Types of Functions:- Chapter 1

2nd Year Objective

I. Algebraic Functions:-

The functions defined by Algebraic expressions are named as Algebraic functions.

(i) Polynomial Function:-

A function of the form

$P(x) = a_0 + a_1x + a_2x^2 + \cdots + a_nx^n$ where $a_0, a_1, a_2, \ldots, a_n$ are called Coefficients and are real numbers. n is non-negative integer. If $a_n \neq 0$ then "n" is called degree of $P(x)$ and a_n is called Leading Coefficient.

e.g. $P(x) = 3x^4 + 7x - 19$ is polynomial of degree 4 and leading Coefficient 3.

(ii) Linear Function:-

A polynomial function of degree One is called Linear function. It is of the form $f(x) = ax + b$ where $a \neq 0$. Domain and range of Linear function is set of real numbers.

(iii) Identity Function:-

A function $I: X \to X$ defined by $I(x) = x \ \forall x \in X$ is called identity function. It may be denoted by $f(x) = x$. Identity function is bijective (one-one and onto) function.

(iv) Constant Function:-

A function $C: X \to Y$ defined by $C(x) = a$ is called Constant function $\forall x \in X$ and $a \in Y$ where "a" is fixed in Y.

e.g. $f(x) = \pi$, $C(x) = 10$, $g(x) = e$ are Constant functions.

(v) Rational Function:-

A function of the form $f(x) = \dfrac{P(x)}{Q(x)}$ where $P(x)$ & $Q(x)$ are polynomials and $Q(x) \neq 0$ is called rational function.

$D_f = \mathbb{R} - \{x \mid Q(x) = 0\}$.

II. Trigonometric Functions:-

Functions	Domains	Ranges:
$y = \sin x$	$\mathbb{R} = (-\infty, \infty)$	$-1 \le y \le 1 = [-1, 1]$
$y = \cos x$	$\mathbb{R} = (-\infty, \infty)$	$-1 \le y \le 1 = [-1, 1]$
$y = \tan x$	$\mathbb{R} - \{x \mid x = (2n+1)\frac{\pi}{2}, n \in \mathbb{Z}\}$	$\mathbb{R} = (-\infty, \infty)$

$y = \cot x$	$\mathbb{R} - \{x \mid x = n\pi, n \in Z\}$	$\mathbb{R} = (-\infty, \infty)$
$y = \sec x$	$\mathbb{R} - \{x \mid x = (2n+1)\frac{\pi}{2}, n \in Z\}$	$y \le -1 \wedge y \ge 1$
$y = \operatorname{cosec} x$	$\mathbb{R} - \{x \mid x = n\pi, n \in Z\}$	$y \le -1 \wedge y \ge 1$

III. Inverse Trigonometric Functions:-

Functions	Domains	Ranges
$y = \sin^{-1} x \Leftrightarrow x = \sin y$	$[-1, 1]$	$[-\frac{\pi}{2}, \frac{\pi}{2}]$
$y = \cos^{-1} x \Leftrightarrow x = \cos y$	$[-1, 1]$	$[0, \pi]$
$y = \tan^{-1} x \Leftrightarrow x = \tan y$	$\mathbb{R} = (-\infty, \infty)$	$(-\frac{\pi}{2}, \frac{\pi}{2})$
$y = \cot^{-1} x \Leftrightarrow x = \cot y$	$\mathbb{R} = (-\infty, \infty)$	$[-\frac{\pi}{2}, \frac{\pi}{2}] - \{0\}$
$y = \sec^{-1} x \Leftrightarrow x = \sec y$	$x \le -1 \wedge x \ge 1$	$[0, \pi] - \{\frac{\pi}{2}\}$
$y = \operatorname{cosec}^{-1} x \Leftrightarrow x = \operatorname{cosec} y$	$x \le -1 \wedge x \ge 1$	$[-\frac{\pi}{2}, \frac{\pi}{2}] - \{0\}$.

