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M.Sc Math
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EXERCISE: 8.3

Q.1 Expand upto 4 terms:

i) $(1-x)^{1/2}$

$$= 1+\frac{1}{2}(-x)+\frac{\frac{1}{2}\left(\frac{1}{2}-1\right)}{\lfloor 2}(-x)^2+\frac{\frac{1}{2}\left(\frac{1}{2}-1\right)\left(\frac{1}{2}-2\right)}{\lfloor 3}(-x)^3+\cdots$$

$$= 1-\frac{x}{2}+\frac{\frac{1}{2}\left(-\frac{1}{2}\right)}{2}x^2+\frac{\frac{1}{2}\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{6}(-x^3)+\cdots$$

$$= 1-\frac{x}{2}-\frac{x^2}{8}-\frac{x^3}{16}+\cdots$$

Validity: $|-x|<1 \Rightarrow |x|<1$

ii) $(1+2x)^{-1}$

$$= 1+(-1)2x+\frac{(-1)(-2)}{\lfloor 2}(2x)^2+\frac{(-1)(-2)(-3)}{\lfloor 3}(2x)^3+\cdots$$

$$= 1-2x+4x^2-8x^3+\cdots$$

Validity: $|2x|<1 \Rightarrow |x|<\frac{1}{2}$

iii) $(1+x)^{-1/3}$

$$= 1-\frac{1}{3}(x)+\frac{-\frac{1}{3}\left(-\frac{1}{3}-1\right)}{\lfloor 2}x^2+\frac{-\frac{1}{3}\left(-\frac{1}{3}-1\right)\left(-\frac{1}{3}-2\right)}{\lfloor 3}x^3+\cdots$$

$$= 1-\frac{x}{3}+\frac{\frac{1}{3}\left(\frac{4}{3}\right)}{2}x^2-\frac{\frac{1}{3}\left(\frac{4}{3}\right)\left(\frac{7}{3}\right)}{6}x^3+\cdots$$

$$= 1-\frac{x}{3}+\frac{2}{9}x^2-\frac{14}{81}x^3+\cdots$$

Validity: $|x|<1$

iv) $(4-3x)^{1/2} = 4^{1/2}\left(1-\frac{3}{4}x\right)^{1/2}$

$$= 2\left[1+\frac{1}{2}\left(-\frac{3}{4}x\right)+\frac{\frac{1}{2}\left(\frac{1}{2}-1\right)}{\lfloor 2}\left(-\frac{3x}{4}\right)^2+\frac{\frac{1}{2}\left(\frac{1}{2}-1\right)\left(\frac{1}{2}-2\right)}{\lfloor 3}\left(-\frac{3x}{4}\right)^3+\cdots\right]$$

$$= 2\left[1-\frac{3x}{8}+\frac{\frac{1}{2}\left(-\frac{1}{2}\right)}{2}\frac{9x^2}{16}+\frac{\frac{1}{2}\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{6}\left(-\frac{27x^3}{64}\right)+\cdots\right]$$

$$= 2\left[1-\frac{3x}{8}-\frac{9x^2}{128}-\frac{27x^3}{1024}+\cdots\right]$$

$$= 2-\frac{3x}{4}-\frac{9x^2}{64}-\frac{27x^3}{512}+\cdots$$

Validity:- $\left|-\frac{3x}{4}\right|<1 \Rightarrow |x|<\frac{4}{3}$

v) $(8-2x)^{-1} = 8^{-1}\left[1-\frac{2}{8}x\right]^{-1}$

$$= \frac{1}{8}\left[1-\frac{x}{4}\right]^{-1}$$

$$= \frac{1}{8}\left[1+(-1)\left(-\frac{x}{4}\right)+\frac{(-1)(-2)}{\lfloor 2}\left(-\frac{x}{4}\right)^2+\frac{(-1)(-2)(-3)}{\lfloor 3}\left(-\frac{x}{4}\right)^3+\cdots\right]$$

$$= \frac{1}{8}\left[1+\frac{x}{4}+\frac{x^2}{16}+\frac{x^3}{64}+\cdots\right]$$

$$= \frac{1}{8}+\frac{x}{32}+\frac{x^2}{128}+\frac{x^3}{512}+\cdots$$

Validity: $\left|-\frac{x}{4}\right|<1 \Rightarrow |x|<4$

vi) $(2-3x)^{-2} = 2^{-2}\left(1-\frac{3x}{2}\right)^{-2}$

$$= \frac{1}{4}\left[1+2\cdot\frac{3x}{2}+\frac{(-2)(-3)}{\lfloor 2}\left(-\frac{3x}{2}\right)^2+\frac{(-2)(-3)(-4)}{\lfloor 3}\left(-\frac{3x}{2}\right)^3+\cdots\right]$$

$$= \frac{1}{4}\left[1+3x+\frac{27x^2}{4}+\frac{27x^3}{2}+\cdots\right]$$

$$= \frac{1}{4}+\frac{3x}{4}+\frac{27x^2}{16}+\frac{27x^3}{8}+\cdots$$

Validity: $\left|-\frac{3x}{2}\right|<1 \Rightarrow |x|<\frac{2}{3}$

vii) $\frac{(1-x)^{-1}}{(1+x)^2} = (1-x)^{-1}(1+x)^{-2}$

$$= \left\{1+(-1)(-x)+\frac{(-1)(-2)}{\lfloor 2}(-x)^2+\frac{(-1)(-2)(-3)}{\lfloor 3}(-x)^3+\cdots\right\}\cdot\left\{1+(-2)(x)+\frac{(-2)(-3)}{\lfloor 2}x^2+\frac{(-2)(-3)(-4)}{\lfloor 3}x^3+\cdots\right\}$$

$$= \left[1+x+x^2+x^3+\cdots\right]\left[1-2x+3x^2-4x^3+\cdots\right]$$

$$= \left[1+x+x^2+x^3-2x-2x^2-2x^3-2x^4+3x^2+3x^3+3x^4+3x^5-4x^3-4x^4+\cdots\right]$$

$$= 1-x+2x^2-2x^3+\cdots$$

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viii) $\frac{\sqrt{1+2x}}{(1-x)} = (1+2x)^{1/2}(1-x)^{-1}$

$= \left\{1+\frac{1}{2}(2x)+\frac{\frac{1}{2}(\frac{1}{2}-1)}{\underline{|2}}(2x)^2+\frac{\frac{1}{2}(\frac{1}{2}-1)(\frac{1}{2}-2)}{\underline{|3}}(2x)^3+\cdots\right\}\left\{1+(-1)(-x)+\frac{(-1)(-2)}{\underline{|2}}(-x)^2+\frac{(-1)(-2)(-3)}{\underline{|3}}(-x)^3+\cdots\right\}$

