

# Exercise: 6.8

(30)

Q.1 G.P =  $1, \frac{1}{3}, \frac{1}{9}, \dots$ , 15 terms

$a = 1$     $r = \frac{1}{3}$     $n = 15$

$$S_n = \frac{a(1-r^n)}{1-r} \quad r < 1$$

$$S_{15} = \frac{1(1-(\frac{1}{3})^{15})}{1-\frac{1}{3}}$$

$$S_{15} = \frac{3(1-(0.33)^{15})}{2} = \frac{3(0.999993)}{2}$$

$$S_{15} = 1.49999895$$

## Convergence of Infinite Geometric Series

An infinite Geometric Series is

Convergent if  $|r| < 1$  then  $r^n \rightarrow 0$  as  $n \rightarrow \infty$

$$\Rightarrow S_{\infty} = \lim_{n \rightarrow \infty} \frac{a_1(1-r^n)}{1-r}$$

$$S_{\infty} = \frac{a_1(1-0)}{1-r}$$

$$S_{\infty} = \frac{a_1}{1-r}$$

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Q Sum upto n terms of:

$0.2 + 0.22 + 0.222 + \dots + n \text{ terms}$

Soln:-

$0.2 + 0.22 + 0.222 + \dots + n \text{ terms}$

$$= \frac{2}{10} + \frac{22}{100} + \frac{222}{1000} + \dots + n \text{ terms}$$

$$= 2 \left\{ \frac{1}{10} + \frac{11}{100} + \frac{111}{1000} + \dots + n \text{ terms} \right\}$$

Multiply and dividing by 9, we get

$$= \frac{2}{9} \left\{ \frac{9}{10} + \frac{99}{100} + \frac{999}{1000} + \dots + n \text{ terms} \right\}$$

$$= \frac{2}{9} \left\{ \left(1 - \frac{1}{10}\right) + \left(1 - \frac{1}{100}\right) + \left(1 - \frac{1}{1000}\right) + \dots \right\}$$

$$= \frac{2}{9} \left\{ \underbrace{(1+1+1+\dots+n)}_{\text{terms}} - \underbrace{\left\{ \frac{1}{10} + \frac{1}{100} + \frac{1}{1000} + \dots \right\}}_{\text{nterms}} \right\}$$

$$= \frac{2}{9} \left\{ n - \left( \frac{1}{10} + \frac{1}{100} + \frac{1}{1000} + \dots \right) \right\}$$

$$a = \frac{1}{10} \quad r = \frac{1/100}{1/10} = \frac{1}{10}$$

$$S_n = \frac{\frac{1}{10} \left( 1 - \left( \frac{1}{10} \right)^n \right)}{1 - \frac{1}{10}}$$

$$S_n = \frac{\frac{1}{10} \left( \frac{10^n - 1}{10^n} \right)}{\frac{10-1}{10}}$$

$$S_n = \frac{\left( \frac{10^n - 1}{10^n} \right)}{9} = \frac{1}{9} \left( \frac{10^n - 1}{10^n} \right)$$

$$S_n = \frac{1}{9} \left( 1 - \frac{1}{10^n} \right)$$

$$\text{So } S_n = \frac{2}{9} \left\{ n - \frac{1}{9} \left( 1 - \frac{1}{10^n} \right) \right\}$$

(ii)  $3 + 33 + 333 + \dots + n \text{ terms}$

$$S_n = \frac{1}{3} \left\{ 9 + 99 + 999 + \dots + n \text{ terms} \right\}$$

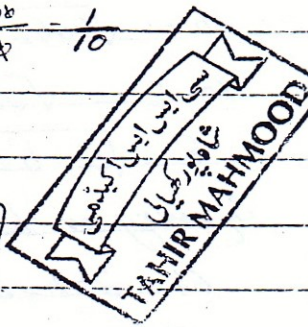
$$S_n = \frac{1}{3} \left\{ (10-1) + (100-1) + (1000-1) + \dots + n \text{ terms} \right\}$$

$$S_n = \frac{1}{3} \left\{ (10+100+1000+\dots) - (1+1+1+\dots) \right\}$$

$$S_n = \frac{1}{3} \left\{ \frac{10(10^n-1)}{10-1} - n \right\}$$

$$S_n = \frac{1}{3} \left\{ \frac{10(10^n-1)}{9} - n \right\}$$

$$S_n = \frac{1}{3} \left\{ \frac{10}{9}(10^n-1) - n \right\}$$



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Q3 Sum to n terms:

