TAHIR MEHMOOD
M.Sc Math
0345-6510779

2nd year

CH # 2

معروضی (ریاضی)

Derivative of a function:-

Let f be a real valued function continous in its domain D_f then $\lim_{\Delta x \to 0} \dfrac{f(x+\Delta x) - f(x)}{\Delta x}$ is called derivative of f at x denoted by $f'(x)$ or $\dfrac{df}{dx}$ or $Df(x)$ or $\dot{f}(x)$.

* The process of finding derivative is called differentiation.

Formulas for Derivatives:-

i) $\dfrac{d}{dx}[c] = 0$

ii) $\dfrac{d}{dx}[x] = 1$

iii) $\dfrac{d}{dx}[x^m] = m x^{m-1}$ for $m \in \mathbb{R}$

iv) $\dfrac{d}{dx}[u \pm v] = \dfrac{du}{dx} \pm \dfrac{dv}{dx}$

v) $\dfrac{d}{dx}[u.v] = u.\dfrac{dv}{dx} + \dfrac{du}{dx}.v$

vi) $\dfrac{d}{dx}\left[\dfrac{u}{v}\right] = \dfrac{v\dfrac{du}{dx} - u\dfrac{dv}{dx}}{v^2}$ if $v \neq 0$

vii) $\dfrac{d}{dx}[c f(x)] = c\dfrac{df(x)}{dx}$

viii) $\dfrac{d}{dx}\left[\dfrac{1}{g(x)}\right] = \dfrac{-g'(x)}{[g(x)]^2}$ for $g(x) \neq 0$

ix) $\dfrac{d}{dx}[f(g(x))] = f'(g(x)) \cdot g'(x)$.

x) $\dfrac{dy}{dx} = \dfrac{1}{dx/dy}$

xi) $\dfrac{d}{dx}[\sin x] = \cos x$

xii) $\dfrac{d}{dx}[\cos x] = -\sin x$

xiii) $\dfrac{d}{dx}[\tan x] = \sec^2 x$

xiv) $\dfrac{d}{dx}[\cot x] = -\csc^2 x$.

xv) $\dfrac{d}{dx}[\sec x] = \sec x \cdot \tan x$.

xvi) $\dfrac{d}{dx}[\csc x] = -\csc x \cdot \cot x$

xvii) $\dfrac{d}{dx}[\sin^{-1} x] = \dfrac{1}{\sqrt{1-x^2}}$

xviii) $\dfrac{d}{dx}[\cos^{-1} x] = \dfrac{-1}{\sqrt{1-x^2}}$

xix) $\dfrac{d}{dx}[\tan^{-1} x] = \dfrac{1}{1+x^2}$

xx) $\dfrac{d}{dx}[\cot^{-1} x] = \dfrac{-1}{1+x^2}$

xxi) $\dfrac{d}{dx}[\sec^{-1} x] = \dfrac{1}{x\sqrt{x^2-1}}$

xxii) $\dfrac{d}{dx}[\csc^{-1} x] = \dfrac{-1}{x\sqrt{x^2-1}}$

xxiii) $\dfrac{d}{dx}[a^x] = a^x . \ln a$

xxiv) $\dfrac{d}{dx}[e^x] = e^x$

xxv) $\dfrac{d}{dx}[e^{f(x)}] = e^{f(x)} \cdot f'(x)$

xxvi) $\dfrac{d}{dx}[\log_a x] = \dfrac{1}{x \ln a}$

xxvii) $\dfrac{d}{dx}[\ln x] = \dfrac{1}{x}$

xxviii) $\dfrac{d}{dx}[\ln f(x)] = \dfrac{f'(x)}{f(x)}$

xxix) $\dfrac{d}{dx}[\sinh x] = \cosh x$

xxx) $\dfrac{d}{dx}[\cosh x] = \sinh x$

xxxi) $\dfrac{d}{dx}[\tanh x] = \text{sech}^2 x$

xxxii) $\dfrac{d}{dx}[\coth x] = -\text{csch}^2 x$

xxxiii) $\dfrac{d}{dx}[\text{sech}\, x] = -\text{sech}\, x \cdot \tanh x$

xxxiv) $\dfrac{d}{dx}[\text{csch}\, x] = -\text{csch}\, x \cdot \coth x$

xxxv) $\dfrac{d}{dx}[a^{f(x)}] = a^{f(x)} \ln a \; f'(x)$

xxxvi) $\frac{d}{dx}[\sinh^{-1}x] = \frac{1}{\sqrt{1+x^2}}$ xxxvii) $\frac{d}{dx}[\cosh^{-1}x] = \frac{1}{\sqrt{x^2-1}}$

xxxviii) $\frac{d}{dx}[\tanh^{-1}x] = \frac{1}{1-x^2}$ $|x|<1$ xxxix) $\frac{d}{dx}[\coth^{-1}x] = \frac{1}{1-x^2}$ $|x|>1$

xxxx) $\frac{d}{dx}[\operatorname{sech}^{-1}x] = \frac{-1}{x\sqrt{1-x^2}}$ xxxxi) $\frac{d}{dx}[\operatorname{cosech}^{-1}x] = \frac{-1}{x\sqrt{1+x^2}}$