IV. EXPONENTIAL FUNCTIONS:-

A function inwhich variable appears as exponent (Power) is called exponential Function. These are:

$y = a^x$ for $a > 0$ and $a \ne 1$	$y = e^x$
Common Exponential Function	Natural Exponential function
a = base of Function	e = Natural Base $\approx 2.7183\ldots$
Domain = $\mathbb{R}$	Domain = $\mathbb{R}$
Range = Positive real Numbers (R^+)	Range = $\mathbb{R}^+$

* Exponential function can never be zero or negative.

V. LOGARITHMIC FUNCTIONS:-

A function defined by

$y = \log_a x$ iff $x = a^y$ where $a > 0$ and $a \ne 1$ is called Common Logrithemic function.

$y = \ln x$ iff $x = e^y$ is called natural exponential Function.

* If $a = 10$ then Function is called Brigg's logrithemic Function.

* Natural Logarithemic function is also called Naprian Logrithemic function

Domain = Positive real numbers = $\mathbb{R}^+$

Range = $\mathbb{R}$.

* Base of Logarithemic function should always be +ve & $\ne 1$.

VI Hyperbolic functions:-

Functions	Domains	Ranges.
$\sinh x = \frac{e^x - e^{-x}}{2}$	$\mathbb{R}$	$[0,\infty)$
$\cosh x = \frac{e^x + e^{-x}}{2}$	$\mathbb{R}$	$[1,\infty)$
$\tanh x = \frac{\sinh x}{\cosh x} = \frac{e^x - e^{-x}}{e^x + e^{-x}}$	$\mathbb{R}$	$[0,\infty)$
$\coth x = \frac{\cosh x}{\sinh x} = \frac{e^x + e^{-x}}{e^x - e^{-x}}$	$\mathbb{R}-\{0\}$	$(0,\infty)$
$\operatorname{sech} x = \frac{2}{e^x + e^{-x}} = \frac{1}{\cosh x}$	$\mathbb{R}$	$(0,1]$
$\operatorname{cosech} x = \frac{1}{\sinh x} = \frac{2}{e^x - e^{-x}}$	$\mathbb{R}-\{0\}$	$(0,\infty)$

VII Inverse Hyperbolic functions:-

Functions	Domains	Ranges.
$\sinh^{-1}x = \ln(x+\sqrt{x^2+1})$	$\mathbb{R}$	$(0,\infty)=\mathbb{R}^+$
$\cosh^{-1}x = \ln(x+\sqrt{x^2-1})$	$[1,\infty)$	$(0,\infty)=\mathbb{R}^+$
$\tanh^{-1}x = \frac{1}{2}\ln\left(\frac{1+x}{1-x}\right)$	$(-1,1)$	$(0,\infty)=\mathbb{R}^+$
$\coth^{-1}x = \frac{1}{2}\ln\left(\frac{x+1}{x-1}\right)$	$(-\infty,-1)\cup(1,\infty)$	$(0,\infty)=\mathbb{R}^+$
$\operatorname{sech}^{-1}x = \ln\left(\frac{1+\sqrt{1-x^2}}{x}\right)$	$(0,1]$	$(0,\infty)=\mathbb{R}^+$
$\operatorname{cosech}^{-1}x = \ln\left(\frac{1}{x}+\frac{\sqrt{1+x^2}}{\lvert x\rvert}\right)$	$(-\infty,0)\cup(0,\infty)$ $=\mathbb{R}-\{0\}$.	$(0,\infty)=\mathbb{R}^+$

VIII EXPLICIT FUNCTION:-

A function of the form $y=f(x)$ is called explicit function.

e.g $y=\sin x$, $y=x^2-1$, $y=\log_a x$, $y=\tan^{-1}x$ etc.

IX Implicit function:-

A function of the form $f(x,y)=0$ is called implicit function.

e.g. $\sin xy - x^2 + y^3 = 0$, $\ln(xy)+e^{-xy}+1=0$.

