$81 - 81r^2 = 5(81 + 81r^2 - 162r)$

$81 - 81r^2 = 405 + 405r^2 - 810r$

$405r^2 + 81r^2 - 810r - 81 + 405 = 0$

$486r^2 - 810r + 324 = 0$

$162[3r^2 - 5r + 2] = 0 \qquad 162 \neq 0$

$3r^2 - 5r + 2 = 0$

$3r^2 - 3r - 2r + 2 = 0$

$3r(r-1) - 2(r-1) = 0$

$(r-1)(3r-2) = 0$

$r = 1 \quad , \quad r = 2/3$

Now $a = 9 - 9r$	Now $a = 9 - 9r$
$a = 9 - 9(1)$	$a = 9 - 9(2/3)$
$a = 9 - 9$	$a = 9 - 6$
$a = 0$	$a = 3$

There is no series for $a = 0$

So the series is

$3 + 3\left(\frac{2}{3}\right) + 3\left(\frac{2}{3}\right)^2 + 3\left(\frac{2}{3}\right)^3 + \cdots$

$3 + 2 + 4/3 + 8/9 + \cdots$

Exercise: 6.9

Q.1 $8, 24, 72, \ldots$ G.P

$S_5 = ? \quad a = 8 \quad r = \frac{24}{8} = 3$

$S_5 = \frac{a(r^5 - 1)}{r-1} = \frac{8(3^5 - 1)}{3-1}$

$S_5 = \frac{8(243-1)}{2} = 4(242)$

$S_5 =$ Rs. 968

So He will deposite 968 rupees after 5 years.

Q.2 $S_n = 32760$

$r = 2 \qquad a = 8$

$\because S_n = \frac{a(r^n - 1)}{r-1} \qquad r > 1$

$32760 = \frac{8(2^n - 1)}{2-1}$

$32760 = 8(2^n - 1)$

$2^n - 1 = \frac{32760}{8} = 4095$

$2^n = 4095 + 1 = 4096$

$2^n = (2)^{12} \Rightarrow n = 12$ ✓

Last installment

$a_n = ar^{n-1}$

$a_{12} = 8(2)^{11} = 8 \times 2048$

$a_{12} = 16384$

Q.3 $a = 62500 \qquad n = 4$

$r = 1 + \frac{4}{100} = \frac{100+4}{100} = \frac{104}{100} = 1.04$

$\because a_4 = ar^3$

$a_4 = 62500(1.04)^3$

$a_4 = 62500 \times 1.124864$

$a_4 = 70304$

Q.4 According to the Statement

1970, 1978, 1986, 1994

$a, 2a, 4a, 8a$

$a_4 = 6000 \quad r = \frac{2a}{a} = 2 \quad n = 4$

$a_4 = ar^3 \Rightarrow 6000 = a(2)^3$

$6000 = a8 \Rightarrow a = \frac{6000}{8}$

$a = 750$

Q.5 According to Statement

$\frac{1}{2}$ hours, 1 hours, $\frac{3}{2}$ hours, 2 hrs and so on the Complete becteria 2A, 4A, 8A, 16A... In Complete hours.

1, 2, 3, 4 and so on Complete bekterias will be 4A, 16A, 64A, ...

$a = 4A \quad r = \frac{16A}{4A} = 4 \quad n = n$

$a_n = ar^{n-1}$

$a_n = (4A).(4)^{n-1}$

$a_n = A.4^{n-1+1} \Rightarrow A4^n$

$a_n = A2^{2n}$ beterias.

Q.6 Parameter of origonal $\Delta = 3/2$

Parameter of equiletral Triangles are

$\frac{3}{2}, \frac{3/2}{2}, \frac{3/2}{2\times2}, \frac{3/2}{2\times2\times2}, \ldots$

$\frac{3}{2}, \frac{3}{4}, \frac{3}{8}, \frac{3}{16}, \ldots$ G.P.

$a = 3/2 \quad r = \frac{3/4}{3/2} = \frac{3}{4}\times\frac{2}{3} = \frac{1}{2} \quad S_\infty = ?$

$S_\infty = \frac{a}{1-r}$

$S_\infty = \frac{3/2}{1-1/2} = \frac{3/2}{\frac{2-1}{2}} = \frac{3}{1}$

$S_\infty = 3$

Hence the Sum of all the the equilateral triangles parameters is 3

Tahir Mahmood. TAHIR

Hormonic Sequence (H.P)

"A sequence whose number reciprocals form an Arithmetic Sequence is called Hormonic Sequence or Hormonic Progression"

we can expand H.P by using

$H_n = \frac{1}{a+(n-1)d}$

Note: Zero cannot be the term of Hormonic Progression.

Hormonic Mean: (H.M.)

A number H is called a Hormonic Mean between a and b if a, H, b are in H.P.

So $\frac{1}{a}, \frac{1}{H}, \frac{1}{b}$ are in A.P.

$\frac{1}{H} = \frac{\frac{1}{a}+\frac{1}{b}}{2} = \frac{\frac{a+b}{ab}}{2}$

$\frac{1}{H} = \frac{a+b}{2ab}$

So $H = \frac{2ab}{a+b}$ TAHIR

Relation b/w (A.M) and (G.M), (H.M)

For any two numbers a, b

$A = \frac{a+b}{2}$

$G = \pm\sqrt{ab} \Rightarrow G^2 = ab$

$H = \frac{2ab}{a+b}$

$A\times H = \frac{\cancel{a+b}}{\cancel{2}}\times\frac{\cancel{2}ab}{\cancel{a+b}} = ab$

$\boxed{A\times H = G^2}$