(i)  $1 + (a+b) + (a^2+ab+b^2) + (a^3+ab^2+ab^2+b^3) + \dots$

$S_n = 1 + (a+b) + (a^2+ab+b^2) + (a^3+ab^2+ab^2+b^3) + \dots$

Multiply and divide by  $(a-b)$

$S_n = \frac{1}{a-b} \{ (a-b) + (a-b)(a+b) + (a-b)(a^2+ab+b^2) + (a-b)(a^3+ab^2+ab^2+b^3) + \dots \}$

$S_n = \frac{1}{a-b} \{ a-b + a^2-b^2 + a^3-b^3 + a^4-b^4 + \dots \}$

$S_n = \frac{1}{a-b} \{ (a+a^2+a^3+a^4+\dots) - (b+b^2+b^3+b^4+\dots) \}$

$S_n = \frac{1}{a-b} \left\{ \frac{a(a^n-1)}{a-1} - \frac{b(b^n-1)}{b-1} \right\}$

$S_n = \frac{1}{(a-b)} \left\{ \frac{a(a^n-1)}{(a-1)} - \frac{b(b^n-1)}{(b-1)} \right\}$

$S_n = \frac{a(b-1)(a^n-1) - b(a-1)(b^n-1)}{(a-b)(a-1)(b-1)}$

(ii)  $r + (1+k)r^2 + (1+k+k^2)r^3 + \dots$

$S_n = r + (1+k)r^2 + (1+k+k^2)r^3 + \dots$

$S_n = \frac{1}{1-k} \{ (1-k)r + (1-k^2)r^2 + (1-k^3)r^3 + \dots \}$   
By multiply and dividing  $(1-k)$

$S_n = \frac{1}{1-k} \{ r - kr + r^2 - k^2r^2 + r^3 - k^3r^3 + \dots \}$

$S_n = \frac{1}{1-k} \{ (r+r^2+r^3+\dots) - (kr+k^2r^2+k^3r^3+\dots) \}$

$S_n = \frac{1}{1-k} \left\{ \frac{r(r^n-1)}{r-1} - \frac{kr(k^n r^n-1)}{kr-1} \right\}$

$S_n = \frac{r}{1-k} \left\{ \frac{r^n-1}{r-1} - \frac{k(k^n r^n-1)}{kr-1} \right\}$

$S_n = \frac{r}{1-k} \left\{ \frac{r^n-1}{r-1} - \frac{k(k^n r^n-1)}{kr-1} \right\}$

Q4  $2 + (-i) + \frac{1}{i} + \dots$  8 terms. (31)

$a = 2$   $r = \frac{1-i}{2} \times \frac{1+i}{1+i} = \frac{1-i^2}{2(1+i)} = \frac{2}{2(1+i)} = \frac{1}{1+i}$   
 $n = 8$

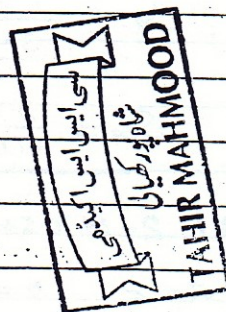
$S_8 = \frac{2 \left( 1 - \left( \frac{1}{1+i} \right)^8 \right)}{1 - \frac{1}{1+i}}$

$S_8 = \frac{2 \left( \frac{(1+i)^8 - 1}{(1+i)^8} \right)}{\frac{(1+i) - 1}{(1+i)}}$

$S_8 = \frac{2 \left( (1+i)^8 - 1 \right) (1+i)}{(1+i)^8 \cdot i}$

$S_8 = \frac{2 \left( (1+i)^8 - 1 \right)}{i(1+i)^7}$

$S_8 = \frac{2}{i} \left[ \frac{(1+i)^8 - 1}{(1+i)^7} \right]$  **TAHIR**



Q.5 Sum of infinite G. Series:

(i)  $\frac{1}{5} + \frac{1}{25} + \frac{1}{125} + \dots$

$a = \frac{1}{5}$   $r = \frac{1/25}{1/5} = \frac{5}{25} = \frac{1}{5}$   $n = \infty$

$S_{\infty} = \frac{a}{r-1} = \frac{a}{1-r}$

$= \frac{1/5}{1 - 1/5} = \frac{1/5}{(5-1)/5} = \frac{1}{4}$

(ii)  $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \dots$

$a = \frac{1}{2}$   $r = \frac{1/4}{1/2} = \frac{2}{4} = \frac{1}{2}$   $n = \infty$