X Parametric Functions:-

A function of x and y is called parametric function if x and y are expressed as function of a third variable θ or t.

e.g $x=a\cos\theta$, $y=a\sin\theta$ for $x^2+y^2=a^2$.

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XI Even Function:-

A function "f" is said to be even function if $f(-x) = f(x)$ where $x, -x \in D_f$.

e.g $f(x) = x^2$, $f(x) = \frac{1}{x^2}$

$f(x) = \cos x$, $f(x) = \sec x$

$f(x) = \cosh x$, $f(x) = \text{sech}\, x$ are even functions.

XII Odd Function:-

A function "f" is said to be odd function if $f(-x) = -f(x)$ where $x, -x \in D_f$

e.g. $f(x) = x$, $f(x) = x^3$

$f(x) = \sin x$, $f(x) = \tan x$

$f(x) = \sinh x$, $f(x) = \text{cosech}\, x$ are odd functions.

Smooth and Piecewise (Sectional) function:-

A function $y = f(x)$ expressed by a single curve is called smooth function.

Such as $f(x) = \sin x$, $f(x) = x^2$

A function defined by sections (Peices of functions) is called Piecewise or Sectional function.

Such as:

$$f(x) = \begin{cases} x & \text{if } x \le 0 \\ x^2 - 1 & \text{if } 0 < x < 3 \\ \sin x & \text{if } 3 \le x \end{cases} \quad \text{etc.}$$

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Show that (i) $\cosh^2 x - \sinh^2 x = 1$ (ii) $\cosh^2 x + \sinh^2 x = \cosh 2x$.

(i)

$LHS = \cosh^2 x - \sinh^2 x$

$= \left(\frac{e^x + e^{-x}}{2}\right)^2 - \left(\frac{e^x - e^{-x}}{2}\right)^2$

$= \frac{e^{2x} + e^{-2x} + 2}{4} - \frac{e^{2x} + e^{-2x} - 2}{4}$

$= \frac{e^{2x} + e^{-2x} + 2 - e^{2x} - e^{-2x} + 2}{4}$

$= \frac{4}{4} = 1 = RHS.$

Thus

$\cosh^2 x - \sinh^2 x = 1$ (Proved)

(ii)

$LHS = \cosh^2 x + \sinh^2 x$

$= \left(\frac{e^x + e^{-x}}{2}\right)^2 + \left(\frac{e^x - e^{-x}}{2}\right)^2$

$= \frac{e^{2x} + e^{-2x} + 2}{4} + \frac{e^{2x} + e^{-2x} - 2}{4}$

$= \frac{e^{2x} + e^{-2x} + 2 + e^{2x} + e^{-2x} - 2}{4}$

$= \frac{2[e^{2x} + e^{-2x}]}{4} = \frac{e^{2x} + e^{-2x}}{2}$

$= \cosh 2x = RHS.$

Thus $\cosh^2 x + \sinh^2 x = \cosh 2x$ (Proved)

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M.C.Q's IInd Year CH# 1

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(i) Domain of $f(x)=x^2$ is ______.
a- $\mathbb{R}$ ✓ b- $\mathbb{R}^-$ c- $\mathbb{R}^+$ d- $[0,\infty)$

(ii) Range of $f(x)=x^2$ is ______.
a- $\mathbb{R}$ b- $(-\infty,0]$ ✓c- $[0,\infty)$ d- $[-1,1]$.

(iii) $f(x)=x$ is known as ______ function.
a- Constant ✓b- Identity c- Scaling d- Even

(iv) Graph of $f(x)=ax+b$ where $a\neq0$ is ______.
a- Straight Line ✓ b- Parabolic c- Cubic d- Elliptic.

(v) $f(x)=ax+b$ will be identity function if ______.
a- $a=0$ b- $b=0$ c- $a\neq0$ ✓d- $a=1 \wedge b=0$

(vi) Degree of Polynomial function is ______ number.
a- Real b- Rational c- Even ✓d- Whole

(vii) Range of $C(x)=2$ is ______.
a- $\mathbb{R}$ b- $\mathbb{R}^+$ c- $\mathbb{R}^-$ ✓d- $\{2\}$.