$= \left[1+x+\frac{\frac{1}{2}(-\frac{1}{2})}{2}\cdot 4x^2+\frac{\frac{1}{2}(-\frac{1}{2})(-\frac{3}{2})}{6}8x^3+\cdots\right]\left[1+x+x^2+x^3+\cdots\right]$

$= \left[1+x-\frac{x^2}{2}+\frac{x^3}{2}+\cdots\right]\left[1+x+x^2+x^3+\cdots\right]$

$= 1+x-\frac{x^2}{2}+\cancel{\frac{x^3}{2}}+x+x^2-\cancel{\frac{x^3}{2}}+\frac{x^4}{2}+x^2+x^3-\frac{x^4}{2}+\frac{x^5}{2}+x^3+x^4-\frac{x^5}{2}+\frac{x^6}{2}+\cdots$

$= 1+2x+\frac{3}{2}x^2+2x^3+\cdots$

ix) $\frac{(4+2x)^{1/2}}{(2-x)} = (4+2x)^{1/2}\cdot(2-x)^{-1} = 4^{1/2}\left[1+\frac{x}{2}\right]^{1/2}\cdot 2^{-1}\left(1-\frac{x}{2}\right)^{-1}$

$= 2\cdot\frac{1}{2}\left[1+\frac{x}{2}\right]^{1/2}\cdot(1-x/2)^{-1} = \left(1+\frac{x}{2}\right)^{1/2}\cdot\left(1-x/2\right)^{-1}$

$= \left[1+\frac{1}{2}\left(\frac{x}{2}\right)+\frac{\frac{1}{2}(\frac{1}{2}-1)}{\underline{|2}}\left(\frac{x}{2}\right)^2+\frac{\frac{1}{2}(\frac{1}{2}-1)(\frac{1}{2}-2)}{\underline{|3}}\left(\frac{x}{2}\right)^3+\cdots\right]\left[1+(-1)\left(\frac{-x}{2}\right)+\frac{(-1)(-2)}{\underline{|2}}\left(\frac{-x}{2}\right)^2+\frac{(-1)(-2)(-3)}{\underline{|3}}\left(\frac{-x}{2}\right)^3+\cdots\right]$

$= \left[1+\frac{x}{4}-\frac{x^2}{32}+\frac{x^3}{128}+\cdots\right]\left[1+\frac{x}{2}+\frac{x^2}{4}+\frac{x^3}{8}+\cdots\right]$

$= 1+\frac{x}{4}-\frac{x^2}{32}+\frac{x^3}{128}+\frac{x}{2}+\frac{x^2}{8}-\frac{x^3}{64}+\frac{x^4}{512}+\frac{x^2}{4}+\frac{x^3}{16}-\frac{x^4}{132}+\frac{x^3}{8}+\frac{x^4}{32}+\cdots$

$= 1+\frac{3x}{4}+\frac{11x^2}{32}+\frac{23x^3}{128}+\cdots$

x) $(1+x-2x^2)^{1/2} = \left[1+(x-2x^2)\right]^{1/2}$

$= 1+\frac{1}{2}(x-2x^2]+\frac{\frac{1}{2}(\frac{1}{2}-1)}{\underline{|2}}(x-2x^2)^2+\frac{\frac{1}{2}(\frac{1}{2}-1)(\frac{1}{2}-2)}{\underline{|3}}(x-2x^2)^3+\cdots$

$= 1+\frac{x}{2}-x^2-\frac{1}{8}\left[x^2+4x^4-4x^3\right]+\frac{1}{16}\left[x^3-8x^6-6x^4+12x^5\right]+\cdots$

$= 1+\frac{x}{2}-x^2-\frac{x^2}{8}+\frac{x^3}{2}-\frac{x^4}{2}+\frac{x^3}{16}-\frac{x^6}{2}-\frac{3x^4}{8}+\frac{3x^5}{4}+\cdots$

$= 1+\frac{x}{2}-\frac{9x^2}{8}+\frac{9x^3}{8}+\cdots$

xi) $(1-2x+3x^2)^{1/2} = \left[1-(2x-3x^2)\right]^{1/2}$

$= 1-\frac{1}{2}(2x-3x^2)+\frac{\frac{1}{2}(\frac{1}{2}-1)}{\underline{|2}}(2x-3x^2)^2-\frac{\frac{1}{2}(\frac{1}{2}-1)(\frac{1}{2}-2)}{\underline{|3}}(2x-3x^2)^3+\cdots$

$= 1-x+\frac{3x^2}{2}-\frac{1}{8}\left[4x^2+9x^4-12x^3\right]-\frac{1}{16}\left[8x^3-27x^8+\cdots\right]+\cdots$

$= 1-x+\frac{3}{2}x^2-\frac{x^2}{2}-\frac{9}{8}x^4+\frac{3}{2}x^3-\frac{x^3}{2}+\cdots$

$= 1-x+x^2+x^3+\cdots$

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Q.2 Find the value using Binomial Series.

i) $\sqrt{99} = (100-1)^{1/2} = 100^{1/2}\left(1-\frac{1}{100}\right)^{1/2}$

$= 10[1-0.01]^{1/2}$

$= 10\left[1+\frac{1}{2}(-0.01)+\frac{\frac{1}{2}(\frac{1}{2}-1)}{\underline{|2}}(-0.01)^2+\cdots\right]$

$= 10[1-0.005-0]$

$= 10[0.995]$

$= 9.950$ Ans

ii) $(0.98)^{1/2} = [1-0.02]^{1/2}$

$= 1+\frac{1}{2}(-0.02)+\frac{\frac{1}{2}(\frac{1}{2}-1)}{\underline{|2}}(-0.02)^2+\cdots$

$= 1-0.010-0$

$= 0.990$ Ans.

iii) $(1.03)^{1/3} = [1+0.03]^{1/3}$

$= 1+\frac{1}{3}(0.03)+\frac{\frac{1}{3}(\frac{1}{3}-1)}{\underline{|2}}(0.03)^2+\cdots$

$= 1+0.010+0$

$= 1.010$ Ans.

iv) $\sqrt[3]{65} = (64+1)^{1/3}$

$= 64^{1/3}\left(1+\frac{1}{64}\right)^{1/3}$

$= 4[1+0.016]^{1/3}$

$= 4\left[1+\frac{1}{3}(0.016)+\frac{\frac{1}{3}(\frac{1}{3}-1)}{\underline{|2}}(0.016)^2+\cdots\right]$

$= 4[1+0.005+0]$

$= 4[1.005] = 4.020$ Ans

v) $\sqrt[4]{17} = (16+1)^{1/4}$.