$S_n = \frac{a}{1-r} = \frac{1/2}{1 - 1/2}$

$S_n = \frac{1/2}{2-1} = \frac{1}{2-1} = \frac{1}{1}$

$S_n = 1$  **Ans.**

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$$(iii) \frac{9}{4} + \frac{3}{2} + 1 + \frac{2}{3} + \dots$$

$$a = \frac{9}{4} \quad r = \frac{3/2}{9/4} = \frac{3/2 \times 4}{9} = \frac{2}{3}$$

$$S_{\infty} = \frac{a}{1-r} = \frac{9/4}{1-2/3}$$

$$S_{\infty} = \frac{9/4}{\frac{3-2}{3}} = \frac{3 \times 9}{4(3-2)}$$

$$S_{\infty} = \frac{27}{4(1)} = \frac{27}{4}$$

$$(iv) 2 + 1 + 0.5 + \dots$$

$$a = 2 \quad r = \frac{1}{2}$$

$$S_{\infty} = \frac{a}{1-r} = \frac{2}{1-1/2}$$

$$S_{\infty} = \frac{2}{\frac{2-1}{2}} = \frac{2 \times 2}{2-1}$$

$$S_{\infty} = \frac{4}{1} = 4$$

$$(v) 4 + 2\sqrt{2} + 2 + \sqrt{2} + 1 + \dots$$

$$a = 4 \quad r = \frac{2\sqrt{2}}{4} = \frac{\sqrt{2}}{\sqrt{2}\sqrt{2}} = \frac{1}{\sqrt{2}}$$

$$S_{\infty} = \frac{a}{1-r}$$

$$S_{\infty} = \frac{4}{\left(1 - \frac{1}{\sqrt{2}}\right)} = \frac{4}{\frac{\sqrt{2}-1}{\sqrt{2}}}$$

$$S_{\infty} = \frac{4\sqrt{2}}{\sqrt{2}-1} \times \frac{\sqrt{2}+1}{\sqrt{2}+1}$$

$$S_{\infty} = \frac{4\sqrt{2}(\sqrt{2}+1)}{(\sqrt{2})^2 - (1)^2}$$

$$S_{\infty} = \frac{4(2) + 4\sqrt{2}}{2-1} = 4(2+\sqrt{2})$$

$$S_{\infty} = 4(2+\sqrt{2})$$

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$$(vi) 0.1 + 0.05 + 0.025 + \dots \quad (32)$$

$$a = 0.1 \quad r = \frac{0.05 \times 10}{0.1 \times 100} = \frac{5}{10} = \frac{1}{2}$$

$$S_{\infty} = \frac{a}{1-r}$$

$$S_{\infty} = \frac{0.1}{1-\frac{1}{2}} = \frac{0.1}{\frac{2-1}{2}}$$

$$S_{\infty} = \frac{2 \times 0.1}{2-1} = \frac{0.2}{1}$$

$$S_{\infty} = 0.2$$

Q.6 Find the sum of recurring fractions

$$(i) 1.\overline{34}$$

$$1.34343434\dots$$

$$1 + 0.34343434\dots$$

$$1 + \{0.34 + 0.0034 + 0.000034 + \dots\}$$

$$\{1 + S_{\infty}\} \quad a = 0.34 \quad r = \frac{25}{10000} \times \frac{100}{34} = \frac{1}{100}$$

$$S_{\infty} = \frac{a}{1-r} = \frac{0.34}{1-\frac{1}{100}}$$

$$S_{\infty} = \frac{\frac{34}{100}}{\frac{100-1}{100}} = \frac{34}{99}$$

$$\therefore \text{sum} = 1 + S_{\infty} = 1 + \frac{34}{99}$$

$$S_{\infty} = \frac{99+34}{99} = \frac{133}{99}$$

$$(iii) 0.\overline{7}$$

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$$0.777777\dots$$

$$\text{Sum} = 0.7 + 0.07 + 0.007 + \dots$$

$$a = 0.7 \quad r = \frac{7}{100} \times \frac{10}{7} = \frac{1}{10}$$

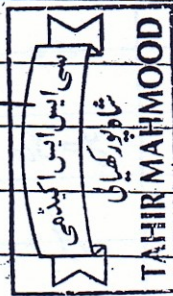


Using the Formula.