(viii) Domain of $y=\tan x$ is ______
a- $\mathbb{R}$ b- $(-\pi/2,\pi/2)$ ✓c- $\mathbb{R}-\{x \mid x=(2n+1)\frac{\pi}{2}, n\in Z\}$ d- $\mathbb{R}-\{x \mid x=n\pi \wedge n\in Z\}$

(ix) $f(x)=\log_a x$ is undefined at $x=$ ______ for $a\neq1$, $a>0$.
✓a- 0 b- e c- 10 d- a

(x) $\sinh^{-1}x=$ ______.
a- $\ln(x+\sqrt{x^2-1})$ ✓b- $\ln(x+\sqrt{1+x^2})$ c- $\ln(x+\sqrt{1-x^2})$ d- $\ln(x-\sqrt{x^2-1})$

(xi) $f(x,y)=0$ denotes ______ Function.
a- Constant b- Explicit ✓c- Implicit d- Odd.

(xii) Range of $\sin x$ is ______
a- $\mathbb{R}$ b- $[1,\infty)$ c- $(-\infty,-1]$ ✓d- $[-1,1]$

(xiii) $\cosh^2x-\sinh^2x=$ ______.
a- $\sinh 2x$ b- $\cosh 2x$ ✓c- 1 d- 0.

(xiv) $\tanh x=$ ______.
a- $\frac{e^x-e^{-x}}{2}$ b- $\frac{e^x+e^{-x}}{(e^x-e^{-x})}$ c- $\frac{e^x+e^{-x}}{2}$ ✓d- $\frac{e^x-e^{-x}}{e^x+e^{-x}}$

(xv) $x=a\cos\theta$ $y=a\sin\theta$ are parametric equations of ______.
a- Circle ✓ b- Ellipse c- Parabola d- Hyperbola.

(xvi) $f(x)=3x^4-2x^2+7$ is ______ function.
a- Linear b- Quadratic ✓c- Even d- Odd

(xvii) If $f(x) = x^2 - x$ then $f(x-1) =$ ______.

a- 0 b- x^2-3x-1 ✓c- x^2-3x+2 d- x^2-3x-2

(xviii) $f(x) = a^x$ where ______ is called Exponential function.

a- $a \neq 0$ b- $a > 1$ c- $a > 0$ ✓d- $a > 0,\ a \neq 1$

(xix) Range of $f(x) = |x-3|$ is ______.

a- $\mathbb{R}$ b- $\mathbb{R}^+$ ✓c- $[0, \infty)$ d- $[0, 3]$

(xx) Domain of $f \circ g(x) =$ ______.

✓a- D_g b- D_f c- R_f d- R_g

(xxi) Range of $f \circ g(x) =$ ______

a- D_f b- D_g ✓c- R_f d- R_g

(xxii) If $f(x) = 2x+1$ and $g(x) = 2x$ then $f \circ g(x) =$ ______

a- $4x$ b- $4x+2$ ✓c- $4x+1$ d- $4x-1$

(xxiii) $f \circ f^{-1}(x) =$ ______ $= f^{-1} \circ f(x)$.

a- $f(x)$ b- $f^{-1}(x)$ ✓c- x d- y

(xxiv) Identity function is denoted by $f(x) =$ ______.

a- $ax+b$ b- ax^2+bx+c ✓c- x d- ax

(xxv) For any function $y = f(x)$, $D_{f^{-1}} =$ ______.

a- D_f ✓b- R_f c- D_g d- $R_{f^{-1}}$

(xxvi) For any function $y = f(x)$, $R_{f^{-1}} =$ ______.

a- R_f b- R_g ✓c- D_f d $D_{f^{-1}}$

(xxvii) $\lim\limits_{x \to 0^+} \frac{1}{x} =$ ______.