$= 16^{1/4}\left(1+\frac{1}{16}\right)^{1/4}$

$= 2[1+0.063]^{1/4}$

$= 2\left[1+\frac{1}{4}[0.063]+\frac{\frac{1}{4}(\frac{1}{4}-1)}{\underline{|2}}(0.063)^2+\cdots\right]$

$= 2[1+0.016+0]$

$= 2[1.016] = 2.032$ Ans.

vi) $\sqrt[5]{31} = (32-1)^{1/5}$

$= 32^{1/5}\left(1-\frac{1}{32}\right)^{1/5}$

$= 2[1-0.031]^{1/5}$

$= 2\left[1+\frac{1}{5}(-0.031)+\frac{\frac{1}{5}(\frac{1}{5}-1)}{\underline{|2}}(-0.031)^2+\cdots\right]$

$= 2[1-0.006+0]$

$= 2[0.994] = 1.988$ Ans.

vii) $\frac{1}{\sqrt[3]{998}} = (1000-2)^{-1/3}$

$= 1000^{-1/3}\left[1-\frac{1}{500}\right]^{-1/3}$

$= \frac{1}{10}\left[1-0.02\right]^{-1/3}$

$= \frac{1}{10}\left[1-\frac{1}{3}(-0.02)+\frac{-\frac{1}{3}(-\frac{1}{3}-1)}{\underline{|2}}(-0.02)^2+\cdots\right]$

$= \frac{1}{10}[1+0.001+0]$

$= \frac{1}{10}[1.001] = 0.100$ Ans.

viii) $\frac{1}{\sqrt[5]{252}} = (243+9)^{-1/5}$

$= 243^{-1/5}\left(1+\frac{9}{243}\right)^{-1/5}$

$= \frac{1}{3}[1+0.037]^{-1/5}$

$= \frac{1}{3}\left[1-\frac{1}{5}(0.037)+\frac{-\frac{1}{5}(-\frac{1}{5}-1)}{\underline{|2}}(0.037)^2+\cdots\right]$

$= \frac{1}{3}[1-0.007+0]$

$= \frac{1}{3}[0.993] = 0.331$ Ans.

ix) $\frac{\sqrt{7}}{\sqrt{8}} = \sqrt{\frac{7}{8}} = \sqrt{0.875} = (1-0.125)^{1/2}$

$= 1+\frac{1}{2}(-0.125)+\frac{\frac{1}{2}(\frac{1}{2}-1)}{\underline{|2}}(-0.125)^2+\cdots$

$= 1-0.063+0$

$= 0.938$ Ans.

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x) $(0.998)^{-1/3} = [1-0.002]^{-1/2}$

$= 1-\frac{1}{2}[-0.002] + \frac{-\frac{1}{2}(-\frac{1}{2}-1)}{\lfloor 2}(-0.002)^2 + \cdots$

$= 1 + 0.001 + 0$

$= 1.001$ Ans.

xi) $\frac{1}{\sqrt[6]{486}} = (729-243)^{-1/6}$

$= 729^{-1/6}\left[1-\frac{243}{729}\right]^{-1/6}$

$= \frac{1}{3}[1-0.333]^{-1/6}$

$= \frac{1}{3}\left[1-\frac{1}{6}(-0.333) + \frac{-\frac{1}{6}(\frac{-1}{6}-1)}{\lfloor 2}(0.333)^2 + \cdots\right]$

$= \frac{1}{3}[1+0.056-0.008+0]$

$= \frac{1}{3}[1.048] = 0.349$ Ans.

xii) $(1280)^{1/4} = (1296-16)^{1/4}$

$= 1296^{1/4}\left[1-\frac{16}{1296}\right]^{1/4}$

$= 6[1-0.012]^{1/4}$

$= 6\left[1+\frac{1}{4}(-0.012) + \frac{\frac{1}{4}(\frac{1}{4}-1)}{\lfloor 2}(-0.012)^2 + \cdots\right]$

$= 6[1-0.003+0]$

$= 6[1-0.003] = 6[0.997]$

$= 5.982$ Ans.

Q.3 Find the Coefficient of x^n:

$\frac{1+x^2}{(1+x)^2} = (1+x^2)[1+x]^{-2}$

$= (1+x^2)\left[1+(-2)x + \frac{(-2)(-3)}{\lfloor 2}(x)^2 + \cdots\cdots + \frac{(-2)(-3)\cdots(-(n-1))}{\lfloor n-2}x^{n-2} + \cdots + \frac{(-2)(-3)\cdots(-(n+1))}{\lfloor n}x^n\right]$

$= (1+x^2)[1-2x+3x^2+\cdots+(-1)^{n-2}(n-1)x^{n-2} + \cdots + (-1)^n x^n(n+1)]$

Coeff. of $x^n = (-1)^n(n+1) + (-1)^{n-2}(n-1)$

$= (-1)^n[n+1+(-1)^{-2}(n-1)]$

$= (-1)^n[n+1+n-1] = 2n(-1)^n$

ii) $\frac{(1+x)^2}{(1-x)^2} = (1+2x+x^2)[1-x]^{-2}$

$= (1+2x+x^2)\left[1+2x+\frac{-2(-3)}{\lfloor 2}x^2 + \cdots + \frac{(-2)(-3)\cdots(-(n-1))}{\lfloor n-2}x^{n-2} + \frac{(-2)(-3)\cdots(-(n))}{\lfloor n-1}x^{n-1} + \frac{(-2)(-3)\cdots(-(n+1))}{\lfloor n}x^n\right]$

$= (1+2x+x^2)[1+2x+3x^2+\cdots + (n-1)x^{n-2} + nx^{n-1} + (n+1)x^n]$

Coefficient of $x^n = (n+1)+2n+(n-1)$

$= n+1+2n+n-1$

$= 4n.$

iii) $\frac{(1+x)^3}{(1-x)^2} = (1+x)^3[1-x]^{-2}$

$= (1+3x+3x^2+x^3)[1+2x+3x^2+\cdots\cdots + (n-2)x^{n-3} + (n-1)x^{n-2} + nx^{n-1} + (n+1)x^n]$

Coeff. of $x^n = (n+1)+3n+3(n-1)+n-2$

$= n+1+3n+3n-3+n-2$

$= 8n-4$

iv) $\frac{(1+x)^2}{(1-x)^3} = (1+2x+x^2)[1-x]^{-3}$

$= (1+2x+x^2)\left[1+3x+\frac{(-3)(-4)}{\lfloor 2}x^2 + \cdots + \frac{(-3)(-4)\cdots(-n)}{\lfloor n-2}x^{n-2} + \frac{(-3)(-4)\cdots(-(n+1))}{\lfloor n-1}x^{n-1} + \frac{(-3)(-4)\cdots(-(n+2))}{\lfloor n}x^n\right]$

