# Exercise: 6.8



**Q.1** G.P $= 1, \frac{1}{3}, \frac{1}{9}, \dots$, 15 terms

$a = 1 \quad r = 1/3 \quad n = 15$

$$S_n = \frac{a(1-r^n)}{1-r} \qquad r<1$$

$$S_{15} = \frac{1(1-(1/3)^{15})}{1-1/3}$$

$$S_{15} = \frac{3(1-(0.33)^{15})}{2} = \frac{3(0.999993)}{2}$$

$$S_{15} = 1.49999895$$

## Convergence of Infinite G. Series

An infinite Geometric Series is Convergent if $|r|<1$ then $r^n \to 0$ as $n \to \infty$

$$\Rightarrow S_\infty = \lim_{n\to\infty} \frac{a_1(1-r^n)}{1-r}$$

$$S_\infty = \frac{a_1(1-0)}{1-r}$$

$$S_\infty = \frac{a_1}{1-r}$$



**Q** Sum upto $n$ terms of:

$0.2 + 0.22 + 0.222 + \dots +$ n terms

Soln:-

$$0.2 + 0.22 + 0.222 + \dots + n\text{ terms}.$$

$$= \frac{2}{10} + \frac{22}{100} + \frac{222}{1000} + \dots + n\text{ terms}$$

$$= 2\left\{\frac{1}{10} + \frac{11}{100} + \frac{111}{1000} + \dots n\text{ terms}\right\}$$

Multiply and dividing by 9, we get

$$= \frac{2}{9}\left\{\frac{9}{10} + \frac{99}{100} + \frac{999}{1000} + \dots + n\text{ terms}\right\}$$

$$= \frac{2}{9}\left\{\left(1-\frac{1}{10}\right) + \left(1-\frac{1}{100}\right) + \left(1-\frac{1}{1000}\right) + \dots + n\text{ terms}\right\}$$

$$= \frac{2}{9}\left\{\underbrace{(1+1+1+\dots+n)}_{\text{terms}} - \underbrace{\left[\frac{1}{10} + \frac{1}{100} + \frac{1}{1000} + \dots\right]}_{n\text{ terms}}\right\}$$

$$= \frac{2}{9}\left\{n - \left(\frac{1}{10} + \frac{1}{100} + \frac{1}{1000} + \dots\right)\right\}$$

$$a = \frac{1}{10} \quad r = \frac{1/100}{1/10} = \frac{1}{10}$$

$$S_n = \frac{\frac{1}{10}\left(1-\left(\frac{1}{10}\right)^n\right)}{1-1/10}$$

$$S_n = \frac{1/10\left(\frac{10^n-1}{10^n}\right)}{\frac{10-1}{10}}$$

$$S_n = \frac{\left(\frac{10^n-1}{10^n}\right)}{9} = \frac{1}{9}\left(\frac{10^n-1}{10^n}\right)$$

$$S_n = \frac{1}{9}\left(1-\frac{1}{10^n}\right)$$

$$\text{so } S_n = \frac{2}{9}\left\{n - \frac{1}{9}\left(1-\frac{1}{10^n}\right)\right\}$$



**(ii)** $3 + 33 + 333 + \dots +$ n terms

$$S_n = \frac{1}{3}\{9 + 99 + 999 + \dots + n\text{ terms}\}$$

$$S_n = \frac{1}{3}\{(10-1) + (100-1) + (1000-1) + \dots + n\text{ terms}\}$$

$$S_n = \frac{1}{3}\{\underbrace{(10+100+1000+\dots)}_{n\text{ terms}} - \underbrace{(1+1+1+\dots)}_{n\text{ terms}}\}$$

$$S_n = \frac{1}{3}\left\{\frac{10((10)^n-1)}{10-1} - n\right\}$$

$$S_n = \frac{1}{3}\left\{\frac{10(10^n-1)}{9} - n\right\}$$

$$S_n = \frac{1}{3}\left\{\frac{10}{9}(10^n-1) - n\right\}$$

**Q3 Sum to n terms:**

(i) $1+(a+b)+(a^2+ab+b^2)+(a^3+a^2b+ab^2+b^3)+\dots$

$S_n = 1+(a+b)+(a^2+ab+b^2)+(a^3+a^2b+ab^2+b^3)+\dots$

Multiply and divide by $(a-b)$

$$S_n = \frac{1}{a-b}\{(a-b)+(a-b)(a+b)+(a-b)(a^2+ab+b^2)+(a-b)(a^3+a^2b+ab^2+b^3)+\dots\}$$