Higher Order Derivatives:-

If $y = f(x)$ be a real valued differentiable function then

$\frac{d^n y}{dx^n} = f^n(x)$ for $n \geq 2$ is called Higher order derivative of $f(x)$.

e.g. $y = \cos(ax+b)$ then $y_4 = a^4\cos(ax+b)$.

Maclaurin's Series:-

Let f be a real valued function such that f and all of its derivatives exist at $x=0$ then

$$f(x) = f(0) + x f'(0) + \frac{x^2}{2!}f''(0) + \frac{x^3}{3!}f'''(0) + \frac{x^4}{4!}f^{iv}(0) + \cdots$$

is called Maclaurin's Series.

e.g.

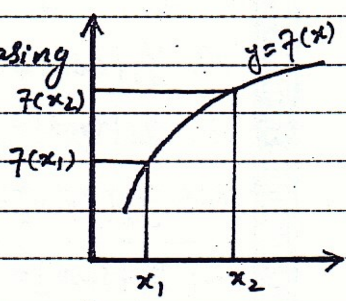
$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots$$

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots$$

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \frac{x^5}{5} - \cdots$$

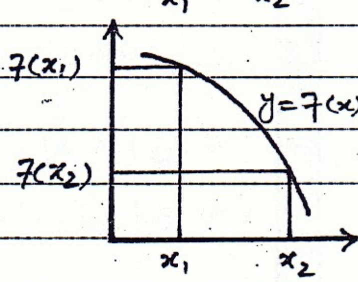
Increasing and Decreasing Function:-



A function "f" is said to be increasing at $x=a$ if $f'(a)>0$

OR A function "f" is called increasing if $f(x_2) > f(x_1)$ when $x_2 > x_1$

e.g $f(x) = e^x$



A function "f" is said to be decreasing at $x=a$ if $f'(a)<0$

OR A function "f" is called decreasing if $f(x_2) < f(x_1)$ if $x_2 > x_1$

e.g. $f(x) = e^{-x}$

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OBJECTIVE (Math)

Existance Theorem of Extremum:-

If a function "f" has extreme (maximum or minimum) value at $x=a$ then $f'(a)=0$.

2nd Derivative Test:-

If "f" is a function such that $f'(a)=0$ then

i) f has maximum value at $x=a$ if $f''(a)<0$

ii) f has minimum value at $x=a$ if $f''(a)>0$

iii) If $f''(a)=0$ then no maximum or minimum exist at $x=a$.

Critical Points and Stationary Points of $f(x)$:-

The point $x=a$ is called Critical point of f if $f'(a)=0$ or $f'(a)$ does not exist.

The point $x=a$ is called Stationary point if $f'(a)=0$.

* Some useful Points of objective. *

* $\frac{d}{dx}[x^m]=0$ if $m=0$

* $\frac{d}{dx}[\tan^{-1}x+\cot^{-1}x]=0$

* $\frac{d}{dx}[\sin^{-1}x+\cos^{-1}x]=0$

* $\frac{d}{dx}[\sec^{-1}x+\operatorname{cosec}^{-1}x]=0$

* $\frac{d}{dx}\left[\lim_{x\to 0}\frac{\sin x}{x}\right]=0$

* $\frac{d}{dx}(ax+b)=a$

* Geometrically derivative at point of function is slope of tangent at that point.

* $\frac{d}{dx}[10^x]=10^x.\ln 10$

* $\frac{d}{dx}[\log_{10}x]=\frac{1}{x\ln 10}$

* $\frac{d}{dx}\left[\lim_{x\to 0}(1+x)^{1/x}\right]=0$

* If $y=e^{ax}$ then $y_4=a^4e^{ax}$

Chain Rule:-

If $x=f(t)$ and $y=g(t)$ then

$$\frac{dy}{dx}=\frac{dy}{dt}\cdot\frac{dt}{dx}=\frac{dy/dt}{dx/dt}$$ is called chain rule of differentiation.