$= (1+2x+x^2)\left[1+\frac{3!}{2}x + \frac{4!}{2\lfloor 2}x^2 + \cdots + \frac{\lfloor n}{2\lfloor n-2}x^{n-2} + \frac{\lfloor n+1}{2\lfloor n-1}x^{n-1} + \frac{\lfloor n+2}{2\lfloor n}x^n\right]$

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Coeffient of x^n =

$$\frac{\lfloor n+2}{2\lfloor n}+\frac{2\lfloor n+1}{2\lfloor n-1}+\frac{\lfloor n}{2\lfloor n-2}$$

$$=\frac{(n+2)(n+1)}{2}+(n+1)n+\frac{n(n-1)}{2}$$

$$=\frac{n^2+3n+2+2n^2+2n+n^2-n}{2}$$

$$=\frac{4n^2+5n-n+2}{2}$$

$$=\frac{4n^2+4n+2}{2}$$

$$=2n^2+2n+1$$

v) $(1-x+x^2-x^3+\cdots)^2$

We know that

$$(1+x)^{-1}=1-x+x^2-x^3+\cdots$$

$$(1-x+x^2-x^3+\cdots)^2=[(1+x)^{-1}]^2$$

$$=(1+x)^{-2}$$

Term involving x^n is

$$T_{n+1}=\frac{(-2)(-3)(-4)\cdots(-2-(n-1))}{n!}x^n$$

$$=\frac{(-1)^n[2.3.4\cdots(2+n-1)]}{n!}x^n$$

$$=\frac{(-1)^n[1.2.3.4\cdots(n+1)]}{n!}x^n$$

$$=\frac{(-1)^n(n+1)!}{n!}x^n$$

$$=(-1)^n\frac{(n+1)\cancel{n!}}{\cancel{n!}}x^n$$

$$T_{n+1}=(-1)^n(n+1)x^n$$

Coeff. of $x^n=(-1)^n(n+1)$.

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Q.4 If x is so small that its square and higher powers are neglected then Show that:

i) $\frac{1-x}{\sqrt{1+x}}\approx 1-\frac{3}{2}x$

$$LHS=\frac{1-x}{\sqrt{1+x}}=(1-x)(1+x)^{-1/2}$$

$$=(1-x)\left[1-\frac{1}{2}x+\text{neglected term}\right]$$

$$=(1-x)\left(1-\frac{x}{2}\right)$$

$$=1-x-\frac{x}{2}+\frac{x^2}{2}$$

$$=1-\frac{3}{2}x \quad \text{neglecting } x^2$$

$$=RHS$$

$$\Rightarrow \frac{1-x}{\sqrt{1+x}}\approx 1-\frac{3}{2}x \quad \text{(Proved)}$$

ii) $\frac{\sqrt{1+2x}}{\sqrt{1-x}}\approx 1+\frac{3}{2}x$

$$LHS=\frac{\sqrt{1+2x}}{\sqrt{1-x}}=(1+2x)^{1/2}(1-x)^{-1/2}$$

$$=\left[1+\frac{1}{2}(2x)+\cdots\right]\left[1-\frac{1}{2}(-x)+\cdots\right]$$

$$=(1+x)\left(1+\frac{x}{2}\right) \quad \text{neglecting } x^2 \text{ terms.}$$

$$\approx 1+x+\frac{x}{2}+\frac{x^2}{2}$$

$$\approx 1+\frac{3}{2}x \quad \text{neglecting } x^2$$

$$\approx RHS$$

$$\Rightarrow \frac{\sqrt{1+2x}}{\sqrt{1-x}}\approx 1+\frac{3}{2}x \quad \text{(Proved)}$$

iii) $\frac{(9+7x)^{1/2}-(16+3x)^{1/4}}{4+5x}\approx\frac{1}{4}-\frac{17}{384}x$

$$LHS=\frac{(9+7x)^{1/2}-(16+3x)^{1/4}}{(4+5x)}$$

$$=\frac{9^{1/2}\left(1+\frac{7}{9}x\right)^{1/2}-16^{1/4}\left(1+\frac{3}{16}x\right)^{1/4}}{4\left[1+\frac{5x}{4}\right]}$$

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$$= \frac{3\left[1+\frac{1}{2}\left(\frac{7}{9}x\right)\right]-2\left[1+\frac{1}{4}\left(\frac{3}{16}x\right)\right]}{4\left[1+\frac{5}{4}x\right]}$$

$$= \frac{\left[3+\frac{7}{6}x\right]-\left[2+\frac{3}{32}x\right]}{4\left[1+\frac{5}{4}x\right]}$$

$$= \frac{1}{4}\left[3+\frac{7}{6}x-2-\frac{3}{32}x\right]\left[1+\frac{5}{4}x\right]^{-1}$$

$$= \frac{1}{4}\left[1+\left(\frac{7}{6}-\frac{3}{32}\right)x\right]\left[1-\frac{5}{4}x\right]$$

$$= \frac{1}{4}\left[1+\frac{112-9}{96}x\right]\left[1-\frac{5}{4}x\right]$$

$$= \frac{1}{4}\left[1+\frac{103}{96}x\right]\left[1-\frac{5}{4}x\right]$$

$$= \frac{1}{4}\left[1+\frac{103}{96}x-\frac{5}{4}x+\text{neglected terms}\right]$$

$$= \frac{1}{4}\left[1+\left(\frac{103}{96}-\frac{5}{4}\right)x\right]$$

$$= \frac{1}{4}\left[1+\frac{103-120}{96}x\right]$$

$$= \frac{1}{4}+\frac{-17}{384}x = \frac{1}{4}-\frac{17}{384}x = \text{RHS}$$

$\because x^2$ and higher powers neglecting.