$$S_n = \frac{1}{a-b}\{a-b+a^2-b^2+a^3-b^3+a^4-b^4+\dots\}$$

$$S_n = \frac{1}{a-b}\{(a+a^2+a^3+a^4+\dots)-(b+b^2+b^3+b^4+\dots)\}$$

$$S_n = \frac{1}{a-b}\left\{\frac{a(a^n-1)}{a-1}-\frac{b(b^n-1)}{b-1}\right\}$$

$$S_n = \frac{1}{(a-b)}\left\{\frac{a(b-1)(a^n-1)-b(a-1)(b^n-1)}{(a-1)(b-1)}\right\}$$

$$S_n = \frac{a(b-1)(a^n-1)-b(a-1)(b^n-1)}{(a-b)(a-1)(b-1)}$$

(ii) $r+(1+k)r^2+(1+k+k^2)r^3+\dots$

$S_n = r+(1+k)r^2+(1+k+k^2)r^3+\dots$

$$S_n = \frac{1}{1-k}\{(1-k)r+(1-k^2)r^2+(1-k^3)r^3+\dots\}$$

By Multiply and dividing $(1-k)$

$$S_n = \frac{1}{1-k}\{r-kr+r^2-k^2r^2+r^3-k^3r^3+\dots\}$$

$$S_n = \frac{1}{1-k}\{(r+r^2+r^3+\dots)-(kr+k^2r^2+k^3r^3+\dots)\}$$

$$S_n = \frac{1}{1-k}\left\{\frac{r(r^n-1)}{r-1}-\frac{kr(k^nr^n-1)}{kr-1}\right\}$$

$$S_n = \frac{r}{1-k}\left\{\frac{r^n-1}{r-1}-\frac{k(k^nr^n-1)}{kr-1}\right\}$$

$$S_n = \frac{r}{1-k}\left\{\frac{r^n-1}{r-1}-\frac{k(k^nr^n-1)}{kr-1}\right\}$$

**TAHIR**

**Q.4** $2+(1-i)+\frac{1}{i}+\dots$ 8 terms.

$a=2 \quad r=\frac{1-i}{2}\times\frac{1+i}{1+i}=\frac{1-i^2}{2(1+i)}=\frac{2}{2(1+i)}=\frac{1}{(1+i)}$

$n=8$

$$S_8 = \frac{2\left[1-\left(\frac{1}{1+i}\right)^8\right]}{1-\frac{1}{1+i}}$$

$$S_8 = \frac{2\left[\frac{(1+i)^8-1}{(1+i)^8}\right]}{\frac{(1+i-1)}{(1+i)}}$$

$$S_8 = \frac{2[(1+i)^8-1](1+i)}{(1+i)^8\, i}$$

$$S_8 = \frac{2((1+i)^8-1)}{i(1+i)^7}$$

$$S_8 = \frac{2}{i}\left[\frac{(1+i)^8-1}{(1+i)^7}\right]$$

**TAHIR**



**Q.5 Sum of infinite G. Series:**

(i) $\frac{1}{5}+\frac{1}{25}+\frac{1}{125}+\dots$

$a=1/5 \quad r=\frac{1/25}{1/5}=\frac{5}{25}=1/5 \quad n=\infty$

$$S_\infty = \frac{a}{r-1}=\frac{a}{1-r}$$

$$=\frac{1/5}{1-1/5}=\frac{1/5}{\frac{(5-1)}{5}}=\frac{1}{4}$$

(ii) $\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\dots$

$a=1/2 \quad r=\frac{1/4}{1/2}=\frac{2}{4}=1/2 \quad n=\infty$

$$S_n = \frac{a}{1-r}=\frac{1/2}{1-1/2}$$

$$S_n = \frac{1/2}{\frac{2-1}{2}}=\frac{1}{2-1}=\frac{1}{1}$$

$S_n = 1$ Ans.