a- 0 ✓b- $+\infty$ c- $-\infty$ d- 0^+

(xxviii) $\lim\limits_{x \to 0^-} \frac{1}{x} =$ ______

a- 0^- b- 0 ✓c- $-\infty$ d- $+\infty$

(xxix) $\lim\limits_{x \to 0} \frac{a^x - 1}{x} =$ ______.

a- $\ln x$ ✓b- $\ln a$ c- $\ln e^x$ d- 1

(xxx) $\lim\limits_{x \to 0} \frac{5^x - 1}{x} =$ ______.

a- $\ln a$ b- $\ln e$ c- $\ln x$ ✓d- $\ln 5$.

(xxxi) $\lim\limits_{x \to 0} \frac{e^x - 1}{x} =$ ______.

a- $\ln x$ b- $\ln a$ c- 0 ✓d- 1

(xxxii) $\lim\limits_{x \to \infty} \frac{1}{2x} =$ ______

✓a- 0 b- $-\infty$ c- $+\infty$ d- $\frac{1}{2}$

(xxxiii) $\lim\limits_{x \to 2} (x^3) =$ ______.

a- 2 b- 2^2 ✓c- 8 d- $\sqrt[3]{2}$

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Chapter 1 (IInd Year)

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(xxxiv) $\underset{x \to c}{Lt} P(x) =$ ______.

a- $P(x)$ ✓b- $P(c)$ c- $P(a)$ d- $P(x/c)$

(xxxv) $\underset{x \to 0}{Lt} (1+x)^{1/x} =$ ______.

a- 0 b- 1 ✓c- e d- $1/e$

(xxxvi) $\underset{n \to \infty}{Lt} (1+1/n)^{n} =$ ______.

a- 0 b- ∞ c- $-\infty$ ✓d- e

(xxxvii) $\underset{x \to 2}{Lt} 3x =$ ______.

✓a- 6 b- 3 c- 2 d- 5

(xxxviii) $\underset{x \to \infty}{Lt} \frac{a}{x^p} = 0$ if ______

✓a- $p > 0$ b- $-p > 1$ c- $p < 0$ d- $p < -1$

(xx xix) $\underset{x \to +\infty}{Lt} e^x =$ ______.

a- 0 b- $-\infty$ ✓c- $+\infty$ d- 1

(xxxx) $\underset{x \to -\infty}{Lt} e^x =$ ______

✓a- 0 b- $-\infty$ c- $+\infty$ d- 1

(xxxxi) $\underset{x \to \pm\infty}{Lt} \frac{a}{x} =$ ______.

a- $-\infty$ b- $+\infty$ c- 1 ✓d- 0

(xxxxii) $\underset{h \to 0}{Lt} (1+2h)^{1/h} =$ ______

a- e b- $1/e$ ✓c- e^2 d- $1/e^2$

(xxxxiii) $\underset{h \to 0}{Lt} (1-h)^{-h} =$ ______.

✓a- e b- $-e$ c- e^{-1} d- $-e^{-1}$

(xxxxiv) $\underset{\theta \to 0}{Lt} \frac{\sin\theta}{\theta} = 1$ where θ is in ______ measure.

a- Degree b- Minutes c- Seconds ✓d- Radian

(xxxxv) If $f(x) \le g(x) \le h(x)$ and $\underset{x \to a}{Lt} f(x) = L = \underset{x \to a}{Lt} h(x)$ then $\underset{x \to a}{Lt} g(x) =$ ______

a- 2L b- $L/2$ ✓c- L d- L-2

(xxxxvi) $\underset{x \to 0}{Lt} \frac{\sin 7x}{x} =$ ______.

a- 1 b- $1/7$ ✓c- 7 d- $7\frac{1}{7}$

(xxxxvii) If $\underset{x \to a}{Lt} f(x) = f(a)$ then f is called ______ function.

a- Even b- Odd c- Constant ✓d- Continous

(xxxxviii) $y = 2^x$ is ______ function.

a- Constant ✓b- Exponential c- Logrithmic d- Hyperbolic

(xxxxix) If $a > 0$ and $a \neq 1$ then $a^x \to$ ____ if $x \to -\infty$.

a- ∞ b- $-\infty$ ✓c- 0 d- 1.