$\Rightarrow$

iv) $\dfrac{\sqrt{4+x}}{(1-x)^3} = 2+\dfrac{25}{4}x$

$$\text{LHS} = \frac{\sqrt{4+x}}{(1-x)^3} = (4+x)^{1/2}(1-x)^{-3}$$

$$= 4^{1/2}\left[1+\frac{x}{4}\right]^{1/2}(1-x)^{-3}$$

$$= 2\left[1+\frac{1}{2}\left(\frac{x}{4}\right)+\cdots\right]\left[1+3x+\cdots\right]$$

$$= 2\left[1+\frac{x}{8}\right]\left[1+3x\right]$$

$$= \left(2+\frac{x}{4}\right)(1+3x)$$

$$= 2+\frac{x}{4}+6x+\frac{3}{4}x^2$$

$$= 2+\frac{25}{4}x+\text{neglecting terms}$$

$$= 2+\frac{25}{4}x = \text{RHS}$$

neglecting $x^2, x^3, \ldots$

$$\Rightarrow \frac{\sqrt{4+x}}{(1-x)^3} \approx 2+\frac{25}{4}x \text{ (Proved)}$$

v) $\dfrac{(1+x)^{1/2}(4-3x)^{3/2}}{(8+5x)^{1/3}} \approx 4\left[1-\dfrac{5}{6}x\right]$

$$\text{LHS} = \frac{(1+x)^{1/2}(4-3x)^{3/2}}{(8+5x)^{1/3}}$$

$$= \frac{(1+x)^{1/2}\,4^{3/2}\left(1-\frac{3}{4}x\right)^{3/2}}{8^{1/3}\left[1+\frac{5}{8}x\right]^{1/3}}$$

$$= \frac{\left[1+\frac{1}{2}x\right]\cdot 2^3\left[1-\frac{3}{2}\cdot\frac{3}{4}x\right]}{2\left[1+\frac{5}{8}x\right]^{1/3}}$$

$$= 4\left[1+\frac{x}{2}\right]\left[1-\frac{9}{8}x\right]\left[1+\frac{5}{8}x\right]^{-1/3}$$

$$= 4\left[1+\frac{x}{2}\right]\left[1-\frac{9}{8}x\right]\left[1-\frac{5}{24}x\right]$$

$$= 4\left[1+\frac{x}{2}-\frac{9}{8}x+x^2\text{ term}\right]\left[1-\frac{5}{24}x\right]$$

$$= 4\left[1+\left(\frac{1}{2}-\frac{9}{8}\right)x\right]\left[1-\frac{5}{24}x\right]$$

$$= 4\left[1+\frac{4-9}{8}x\right]\left[1-\frac{5}{24}x\right]$$

$$= 4\left[1-\frac{5}{8}x\right]\left[1-\frac{5}{24}x\right]$$

$$= 4\left[1-\frac{5}{8}x-\frac{5}{24}x+x^2\text{ term}\right]$$

$$= 4\left[1-\left(\frac{5}{8}+\frac{5}{24}\right)x\right]$$

$$= 4\left[1-\frac{15+5}{24}x\right] = 4\left[1-\frac{20}{24}x\right]$$

$$= 4\left[1-\frac{5}{6}x\right] = \text{RHS.}$$

x^2 and higher power neg.

vi) $\dfrac{(1-x)^{1/2}(9-4x)^{1/2}}{(8+3x)^{1/3}} \approx \dfrac{3}{2}-\dfrac{61}{48}x$

$$\text{LHS} = \frac{(1-x)^{1/2}\cdot(9-4x)^{1/2}}{(8+3x)^{1/3}}$$

$$= \frac{(1-x)^{1/2}\,9^{1/2}\left[1-\frac{4}{9}x\right]^{1/2}}{8^{1/3}\left[1+\frac{3}{8}x\right]^{1/3}}$$

$$= \frac{\left[1-\frac{1}{2}x\right]3\left[1-\frac{1}{2}\cdot\frac{4}{9}x\right]}{2}\cdot\left(1+\frac{3}{8}x\right)^{-1/3}$$

neglecting $x^2, x^3, \ldots$

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$$\approx \frac{3}{2}\left(1-\frac{x}{2}\right)\left(1-\frac{2}{9}x\right)\left(1-\frac{1}{3}\left(\frac{3}{8}x\right)\right)$$

$$\approx \frac{3}{2}\left[1-\frac{x}{2}\right]\left[1-\frac{2}{9}x\right]\left[1-\frac{x}{8}\right]$$

$$\approx \frac{3}{2}\left[1-\frac{x}{2}-\frac{2x}{9}+x^2\text{term}\right]\left[1-\frac{x}{8}\right]$$

$$= \frac{3}{2}\left[1-\frac{9x+4x}{18}\right]\left[1-\frac{x}{8}\right]$$

$$\approx \frac{3}{2}\left[1-\frac{13}{18}x\right]\left[1-\frac{x}{8}\right]$$

$$\approx \frac{3}{2}\left[1-\frac{13}{18}x-\frac{x}{8}+x^2\text{term}\right]$$

$$\approx \frac{3}{2}\left[1-\frac{52x+9x}{72}\right]$$

$$\approx \frac{3}{2}\left[1-\frac{61}{72}x\right]$$

$$\approx \frac{3}{2}-\frac{61}{72}\times\frac{3}{2}x$$

$$\approx \frac{3}{2}-\frac{61}{48}x = \text{RHS}.$$

neglecting x^2 and higher Powers.

vii) $\dfrac{\sqrt{4-x}+(8-x)^{1/3}}{(8-x)^{1/3}} \approx 2-\dfrac{x}{12}$

$$\text{LHS} = \frac{\sqrt{4-x}}{(8-x)^{1/3}}+\cancel{\frac{(8-x)^{1/3}}{(8-x)^{1/3}}}$$

$$\approx 1+(4-x)^{1/2}(8-x)^{-1/3}$$

$$\approx 1+4^{1/2}\left[1-\frac{x}{4}\right]^{1/2}8^{-1/3}\left[1-\frac{x}{8}\right]^{-1/3}$$

$$\approx 1+\cancel{2}\left[1-\frac{x}{8}+x^2\text{term}\right]\frac{1}{\cancel{2}}\left[1+\frac{x}{24}+x^2\text{term}\right]$$

$$\approx 1+\left(1-\frac{x}{8}\right)\left(1+\frac{x}{24}\right)$$

$$\approx 1+1-\frac{x}{8}+\frac{x}{24}+x^2\text{term}$$

$$\approx 2+\frac{x-3x}{24}$$

$$\approx 2-\frac{2}{24}x = 2-\frac{1}{12}x = \text{RHS}$$

neglecting $x^2, x^3, \ldots$

Q.5 If x is so small that x^3 and higher Powers are negligible, show that:

i) $\sqrt{1-x-2x^2} \approx 1-\frac{1}{2}x-\frac{9}{8}x^2$

$$\text{LHS} = \sqrt{1-x-2x^2} = \left[1-(x+2x^2)\right]^{1/2}$$

$$\approx 1-\frac{1}{2}(x+2x^2)+\frac{\frac{1}{2}\left(\frac{1}{2}-1\right)}{\underline{|2}}\left[-(x+2x^2)\right]^2+\cdots$$