(iii) $\frac{9}{4}+\frac{3}{2}+1+\frac{2}{3}+\ldots$

$a=\frac{9}{4}$ $r=\frac{3/2}{9/4}=\frac{3}{2}\times\frac{4}{9}=\frac{2}{3}$

$S_\infty=\frac{a}{1-r}=\frac{9/4}{1-2/3}$

$S_\infty=\frac{9/4}{\frac{3-2}{3}}=\frac{3\times 9}{4(3-2)}$

$S_\infty=\frac{27}{4(1)}=\frac{27}{4}$

(iv) $2+1+0.5+\ldots$

$a=2$ $r=\frac{1}{2}$

$S_\infty=\frac{a}{1-r}=\frac{2}{1-\frac{1}{2}}$

$S_\infty=\frac{2}{\frac{(2-1)}{2}}=\frac{2\times 2}{(2-1)}$

$S_\infty=\frac{4}{1}=4$

(v) $4+2\sqrt{2}+2+\sqrt{2}+1+\ldots$

$a=4$ $r=\frac{2\sqrt{2}}{4}=\frac{\sqrt{2}}{\sqrt{2}\sqrt{2}}=\frac{1}{\sqrt{2}}$

$S_\infty=\frac{a}{1-r}$

$S_\infty=\frac{4}{(1-\frac{1}{\sqrt{2}})}=\frac{4}{\frac{(\sqrt{2}-1)}{\sqrt{2}}}$

$S_\infty=\frac{4\sqrt{2}}{\sqrt{2}-1}\times\frac{\sqrt{2}+1}{\sqrt{2}+1}$

$S_\infty=\frac{4\sqrt{2}(\sqrt{2}+1)}{(\sqrt{2})^2-(1)^2}$

$S_\infty=\frac{4(2)+4\sqrt{2}}{2-1}=\frac{4(2+\sqrt{2})}{1}$

$S_\infty=4(2+\sqrt{2})$



(vi) $0.1+0.05+0.025+\ldots$

$a=0.1$ $r=\frac{0.05}{0.1}=\frac{5}{100}\times\frac{10}{1}=\frac{5}{10}=\frac{1}{2}$

$S_\infty=\frac{a}{1-r}$

$S_\infty=\frac{0.1}{1-\frac{1}{2}}=\frac{0.1}{\frac{(2-1)}{2}}$

$S_\infty=\frac{2\times 0.1}{2-1}=\frac{0.2}{1}$

$S_\infty=0.2$



Q.6 Find the sum of recurring fractions

(i) $1.\overline{34}$

$1.34343434\ldots$

$1+0.34343434+\ldots$

$1+\{0.34+0.0034+0.000034+\ldots\}$

$\{1+S_\infty\}$ $a=0.34$ $r=\frac{34}{10000}\times\frac{100}{34}=\frac{1}{100}$

$S_\infty=\frac{a}{1-r}=\frac{0.34}{1-\frac{1}{100}}$

$S_\infty=\frac{34/100}{\frac{100-1}{100}}=\frac{34}{99}$

So sum $=1+S_\infty=1+\frac{34}{99}$

$S_\infty=\frac{99+34}{99}=\frac{133}{99}$





(ii) $0.\overline{7}$

$0.777777\ldots$

Sum $=0.7+0.07+0.007+\ldots$

$a=0.7$ $r=\frac{7}{100}\times\frac{10}{7}=\frac{1}{10}$

Using the Formula

$$S_\infty = \frac{a}{1-r}$$

$$S_\infty = \frac{0.7}{1-\frac{1}{10}} = \frac{7/10}{\frac{10-1}{10}}$$

$$S_\infty = \frac{7}{9}$$



(iii) $0.\overline{259}$

$= 0.259 + 0.000259 + 0.000000259 + \dots$

$a = \frac{259}{1000}$ $\quad r = \frac{259}{1000000} \times \frac{1000}{259} = \frac{1}{1000}$