* Any point where f is neither increasing nor decreasing is called Stationary point povided $f'(x)=0$ at that point.

i) The process of finding derivative is called ______.

a- Extremization b- Integration ✓c- Differentiation d- Antiderivation

ii) Derivative of Constant function is ____.

a- x ✓b- 0 c- 1 d- Constant

iii) If $f(x) = \sin x$ then $f'(\cos^{-1} x) =$ ______.

a- 1 b- 0 c- ✓ x d- $\sqrt{1-x^2}$

iv) $\frac{d}{dx}[\tan^{-1} x + \cot^{-1} x] =$ ______. a- ✓ 0 b- 1 c- x d- $2x$

v) $\frac{d}{dx}[x^0] =$ ______. a- ✓ 0 b- 1 c- x d- x^{-1}

vi) ______ $= x - \frac{x^3}{\lfloor 3} + \frac{x^5}{\lfloor 5} - \frac{x^7}{\lfloor 7} + \cdots$

a- e^x b- $\ln(1+x)$ c- ✓ $\sin x$ d- $\cos x$

vii) $\frac{d}{dx}(ax+b)^n =$ ______. a- $n(ax+b)^{n-1}$ b- $n(ax+b)^{n+1}$ c- ✓ $na(ax+b)^{n-1}$ d- None

viii) $\lim\limits_{\delta x \to 0} \frac{\delta y}{\delta x} =$ ______. a- dy b- $f'(x)\,dx$ c- ✓ $\frac{dy}{dx}$ d- $\frac{dx}{dx}$

ix) If $g(x) \neq 0$ then $\frac{d}{dx}\left[\frac{1}{g(x)}\right] =$ ______.

a- $g'(x)$ b- $g(x)\,g'(x)$ c- $\frac{g'(x)}{g(x)}$ d- ✓ $\frac{g'(x)}{[g(x)]^2}$

x) $\frac{d}{dx}[f(g(x))] =$ ______.

a- $f'(g(x))$ b- $f[g'(x)]$ c- $f[g(x)]\,g'(x)$ d- ✓ $f'(g(x)) \cdot g'(x)$

xi) $\frac{d}{dx}[0] =$ ______ a- ✓ 0 b- 1 c- -1 d- infinity

xii) $\frac{d}{dx}[\cos(ax+b)] =$ ______.

a- $-\sin(ax+b)$ b- $\sin(ax+b)$ c- $a\sin(ax+b)$ d- ✓ $-a\sin(ax+b)$

xiii) $\frac{d}{dx}[$ ______ $] = -\operatorname{cosec} x \cdot \cot x.$

a- $\cot x$ b- $\tan x$ c- $\sec x$ d- ✓ $\operatorname{cosec} x$

xiv) $\frac{d}{dx}[$ ______ $] = e^{\sin x}(x\cos x + 1)$

a- $e^{\sin x}$ b- ✓ $xe^{\sin x}$ c- $e^{\sin x}\cos x$ d- $xe^{\cos x}$

xv) $\frac{d}{dx}[\tanh^{-1} x] =$ ______ for $|x| < 1$.

a- $1/1+x^2$ b- $1/x^2-1$ c- ✓ $1/1-x^2$ d- $-1/1+x^2$

xvi) $\frac{d}{dx}(\sin^{-1} x) =$ ______.

a- $\cos^{-1} x$ b- $-\cos^{-1} x$ c- ✓ $1/\sqrt{1-x^2}$ d- $-1/\sqrt{1-x^2}$

xvii) $\frac{d}{dx}[e^{\sin x}] =$ ______.

a- $e^{\sin x}$ b- $e^{\cos x}$ c- ✓ $e^{\sin x}\cos x$ d- $e^{\cos x}\sin x$

xviii) $\frac{d}{dx}[\ln e^{2x}] =$ ______. a- e^{2x} b- $2e^{2x}$ c- $\frac{1}{2e^{2x}}$ d- ✓ 2

xix) If f has maximum value at $x=a$ then ______.

a- $f'(a) < 0$ b- $f''(a) = 0$ c- $f'(a) > 0$ d- ✓ $f'(a) = 0$

xx) If $y = e^{ax}$ then $y_4 =$ ______.

a- $a^4 e^x$ b- e^{ax} c- ✓ $a^4 e^{ax}$ d- $4a^3 e^{ax}$.

xxi) A function f such that $f'(a) = 0$ has maximum at $x=a$ if ______.

a- $f'(a) < 0$ b- $f'(a) > 0$ c- ✓ $f''(a) < 0$ d- $f''(a) > 0$

xxii) If $y = \tanh^{-1}(\sin x)$ then $y_1 =$ ______.

a- $\cos x$ b- $\sin x$ c- ✓ $\sec x$ d- $\operatorname{cosec} x$

xxiii) $f(x) = f(0) + x f'(0) + \frac{x^2}{\lfloor 2} f''(0) + \frac{x^3}{\lfloor 3} f'''(0) + \cdots$ is called ______ Series.

a- Binomial b- Taylor's c- ✓ Maclaurin's d- Geometric.

xxiv) f is increasing at $x=a$ if ______. a- $f'(a)=0$ b- $f'(a)<0$ c- ✓ $f'(a)>0$ d- None