$$\approx 1-\frac{x}{2}-x^2-\frac{1}{8}\left[x^2+4x^4+4x^3\right]+\cdots$$

$$\approx 1-\frac{x}{2}-x^2-\frac{x^2}{8}+x^3\text{terms}.$$

$$\approx 1-\frac{x}{2}-\frac{9}{8}x^2 = \text{RHS}.$$

neglecting $x^3, x^4, \ldots$

ii) $\sqrt{\dfrac{1+x}{1-x}} \approx 1+x+\frac{1}{2}x^2$

$$\text{LHS} = \sqrt{\frac{1+x}{1-x}} = (1+x)^{1/2}(1-x)^{-1/2}$$

$$= \left[1+\frac{x}{2}+\frac{\frac{1}{2}\left(\frac{1}{2}-1\right)}{\underline{|2}}x^2+\cdots\right]\left[1+\frac{x}{2}+\frac{-\frac{1}{2}\left(-\frac{1}{2}-1\right)}{\underline{|2}}x^2+\cdots\right]$$

$$\approx \left[1+\frac{x}{2}-\frac{x^2}{8}\right]\left[1+\frac{x}{2}+\frac{3x^2}{8}\right]$$

$$\approx 1+\frac{x}{2}+\frac{3x^2}{8}+\frac{x}{2}+\frac{x^2}{4}-\frac{x^2}{8}+x^3\text{terms}.$$

$$\approx 1+x+\left(\frac{3}{8}-\frac{1}{8}+\frac{1}{4}\right)x^2$$

$$\approx 1+x+\frac{3-1+2}{8}x^2$$

$$\approx 1+x+\frac{4}{8}x^2$$

$$\approx 1+x+\frac{1}{2}x^2 = \text{RHS}$$

neglecting $x^3, x^4, \ldots$

Q.6 If x is nearly equal to 1, show that $px^p - qx^q \approx (p-q)x^{p+q}$

Sol: Let $x = 1+h$ where h^2 and higher Powers are negligible.

$$px^p - qx^q = p(1+h)^p - q(1+h)^q$$

$$\approx p(1+ph)-q(1+qh)$$

$$\approx p+p^2h-q-q^2h$$

$$\approx (p-q)+(p^2-q^2)h$$

$$\approx (p-q)\left[1+(p+q)h\right]$$

$$\approx (p-q)\left[1+h\right]^{p+q}$$

$$\approx (p-q)\,x^{p+q}$$

$$\Rightarrow px^p - qx^q \approx (p-q)x^{p+q}$$

(Proved)

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Q.7 If $p-q$ is very small when compared with p and q, show that

$$\frac{(2n+1)p+(2n-1)q}{(2n-1)p+(2n+1)q}=\left(\frac{p+q}{2q}\right)^{1/n}$$

Sol:- Let $p-q=h \Rightarrow p=q+h$

where h^2 and higher power are negligible

$$LHS=\frac{(2n+1)p+(2n-1)q}{(2n-1)p+(2n+1)q}$$

$$=\frac{(2n+1)(q+h)+(2n-1)q}{(2n-1)(q+h)+(2n+1)q}$$

$$=\frac{2nq+2nh+\cancel{q}+h+2nq-\cancel{q}}{2nq+2nh-\cancel{q}-h+2nq+\cancel{q}}$$

$$=\frac{4nq+(2n+1)h}{4nq+(2n-1)h}$$

$$=\frac{\cancel{4nq}\left[1+\left(\frac{2n+1}{4nq}\right)h\right]}{\cancel{4nq}\left[1+\left(\frac{2n-1}{4nq}\right)h\right]}$$

$$=\left[1+\left(\frac{2n+1}{4nq}\right)h\right]\left[1+\left(\frac{2n-1}{4nq}\right)h\right]^{-1}$$

$\because h^2$ and higher powers are negligible.

$$=\left[1+\left(\frac{2n+1}{4nq}\right)h\right]\left[1-\left(\frac{2n-1}{4nq}\right)h\right]$$

$$=1+\frac{2n+1}{4nq}h-\frac{2n-1}{4nq}h+h^2\text{ term}$$

$$=1+\frac{h}{4nq}\left[\cancel{2n}+1-\cancel{2n}-1\right]$$

$$=1+\frac{2h}{4nq}=1+\frac{h}{2nq}$$

$$=1+\left(\frac{1}{n}\right)\frac{h}{2q}=\left(1+\frac{h}{2q}\right)^{1/n}$$

$$=\left(\frac{2q+h}{2q}\right)^{1/n}=\left(\frac{2q+p-q}{2q}\right)^{1/n}$$

$$=\left(\frac{p+q}{2q}\right)^{1/n}=RHS.$$

Q.8 Show $\left[\frac{n}{2(n+N)}\right]^{1/2}=\frac{8n}{9n-N}-\frac{n+N}{4n}$

where n and N are nearly equal.

Sol:- Let $N=n+h$ where h^2 and higher powers are negligible.

$$LHS=\left[\frac{n}{2(n+N)}\right]^{1/2}=\left[\frac{n}{2(n+n+h)}\right]^{1/2}$$

$$=\left[\frac{n}{2(2n+h)}\right]^{1/2}=\frac{n^{1/2}}{2^{1/2}}\left[2n+h\right]^{-1/2}$$

$$=\frac{n^{1/2}}{2^{1/2}}(2n)^{-1/2}\left[1+\frac{h}{2n}\right]^{-1/2}$$

$$=\frac{\cancel{n^{1/2}}}{2^{1/2}\cdot 2^{1/2}\cancel{n^{1/2}}}\left[1-\frac{h}{4n}\right]$$

$$=\frac{1}{2}\left[1-\frac{h}{4n}\right] \quad \text{—— (1)}$$

$$RHS=\frac{8n}{9n-N}-\frac{n+N}{4n}$$

$$=\frac{8n}{9n-n-h}-\frac{n+n+h}{4n}$$

$$=\frac{8n}{8n-h}-\frac{2n+h}{4n}$$

$$=\frac{32n^2-(2n+h)(8n-h)}{4n(8n-h)}$$

$$=\frac{32n^2-16n^2+2nh-8nh+h^2}{4n(8n-h)}$$

neglecting h^2

$$=\frac{16n^2-6nh}{4n(8n-h)}=\frac{2n[8n-3h]}{4n[8n-h]}$$

$$=\frac{1}{2}\frac{[8n-h-2h]}{[8n-h]}$$

$$=\frac{1}{2}\left[1-\frac{2h}{8n-h}\right]$$

$\because h$ is so small so $8n-h\approx 8n$

$$=\frac{1}{2}\left[1-\frac{2h}{8n}\right]$$

$$=\frac{1}{2}\left[1-\frac{h}{4n}\right] \quad \text{—— (2)}$$

From (1) and (2)

$$LHS=RHS$$

Q.9 Identify and Sum the Series:

i) $1-\frac{1}{2}\left(\frac{1}{4}\right)+\frac{1\cdot 3}{2!\,4}\left(\frac{1}{4}\right)^2+\cdots$