$$S_\infty = \frac{a}{1-r} = \frac{259/1000}{1-\frac{1}{1000}}$$

$$S_\infty = \frac{259/1000}{\frac{1000-1}{1000}}$$

$$S_\infty = \frac{259}{999}$$



(iv) $1.\overline{53}$

$= 1.53535353\dots$

$= 1 + \{0.53 + 0.0053 + 0.000053 + \dots\}$

$= 1 + S_\infty$

$a = 0.53 = \frac{53}{100}$ $\quad r = \frac{53}{10000} \times \frac{100}{53} = \frac{1}{100}$

$$S_\infty = \frac{a}{1-r} = \frac{53/100}{1-\frac{1}{100}}$$

$$S_\infty = \frac{53/100}{100-1/100} = \frac{53}{99}$$

So

$$\text{Sum} = 1 + \frac{53}{99} = \frac{99+53}{99} = \frac{152}{99}$$

(v) $0.\overline{159}$

$= 0.159 + 0.000159 + 0.000000159 + \dots$

$a = \frac{159}{1000}$ $\quad r = \frac{159}{1000000} \times \frac{1000}{159} = \frac{1}{1000}$

$$S_\infty = \frac{a}{1-r}$$

$$S_\infty = \frac{159/1000}{1-1/1000}$$

$$S_\infty = \frac{159/1000}{\frac{1000-1}{1000}} = \frac{159}{999}$$

(vi) $1.1\overline{47}$

$1.1 + 0.0\overline{47}$

$1.1 + \{0.047 + 0.00047 + 0.0000047 + \dots\}$

$1.1 + S_\infty$

$a = 0.047 = \frac{47}{1000}$ $\quad r = \frac{47}{100000} \times \frac{1000}{47} = \frac{1}{100}$

$$S_\infty = \frac{a}{1-r}$$

$$= \frac{47/1000}{1-1/100} = \frac{47/1000}{\frac{100-1}{100}}$$

$$S_\infty = \frac{47}{10(99)} = \frac{47}{990}$$

So

$$\text{Sum} = 1.1 + \frac{47}{990}$$

$$S_\infty = \frac{990 \times 1.1 + 47}{990}$$

$$S_\infty = \frac{1089+47}{990} = \frac{1136}{990}$$

$$S_\infty = \frac{568}{495}$$

**Q.7** $S_\infty = ?$

$r+(1+k)r^2+(1+k+k^2)r^3+\dots\dots$

Multiply and divide by $(1-k)$

$$\frac{1}{(1-k)}\left\{(1-k)r+(1-k^2)r^2+(1-k^3)r^3+\dots\right\}$$

$$\frac{1}{1-k}\left\{(r+r^2+r^3+\dots)-(kr+k^2r^2+k^3r^3+\dots)\right\}$$

$$S_\infty=\frac{a}{1-r}$$

$$S_\infty=\frac{1}{1-k}\left\{\frac{r}{1-r}-\frac{kr}{1-kr}\right\}$$

$$S_\infty=\frac{r}{1-k}\left\{\frac{1-\cancel{kr}-k+\cancel{kr}}{(1-r)(1-kr)}\right\}$$

$$S_\infty\ \frac{r}{\cancel{1-k}}\left\{\frac{\cancel{1-k}}{(1-r)(1-kr)}\right\}$$