$$S_{\infty} = \frac{a}{1-r}$$

$$S_{\infty} = \frac{0.7}{1 - \frac{1}{10}} = \frac{7/10}{\frac{10-1}{10}}$$

$$S_{\infty} = \frac{7}{9}$$



(iii)  $0.\overline{259}$

$$= 0.259 + 0.000259 + 0.000000259 + \dots$$

$$a = \frac{259}{1000} \quad r = \frac{259}{1000000} \times \frac{1000}{259} = \frac{1}{1000}$$

$$S_{\infty} = \frac{a}{1-r} = \frac{259/1000}{1 - \frac{1}{1000}}$$

$$S_{\infty} = \frac{259/1000}{\frac{1000-1}{1000}}$$

$$S_{\infty} = \frac{259}{999}$$

(iv)  $1.\overline{53}$

$$= 1.53535353 \dots$$

$$= 1 + \{0.53 + 0.0053 + 0.000053 + \dots\}$$

$$= 1 + S_{\infty}$$

$$a = 0.53 = \frac{53}{100} \quad r = \frac{53}{10000} \times \frac{100}{53} = \frac{1}{100}$$

$$S_{\infty} = \frac{a}{1-r} = \frac{53/100}{1 - \frac{1}{100}}$$

$$S_{\infty} = \frac{53/100}{\frac{100-1}{100}} = \frac{53}{99}$$

$$S_{\infty} = 1 + \frac{53}{99} = \frac{99+53}{99} = \frac{152}{99}$$

(v)  $0.\overline{159}$

(33)

$$= 0.159 + 0.000159 + 0.000000159 + \dots$$

$$a = \frac{159}{1000} \quad r = \frac{159}{1000000} \times \frac{1000}{159} = \frac{1}{1000}$$

$$S_{\infty} = \frac{a}{1-r}$$

$$S_{\infty} = \frac{159/1000}{1 - \frac{1}{1000}}$$

$$S_{\infty} = \frac{159/1000}{\frac{1000-1}{1000}} = \frac{159}{999}$$

(vi)  $1.\overline{147}$

$$1.1 + 0.047$$

$$1.1 + \{0.047 + 0.00047 + 0.0000047 + \dots\}$$

$$1.1 + S_{\infty}$$

$$a = 0.047 = \frac{47}{1000} \quad r = \frac{47}{1000000} \times \frac{1000}{47} = \frac{1}{1000}$$

$$S_{\infty} = \frac{a}{1-r}$$

$$= \frac{47/1000}{1 - \frac{1}{1000}} = \frac{47/1000}{\frac{1000-1}{1000}}$$

$$S_{\infty} = \frac{47}{10(99)} = \frac{47}{990}$$

$S_{\infty}$

$$Sum = 1.1 + \frac{47}{990}$$

$$S_{\infty} = \frac{990 \times 1.1 + 47}{990}$$

$$S_{\infty} = \frac{1089 + 47}{990} = \frac{1136}{990}$$

$$S_{\infty} = \frac{568}{495}$$

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Q.7  $S_{\infty} = ?$

$$h + (1+k)h^2 + (1+k+k^2)h^3 + \dots$$

Multiply and divide by  $(1-k)$

$$\frac{1}{(1-k)} \{ (1-k)h + (1-k^2)h^2 + (1-k^3)h^3 + \dots \}$$

$$\frac{1}{1-k} \{ (h + h^2 + h^3 + \dots) - (kh + k^2h^2 + k^3h^3 + \dots) \}$$

$$S_{\infty} = \frac{a}{1-r}$$

$$S_{\infty} = \frac{1}{1-k} \left\{ \frac{h}{1-k} - \frac{kh}{1-kh} \right\}$$

$$S_{\infty} = \frac{h}{1-k} \left\{ \frac{1-kh - k + kh}{(1-k)(1-kh)} \right\}$$

$$S_{\infty} = \frac{h}{1-k} \left\{ \frac{1-k}{(1-k)(1-kh)} \right\}$$

$$S_{\infty} = \frac{h}{(1-k)(1-kh)}$$

Q.8  $y = \frac{x}{2} + \frac{x^2}{4} + \frac{x^3}{8} + \dots$

$$a = \frac{x}{2} \quad r = \frac{x/4}{x/2} = \frac{x}{2}$$

$$y = S_{\infty} = \frac{a}{1-r} = \frac{x/2}{1-x/2}$$

$$y = S_{\infty} = \frac{x/2}{2-x} = \frac{x}{2-x}$$

$$y = \frac{x}{2-x} \Rightarrow y(2-x) = x$$

$$2y - xy = x$$

$$2y = x + xy$$

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$$2y = x(1+y)$$

$$x = \frac{2y}{1+y}$$

(Proved)