Compare it with

$$(1+x)^n=1+nx+\frac{n(n-1)}{\underline{2}}x^2+\cdots$$

$$nx = \frac{-1}{8} \quad \text{and} \quad \frac{n(n-1)}{\lfloor 2}x^2 = \frac{1.3}{\lfloor 2\, 4}\left(\frac{1}{4}\right)^2$$

$$x = \frac{-1}{8n} \quad \text{and} \quad \frac{n(n-1)}{\cancel{\lfloor 2}}\left[\frac{-1}{8n}\right]^2 = \frac{3}{\cancel{2}.4}\,\frac{1}{16}$$

$$x = \frac{-1}{8[-\frac{1}{2}]} \qquad \frac{n(n-1)}{64n^2} = \frac{3}{64}$$

$$x = \frac{1}{4} \qquad \Rightarrow \frac{n-1}{n} = 3 \Rightarrow n-1 = 3n$$

$$\Rightarrow 2n = -1 \Rightarrow n = -\frac{1}{2}$$

So

$$1 - \frac{1}{2}\left(\frac{1}{4}\right) + \frac{1.3}{2!\,4}\left(\frac{1}{4}\right)^2 + \cdots = (1+x)^n = \left[1+\frac{1}{4}\right]^{-1/2}$$

$$= \left[\frac{4+1}{4}\right]^{-1/2} = \left(\frac{5}{4}\right)^{-1/2}$$

$$= \left(\frac{4}{5}\right)^{1/2} = \frac{2}{\sqrt{5}}$$

ii) $1 - \frac{1}{2}\left(\frac{1}{2}\right) + \frac{1.3}{2.4}\left(\frac{1}{2}\right)^2 + \cdots$

Compare with

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{\lfloor 2}x^2 + \cdots$$

$$nx = -\frac{1}{2}\left(\frac{1}{2}\right) \quad \text{and} \quad \frac{n(n-1)}{\lfloor 2}x^2 = \frac{1.3}{2.4}\left(\frac{1}{2}\right)^2$$

$$x = \frac{-1}{4n} \quad \text{and} \quad \frac{n(n-1)}{2}\left[\frac{-1}{4n}\right]^2 = \frac{3}{8}\cdot\frac{1}{4}$$

$$x = \frac{-1}{4[-\frac{1}{2}]} \qquad \frac{n(n-1)}{2}\cdot\frac{1}{16n^2} = \frac{3}{32}$$

$$x = \frac{1}{2} \qquad \frac{n-1}{n} = 3 \Rightarrow n-1 = 3n$$

$$2n = -1 \Rightarrow n = -\frac{1}{2}$$

So

$$1 - \frac{1}{2}\left(\frac{1}{2}\right) + \frac{1.3}{2.4}\left(\frac{1}{2}\right)^2 + \cdots = (1+x)^n = \left[1+\frac{1}{2}\right]^{-1/2}$$

$$= \left[\frac{2+1}{2}\right]^{-1/2} = \left[\frac{3}{2}\right]^{-1/2}$$

$$= \sqrt{\frac{2}{3}}$$

iii) $1 + \frac{3}{4} + \frac{3.5}{4.8} + \cdots$

Compare with

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{\lfloor 2}x^2 + \cdots$$

$$nx = \frac{3}{4} \quad \text{and} \quad \frac{n(n-1)}{\lfloor 2}x^2 = \frac{3.5}{4.8}$$

$$x = \frac{3}{4n} \quad \text{and} \quad \frac{n(n-1)}{\lfloor 2}\left(\frac{3}{4n}\right)^2 = \frac{15}{32}$$

$$x = \frac{3}{4[-\frac{3}{2}]} \qquad \frac{\cancel{n}(n-1)}{\cancel{2}}\,\frac{\cancel{9}\,3}{\cancel{16n^2}} = \frac{\cancel{15}\,5}{\cancel{32}}$$

$$3n - 3 = 5n$$

$$x = -\frac{1}{2} \qquad -3 = 2n \Rightarrow n = -\frac{3}{2}$$

So

$$1 + \frac{3}{4} + \frac{3.5}{4.8} + \cdots = (1+x)^n = \left[1-\frac{1}{2}\right]^{-3/2}$$

$$= \left(\frac{2-1}{2}\right)^{-3/2} = \left(\frac{1}{2}\right)^{-3/2}$$

$$= (2)^{3/2} = 2\sqrt{2}$$

iv) $1 - \frac{1}{2}\cdot\frac{1}{3} + \frac{1.3}{2.4}\left(\frac{1}{3}\right)^2 + \cdots$

Compare with

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{\lfloor 2}x^2 + \cdots$$

$$nx = -\frac{1}{2}\cdot\frac{1}{3} \quad \text{and} \quad \frac{n(n-1)}{\lfloor 2}x^2 = \frac{1.3}{2.4}\left(\frac{1}{3}\right)^2$$

$$x = \frac{-1}{6n} \quad \text{and} \quad \frac{n(n-1)}{2}\left[\frac{-1}{6n}\right]^2 = \frac{3}{72}$$

$$x = \frac{-1}{6[-\frac{1}{2}]} \qquad \frac{n(n-1)}{2}\cdot\frac{1}{36n^2} = \frac{3}{72}$$

$$x = \frac{1}{3} \qquad \frac{n-1}{n} = 3 \Rightarrow 2n = -1$$

$$n = -\frac{1}{2}$$

So

$$1 - \frac{1}{2}\cdot\frac{1}{3} + \frac{1.3}{2.4}\left(\frac{1}{3}\right)^2 + \cdots = (1+x)^n$$

$$= \left[1+\frac{1}{3}\right]^{-1/2}$$

$$= \left(\frac{3+1}{3}\right)^{-1/2} = \left(\frac{4}{3}\right)^{-1/2}$$

$$= \left(\frac{3}{4}\right)^{1/2} = \frac{\sqrt{3}}{2}$$

Q.10 Show $1+\frac{1}{4}+\frac{1.3}{4.8}+\cdots=\sqrt{2}$

Compare with

$(1+x)^n = 1+nx+\frac{n(n-1)}{\underline{|2}}x^2+\cdots$

$nx=\frac{1}{4}$ and $\frac{n(n-1)}{\underline{|2}}x^2=\frac{1.3}{4.8}$

$x=\frac{1}{4n}$ and $\frac{n(n-1)}{2}\left(\frac{1}{4n}\right)^2=\frac{3}{32}$

$x=\frac{1}{4[-\frac{1}{2}]}$ | $\frac{n(n-1)}{2.16n^2}=\frac{3}{32}$

$x=\frac{-1}{2}$ | $\frac{n-1}{n}=3 \Rightarrow n-1=3n$

$2n=-1 \Rightarrow n=-\frac{1}{2}$

So

$1+\frac{1}{4}+\frac{1.3}{4.8}+\cdots=(1+x)^n=\left(1-\frac{1}{2}\right)^{-\frac{1}{2}}$