(L) If $\underset{x\to a}{Lt}\ f(x) = L$ and $\underset{x\to a}{Lt}\ g(x) = M$ then $\underset{x\to a}{Lt}\ (fg)(x) =$ ______
a- L b- M c- L/M ✓d- LM

(Li) Domain of $f(x) = \sqrt{x+2}$ is ______.
a- $[-2,0]$ b- $[-2,2]$ c- $\mathbb{R}$ ✓d- $[-2,\infty)$

(Lii) Range of $f(x) = \frac{x^2-16}{x-4}$ is ______.
a- $\mathbb{R}$ b- $\mathbb{R}-\{4\}$ ✓c- $\mathbb{R}-\{8\}$ d- $\mathbb{R}^+$

(Liii) $f^{-1}(x)$ exist if f is ______.
a- One-one b- Onto c- into ✓d- Bijective.

(Liv) Range of $g(x) = \frac{2}{\sqrt{x}}$ is ______.
a- $\mathbb{R}$ b- $\mathbb{R}-\{0\}$ c- $\mathbb{R}^-$ ✓d- $\mathbb{R}^+$

(LV) $f(x) = \sin x$ is ______ function.
a- Even ✓b- Odd c- Constant d- Identity.

(LVi) $f(x) =$ ______ is even function.
a- $\sin x$ b- $\tan x$ ✓c- $\cos x$ d- $\operatorname{cosec} x$

(LVii) $f(x) =$ ______ is odd function.
a- x^2 b- x^4 c- x^2+2 ✓d- x

(LViii) $f\circ g(x)$ ______ $g\circ f(x)$ in general.
a- $=$ ✓b- $\neq$ c- $<$ d- $>$

(Lix) If $f(x) = x$ then $f^2(x) =$ ______.
✓a- x b- x^2 c- $1/x$ d- $1/x^2$

(LX) A function $f(x)$ is called even if ______.
a- $f(-x) = -f(x)$ b- $f(-x)+f(x)=0$ c- $f(-x)-f(x)=0$ ✓d- $f(-x)=f(x)$.

(LXi) $f(x)$ is odd function if ______.
a- $f(-x)=0$ b- $f(x)=0$ c- $f(-x)=f(x)$ ✓d- $f(-x)=-f(x)$

(LXii) $f(x) = \sqrt{x+4}$ then $f(0) =$ ______.
a- 4 b- 0 ✓c- 2 d- 16

(LXiii) Area of Circle as function of radius "r" is $A(r) =$ ______
a- $2\pi r$ b- $2\pi r^2$ c- $4\pi r^2$ ✓d- πr^2

(LXiv) Domain of $y = 5^x$ is ______.
a- $\mathbb{R}$ b- $(-\infty,\infty)$ c- $[0,\infty)$ ✓d- $(0,\infty)$

(LXV) $\operatorname{Cosech}^{-1} x =$ ______.
a- $\frac{e^x+e^{-x}}{2}$ b- $\frac{e^x-e^{-x}}{2}$ c- $\frac{2}{e^x+e^{-x}}$ ✓d- $\frac{2}{e^x-e^{-x}}$

(LXVi) Leading Coefficient in $P(x) = 2x^4-3x^2+2x-1$ is ______.
✓a- 2 b- 3 c- -3 d- -1

LXVii) $\underset{\theta\to 0}{Lt}\ \frac{\sin\theta}{\theta} = 1$ if $\theta \in$ ______ a- $(0,\frac{\pi}{2})$ b- $(-\frac{\pi}{2},0)$ ✓c- $(-\frac{\pi}{2},\frac{\pi}{2})$ d- $(0,\pi)$