Q.9

$$y = \frac{2}{3}x + \frac{4}{9}x^2 + \frac{8}{27}x^3 \quad (34)$$

$$a = \frac{2}{3}x \quad r = \frac{4/9 x^2}{2/3 x} = \frac{2}{3}x$$

$$y = S_{\infty} = \frac{a}{1-r}$$

$$y = \frac{2/3 x}{1 - 2/3 x} = \frac{2x/3}{3-2x}$$

$$y = \frac{2x}{3-2x}$$

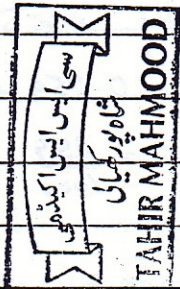
$$y(3-2x) = 2x$$

$$3y - 2xy = 2x$$

$$3y = 2x + 2xy$$

$$2x(1+y) = 3y$$

$$x = \frac{3y}{2(1+y)} \quad (\text{Proved})$$



Q.10  $a-h = 27 \text{ m}$   $r = 2/3$

$$G.S = 27 + 2(27 \times \frac{2}{3}) + 2(27 \times \frac{2}{3} \times \frac{2}{3}) + \dots$$

$$dis = 27 + 2 \left[ 27 \times \frac{2}{3} + 27 \times \left(\frac{2}{3}\right)^2 + \dots \right]$$

$$dis = 27 + 2 \left[ \frac{9 \times 27 \times 2/3}{1 - 2/3} \right]$$

$$dis = 27 + 2 \left[ \frac{18 \times 3}{3-2} \right]$$

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$$= 27 + 2(54)$$

$$= 27 + 2 \times 54 = 27 + 108$$

$$= 135 \text{ meter}$$

where 2 with each terms represent bounce up as well as down distance.



Q.11  $a=b=75 \text{ m/s}$   $r=\frac{2}{5}$

Distance =  $75 + 2(75 \times \frac{2}{5}) + 2(75 \times (\frac{2}{5})^2) + \dots$

$D = 75 + 2(75 \times \frac{2}{5} + 75 \times (\frac{2}{5})^2 + \dots)$

$D = 75 + 2 \left[ \frac{a}{1-r} \right]$   $r = \frac{2}{5}$

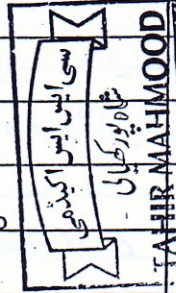
$D = 75 + 2 \left[ \frac{75 \times \frac{2}{5}}{1 - \frac{2}{5}} \right]$

$D = 75 + 2 \left[ \frac{150 \times \frac{2}{5}}{5 - 2 \times \frac{2}{5}} \right]$

$D = 75 + 2 \left[ \frac{150}{3} \right]$

$D = 75 + 2 \times 50 = 75 + 100$

$D = 175 \text{ meter.}$



Q.12  $y = 1 + 2x + 4x^2 + 8x^3 + \dots$

(i)  $y = S_{\infty} = \frac{a}{1-r}$   $\therefore a = 1$   
 $r = \frac{2x}{1} = 2x$

$y = \frac{1}{1-2x} =$

$y(1-2x) = 1$

$y - 2xy = 1$

$y - 1 = 2xy \Rightarrow x = \frac{y-1}{2y}$  (Proved)

(ii)  $|r| = |2x| < 1$  Series will Converge if

$|x| < \frac{1}{2}$

$-\frac{1}{2} < x < \frac{1}{2}$

ie Series will Converge if

$-\frac{1}{2} < x < \frac{1}{2}$

Q.13

(i)  $y = 1 + \frac{x}{2} + \frac{x^2}{4} + \dots$

$y = S_{\infty} = \frac{a}{1-r}$   $a=1$   $r=\frac{x}{2}$

$y = \frac{1}{1-\frac{x}{2}} = \frac{2}{2-x}$

$y = \frac{2}{2-x}$

$\Rightarrow y(2-x) = 2$

$2y - 2xy = 2$

$2y - 2 = 2xy$

$x = \frac{2(y-1)}{y}$  (Proved)

(ii) Interval of Convergence.

$|r| = \left| \frac{x}{2} \right| < 1$

$= |x| < 2$

$-2 < x < 2$

Q.14 Let the  $a$  is first term and

$r$  is Common ratio so

The Series is  $a + ar + ar^2 + \dots$

$S_{\infty} = \frac{a}{1-r} = 9$

$\frac{a}{1-r} = 9$

$a = 9 - 9r$  (1)

Now Squares of Series terms

$a^2 + a^2 r^2 + a^2 r^4 + \dots$

$S_{\infty} = \frac{81}{5} = \frac{a^2}{1-r^2}$

$81 - 81r^2 = 5a^2$

$81 - 81r^2 = 5(9-9r)^2$  from (1)

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