$=\left[\frac{2-1}{2}\right]^{-\frac{1}{2}}=\left(\frac{1}{2}\right)^{-\frac{1}{2}}$

$=2^{\frac{1}{2}}=\sqrt{2}$

$\Rightarrow 1+\frac{1}{4}+\frac{1.3}{4.8}+\cdots=\sqrt{2}$ (Proved)

Q.11 $y=\frac{1}{3}+\frac{1.3}{2!}\left(\frac{1}{3}\right)^2+\cdots$

$1+y=1+\frac{1}{3}+\frac{1.3}{\underline{|2}}\left(\frac{1}{3}\right)^2+\cdots$

Compare with

$(1+x)^n=1+nx+\frac{n(n-1)}{\underline{|2}}x^2+\cdots$

$nx=\frac{1}{3}$ and $\frac{n(n-1)}{\underline{|2}}x^2=\frac{1.3}{\underline{|2}}\left(\frac{1}{3}\right)^2$

$x=\frac{1}{3n}$ and $\frac{n(n-1)}{\underline{|2}}\left[\frac{1}{3n}\right]^2=\frac{3}{\underline{|2}}\left[\frac{1}{3}\right]^2$

$x=\frac{1}{3[-\frac{1}{2}]}$ | $\frac{n(n-1)}{\underline{|2}}\cdot\frac{1}{9n^2}=\frac{3}{\underline{|2}}\cdot\frac{1}{9}$

$x=-\frac{2}{3}$ | $\frac{n-1}{n}=3 \Rightarrow 3n=n-1$

$2n=-1 \Rightarrow n=-\frac{1}{2}$

So $1+y=(1+x)^n$

$\Rightarrow 1+y=\left(1-\frac{2}{3}\right)^{-\frac{1}{2}}$

$\Rightarrow 1+y=\left[\frac{3-2}{3}\right]^{-\frac{1}{2}}$

$\Rightarrow 1+y=\left(\frac{1}{3}\right)^{-\frac{1}{2}}=3^{\frac{1}{2}}$

Squaring both sides

$(1+y)^2=3$

$y^2+2y+1=3$

$y^2+2y-2=0$ (Proved)

Q.12 $2y=\frac{1}{2^2}+\frac{1.3}{2!}\frac{1}{2^4}+\cdots$

$1+2y=1+\frac{1}{2^2}+\frac{1.3}{2!}\frac{1}{2^4}+\cdots$

Compare with

$(1+x)^n=1+nx+\frac{n(n-1)}{\underline{|2}}x^2+\cdots$

$nx=\frac{1}{2^2}$ and $\frac{n(n-1)}{\underline{|2}}x^2=\frac{1.3}{\underline{|2}}\frac{1}{2^4}$

$x=\frac{1}{4n}$ and $\frac{n(n-1)}{\underline{|2}}\left(\frac{1}{4n}\right)^2=\frac{3}{\underline{|2}}\cdot\frac{1}{16}$

$x=\frac{1}{4[-\frac{1}{2}]}$ | $\frac{n(n-1)}{\underline{|2}}\cdot\frac{1}{16n^2}=\frac{3}{\underline{|2}\,16}$

$x=\frac{-1}{2}$ | $\frac{n-1}{n}=3 \Rightarrow 3n=n-1$

$2n=-1 \Rightarrow n=-\frac{1}{2}$

So

$1+2y=(1+x)^n \Rightarrow (1+2y)=\left(1-\frac{1}{2}\right)^{-\frac{1}{2}}$

$1+2y=\left(\frac{2-1}{2}\right)^{-\frac{1}{2}} \Rightarrow 1+2y=\left(\frac{1}{2}\right)^{-\frac{1}{2}}$

$1+2y=2^{\frac{1}{2}}$

$(1+2y)^2=2$

$4y^2+4y+1=2$

$4y^2+4y-1=0$ (Proved)

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Q.13 $y = \frac{2}{5} + \frac{1.3}{\underline{|2}}\left(\frac{2}{5}\right)^2 + \cdots$

$1+y = 1 + \frac{2}{5} + \frac{1.3}{\underline{|2}}\left(\frac{2}{5}\right)^2 + \cdots$

Comparing with $(1+x)^n = 1 + nx + \frac{n(n-1)}{\underline{|2}}x^2 + \cdots$

$nx = \frac{2}{5}$

$x = \frac{2}{5n}$

so $x = \frac{2}{5[-\frac{1}{2}]}$

$x = -\frac{4}{5}$

and $\frac{n(n-1)}{\underline{|2}}x^2 = \frac{1.3}{\underline{|2}}\left(\frac{2}{5}\right)^2$

$\frac{n(n-1)}{\underline{|2}}\left(\frac{2}{5n}\right)^2 = \frac{3}{\underline{|2}}\cdot\left(\frac{2}{5}\right)^2$

$\frac{n(n-1)}{\cancel{\underline{|2}}}\cdot\frac{\cancel{4}}{\cancel{25}n^2} = \frac{3}{\cancel{\underline{|2}}}\cdot\frac{\cancel{4}}{\cancel{25}}$

$\frac{n-1}{n} = 3 \Rightarrow 3n = n-1$

$2n = -1 \Rightarrow n = -\frac{1}{2}$

So $1+y = (1+x)^n$

$1+y = \left(1-\frac{4}{5}\right)^{-\frac{1}{2}} = \left(\frac{5-4}{5}\right)^{-\frac{1}{2}} = \left(\frac{1}{5}\right)^{-\frac{1}{2}}$

$1+y = 5^{\frac{1}{2}}$ squareing both sides

$(1+y)^2 = 5 \Rightarrow y^2+2y+1 = 5$

$y^2+2y-4 = 0$ (Proved)

THE End

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