh x\,dx = \sinh x + c$

xxii) $\int \operatorname{sech}^2 x\,dx = \tanh x + c$ $\qquad$ xxiii) $\int \operatorname{cosech}^2 x\,dx = -\coth x + c$

xxiv) $\int \operatorname{sech} x \tanh x\,dx = -\operatorname{sech} x + c$ $\qquad$ xxv) $\int \operatorname{cosech} x \coth x\,dx = -\operatorname{cosech} x + c$.

* Substitution for $\int \sqrt{a^2-x^2}\,dx$ is $x = a\sin\theta$
* Substitution for $\int \sqrt{x^2-a^2}\,dx$ is $x = a\sec\theta$
* Substitution for $\int \sqrt{a^2+x^2}\,dx$ is $x = a\tan\theta$
* $\int u\,dv = uv - \int v\,du$ is by part integration.
* $\int e^x [f(x) + f'(x)]\,dx = e^x f(x) + c$
* $\int e^{ax}[af(x) + f'(x)]\,dx = e^{ax} f(x) + c$

Fundamental Theorem of Calculus:-

Let f be a function such that $\frac{d}{dx}F(x) = f(x)$ then

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

is called fundamental theorem of Calculus.

* $\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$
* $\int_a^a f(x)\,dx = 0$ * $\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$ where $a \le c \le b$.
* The definite integral gives area under $y = f(x)$ from "a" to "b"

$$A = \int_a^b f(x)\,dx.$$

i) $\int a^x\,dx =$ ________ $+c$

a- a^x b- $a^x \ln a$ ✓c- $\frac{a^x}{\ln a}$ d- $\frac{a^x}{\ln x}$

ii) $\int e^x\left[\frac{1}{x} - \frac{1}{x^2}\right]dx =$ ________.

✓a- $\frac{e^x}{x} + c$ b- $\frac{e^x}{x^2} + c$ c- $\frac{-e^x}{x} + c$ d- $\frac{-e^x}{x^2} + c$

iii) $\int_0^1 \frac{1}{1+x^2}\,dx =$ ________.

a- $\tan^{-1}x + c$ ✓b- $\pi/4$ c- $\pi/2$ d- π

iv) $\int_{-\pi}^{\pi} \sin x\,dx =$ ________.

✓a- 0 b- π c- 2π d- $\pi/2$

v) $\int \frac{e^{\tan^{-1}x}}{1+x^2}\,dx =$ ________ $+c$

✓a- $e^{\tan^{-1}x}$ b- $e^{\tan^{-1}x}.\tan^{-1}x$ c- $e^x \tan^{-1}x$ d- e^x

vi) $\int \sec x\,dx =$ ________.

a- $\tan x + c$ b- $\ln|\sec x| + c$ ✓c- $\ln|\sec x + \tan x| + c$ d- $\ln|\sec x + \cot x| + c$

vii) $\int (ax+b)^n\,dx =$ ________ $+c$

a- $(ax+b)^{n+1}$ b- $\frac{(ax+b)^{n+1}}{n+1}$ ✓c- $\frac{1}{a}\frac{(ax+b)^{n+1}}{n+1}$ d- $\frac{a(ax+b)^{n+1}}{n+1}$

viii) $\int [f(x)]^n . f'(x)\,dx =$ ________ $+c$

a- $[f(x)]^n . f(x)$ b- $[f(x)]^{n+1} f(x)$ ✓c- $\frac{[f(x)]^{n+1}}{n+1}$ d- $(n+1)[f(x)]^{n+1}$

ix) $\int \frac{\sec^2 x}{\tan x}\,dx =$ ________ $+c$

a- $\ln|\sin x|$ b- $\ln|\cos x|$ ✓c- $\ln|\tan x|$ d- $\ln|\cot x|$

x) $\int \tan x\,dx =$ ________.

a- $\sin x + c$ b- $\ln|\cos x| + c$ c- $\ln|\sin x| + c$ ✓d- $\ln|\sec x| + c$

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OBJECTIVE (Math)

xi) $\int \frac{1}{1+x^2}\,dx =$ ________ $+c$

a- $\cot^{-1}x$ b- $\tan^{-1}x$ c- $\cos^{-1}x$ d- $\sin^{-1}x$

xii) Differential of y is $dy =$ ________.

a- $f'(x)$ b- $f(x)\,dx$ c- $f'(x)\,dx$ d- $f(x)$

xiii) $\int \cot x\,dx =$ ________

a- $\ln|\sin x| + c$ b- $\ln|\cos x| + c$ c- $\ln|\sec x| + c$ d- $\ln|\operatorname{cosec} x| + c$

xiv) $\int e^x\left[\tan^{-1}x + \frac{1}{1+x^2}\right]dx =$ ________ $+c$.

a- $e^x \tan^{-1}x$ b- $e^x/1+x^2$ c- $-e^x\tan^{-1}x$ d- $\frac{-e^x}{1+x^2}$

xv) $\int \frac{dx}{x^2+a^2}$ can be evaluated by substituting $x =$ ________.

a- $a\cos\theta$ b- $a\sin\theta$ c- $a\sec\theta$ d- $a\tan\theta$

xvi) $\int \frac{1}{x\ln x}\,dx =$ ________ $+c$

a- $\ln x$ b- $\ln(\ln x)$ c- $\ln(x\ln x)$ d- $(\ln x)^2$

xvii) $\int_a^b f(x)\,dx =$ ________:

a- $F(a) - F(b)$ b- $F(b) - F(a)$ c- $F(a) + F(b)$ d- $2F(a)$

xviii) $\int \frac{dx}{\sqrt{a^2-x^2}} =$ ________ $+c$

a- $a\sin^{-1}x$ b- $a\sin^{-1}(x/a)$ c- $\frac{1}{a}\sin^{-1}(x/a)$ d- $\sin^{-1}(x/a)$

xix) $\int e^x(x+1)\,dx =$ ________ $+c$

a- e^x b- xe^x c- $e^x\left(\frac{x^2}{2}+x\right)$ d- $e^x(x+1)$

xx) $\int \cos 2x\,dx =$ ________ $+c$

a- $\sin 2x$ b- $-\sin 2x$ c- $-\frac{1}{2}\sin 2x$ d- $\frac{1}{2}\sin 2x$

xxi) Order of $y\frac{dy}{dx} + 2x = 0$ is ________.

a- 0 b- 1 c- 2 d- 3

xxii) $\int_0^{\pi/4} \sec^2 x\,dx =$ ________

a- 0 b- 1 c- $\pi/4$ d- $\pi/2$

xxiii) $\int (e^x + 1)\,dx =$ ________ $+c$

a- $e^x + x$ b- e^x c- $e^x + e^{-x}$ d- $e^x + x^3$

xxiv) $\int_0^2 2x\,dx =$ ________. a- 0 b- 2 c- 4 d- 9

xxv) $\int \ln x\,dx =$ ________ $+c$

a- $x\ln x$ b- $x\ln x - x$ c- $(x-1)\ln x$ d- $(x+1)\ln x$

xxvi) $\int \frac{f'(x)}{f(x)}\,dx =$ ________ $+c$

a- $f(x)$ b- $\ln|f(x)|$ c- $[f(x)]^2$ d- $2\ln|f(x)|$

xxvii) $\int_0^{\pi} \cos x\,dx =$ ________.

a- 0 b- 1 c- 2 d- 4