$$S_\infty\ \frac{r}{(1-r)(1-kr)}$$

**Q.8** $y=\frac{x}{2}+\frac{x^2}{4}+\frac{x^3}{8}+\dots$

$$a=\frac{x}{2}\qquad r=\frac{x^2}{4}\times\frac{2}{x}=\frac{x}{2}$$

$$y=S_\infty=\frac{a}{1-r}=\frac{x/2}{1-x/2}$$

$$y=S_\infty=\frac{x/2}{\frac{2-x}{2}}=\frac{x}{2-x}$$

$$y=\frac{x}{2-x}\implies y(2-x)=x$$

$$2y-xy=x$$

$$2y=x+xy$$



$$2y=x(1+y)$$

$$x=\frac{2y}{1+y}\qquad\text{(Proved)}$$

**Q.9**

$$y=\frac{2}{3}x+\frac{4}{9}x^2+\frac{8}{27}x^3$$

$$a=\frac{2}{3}x\qquad r=\frac{4}{9}x^2\times\frac{3}{2x}=\frac{2}{3}x$$

$$y=S_\infty=\frac{a}{1-r}$$

$$y=\frac{2/3\,x}{1-\frac{2x}{3}}=\frac{2x/3}{\frac{3-2x}{3}}$$

$$y=\frac{2x}{3-2x}$$



$$y(3-2x)=2x$$

$$3y-2xy=2x$$

$$3y=2x+2xy$$

$$2x(1+y)=3y$$

$$x=\frac{3y}{2(1+y)}\qquad\text{(Proved)}$$

**Q.10** $a=h=27\ m\qquad r=2/3$

$$G.S=27+2\left(27\times\frac{2}{3}\right)+2\left(27\times\frac{2}{3}\times\frac{2}{3}\right)+\dots$$

$$dis=27+2\left[27\times\frac{2}{3}+27\times\left(\frac{2}{3}\right)^2+\dots\right]$$

$$dis=27+2\left(\frac{27\times 2/3}{1-2/3}\right)$$

$$dis=27+2\left(\frac{18\times 3}{3-2}\right)$$



$$=27+2(54)$$

$$=27+2\times 54=27+108$$

$$=135\ \text{meter}$$

where 2 with each terms represent bounce up as well as down distance.

Q.11 $a = h = 75$ meter $r = 2/5$

Distance $= 75 + 2(75\times 2/5) + 2(75\times(2/5)^2) + \dots$

$D = 75 + 2[75\times 2/5 + 75\times(2/5)^2 + \dots]$

$D = 75 + 2\left(\frac{a}{1-r}\right)$ $r = 2/5$

$D = 75 + 2\left(\frac{75\times 2/5}{1-2/5}\right)$

$D = 75 + 2\left(\frac{150/5}{5-2/5}\right)$

$D = 75 + 2\left(\frac{150}{3}\right)$ (50)

$D = 75 + 2\times 50 = 75 + 100$

$D = 175$ meter.



Q.12 $y = 1 + 2x + 4x^2 + 8x^3 + \dots$

(i) $y = S_\infty = \frac{a}{1-r}$ $\because a = 1$, $r = \frac{2x}{1} = 2x$

$y = \frac{1}{1-2x} =$

$y(1-2x) = 1$



$y - 2xy = 1$

$y - 1 = 2xy \implies x = \frac{y-1}{2y}$ (Proved)

(ii) $|r| = \left|\frac{2x}{1}\right| < 1$ Series will Converge if

$|x| < \frac{1}{2}$

$-\frac{1}{2} < x < \frac{1}{2}$

i.e Series will Converge if

$-\frac{1}{2} < x < \frac{1}{2}$

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Q.13

(i) $y = 1 + \frac{x}{2} + \frac{x^2}{4} + \dots$

$y = S_\infty = \frac{a}{1-r}$ $a = 1$ $r = \frac{x}{2}$

$y = \frac{1}{1-x/2} = \frac{2}{2-x}$

$y = \frac{2}{2-x}$

$\implies y(2-x) = 2$

$2y - xy = 2$

$2y - 2 = xy$

$x = \frac{2(y-1)}{y}$ (Proved)



(ii) Interval of Convergence.

$|r| = \left|\frac{x}{2}\right| < 1$

$= |x| < 2$

$-2 < x < 2$

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Q.14 Let the $a$ is first term and $r$ is Common ratio so

The Series is $a + ar + ar^2 + \dots$

$S_\infty = 9$



$\frac{a}{1-r} = 9$

$a = 9 - 9r$ —— (1)

Now Squares of Series terms

$a^2 + a^2r^2 + a^2r^4 + \dots$

$S_\infty = \frac{81}{5} = \frac{a^2}{1-r^2}$

$81 - 81r^2 = 5a^2$

$81 - 81r^2 = 5(9-9r)^2$ from (1)