Geomatric Means: (G.M)

A number G is called Geometric Mean between two numbers a and b if a, G, b are in G.P and $\frac{G}{a} = \frac{b}{G}$

$G^2 = ab$

$G = \pm\sqrt{ab}$

Exercise: 6.7

Q.1 Find G.M b/w

(i) -2 and 8

$a = -2 \quad b = 8$

$G = \sqrt{(-2)(8)} = \sqrt{16}\,i$

$G = \pm 4i$

(ii) $-2i$ and $8i$

$a = -2i$ and $b = 8i$

$G = \sqrt{(-2i)(8i)} = \sqrt{-2\times 8\, i^2}$

$G = \sqrt{-16(-1)} \quad \because i^2 = (-1)$

$G = \sqrt{16} = \pm 4$

Q.2 (i) Let G_1, G_2 be the two G.M between 1 and 8 so

$1, G_1, G_2, 8$ are in G.P

$a = 1 \quad a_4 = 8 \quad n = 4$

$a_4 = ar^3 \Rightarrow 8 = 1r^3 \Rightarrow r = 2$

$G_1 = ar = (1)2 = 2$

$G_2 = ar^2 = (1)(2)^2 = 4$

(ii) Let G_1, G_2 be the GMs b/w 2 and 16

so $2, G_1, G_2, 16$

$a = 2 \quad a_4 = 16 \quad n = 4$

$a_4 = ar^3 \Rightarrow 16 = 2r^3$

$r^3 = 8 \Rightarrow r = 2$

$G_1 = ar = 2(2) = 4$

$G_2 = ar^2 = 2(2)^2 = 8$

Q.3 (i) Let G_1, G_2, G_3 be the G.Ms b/w 1 and 16

so $1, G_1, G_2, G_3, 16$

$a = 1 \quad a_5 = 16 \quad n = 5$

$a_5 = ar^4 = 16 = 1r^4$

$r = 2$

so $G_1 = ar = 1(2) = 2$

$G_2 = ar^2 = 1(2)^2 = 4$

$G_3 = ar^3 = 1(2)^3 = 8$

(ii) Let G_1, G_2, G_3 are GMs b/w 2 & 32

so $2, G_1, G_2, G_3, 32$

$a = 2 \quad a_5 = 32 \quad n = 5$

$a_5 = ar^4 \Rightarrow 32 = 2r^4 \Rightarrow 16 = r^4$

$r = 2$

$G_1 = ar = 2(2) = 4$

$G_2 = ar^2 = 2(2)^2 = 8$

$G_3 = ar^3 = 2(2)^3 = 16$

Q.4 Let G_1, G_2, G_3, G_4 are G.Ms b/w 3 & 96

$3, G_1, G_2, G_3, G_4, 96$

$a = 3 \quad a_6 = 96 \quad n = 6$

$a_6 = ar^5 \Rightarrow 96 = 3r^5$

$r^5 = 32 \Rightarrow r = 2$

$G_1 = ar = 3(2) = 6$

$G_2 = ar^2 = 3(2)^2 = 3(4) = 12$

$G_3 = ar^3 = 3(2)^3 = 3(8) = 24$

$G_4 = ar^4 = 3(2)^4 = 3(16) = 48$

Q.5 x, y are real numbers

$G = \sqrt{xy}$ is G.M.

$A = \dfrac{x+y}{2}$ (where A is A.M.)

we have to prove that $A > G$

$A - G = \dfrac{x+y}{2} - \sqrt{xy}$

$A - G = \dfrac{x+y-2\sqrt{xy}}{2}$

$A - G = \dfrac{(\sqrt{x}-\sqrt{y})^2}{2} > 0$

So $A - G > 0$ (Square is always Positive)

$A > G$

(Proved.)

Q.6 $n = ?$

$\dfrac{a^n+b^n}{a^{n-1}+b^{n-1}}$ is G.M b/w a & b

So $\dfrac{a^n+b^n}{a^{n-1}+b^{n-1}} = \sqrt{ab}$

$\dfrac{a^n+b^n}{a^{n-1}+b^{n-1}} = (ab)^{1/2}$

$a^n+b^n = (a^{n-1}+b^{n-1})\, a^{1/2}b^{1/2}$

$a^n+b^n = a^{n-1+\frac{1}{2}}b^{1/2} + a^{1/2}b^{n-1+1/2}$

$a^n+b^n = a^{n-\frac{1}{2}}b^{1/2} + a^{1/2}b^{n-1/2}$

$a^n+b^n - a^{n-1/2}b^{1/2} - a^{1/2}b^{n-1/2} = 0$

$(a^{n-\frac{1}{2}+\frac{1}{2}} - a^{n-1/2}b^{1/2}) + (b^{n+\frac{1}{2}-\frac{1}{2}} - a^{1/2}b^{n-1/2}) = 0$

$a^{n-1/2}(a^{1/2}-b^{1/2}) - b^{n-1/2}(a^{1/2}-b^{1/2}) = 0$

$(a^{n-1/2} - b^{n-1/2})(a^{1/2}-b^{1/2}) = 0$

$a^{n-1/2} - b^{n-1/2} = 0 \qquad a^{1/2}-b^{1/2} \neq 0$

$a^{n-1/2} = b^{n-1/2} \qquad a^{1/2} \neq b^{1/2}$

$\dfrac{a^{n-1/2}}{b^{n-1/2}} = 1$

$\left(\dfrac{a}{b}\right)^{n-1/2} = \left(\dfrac{a}{b}\right)^0$

Equating the powers

$n - 1/2 = 0$

$n = 1/2$

Q.7 Let the two numbers are a & b

By the given Condition

$\dfrac{a+b}{2} = \sqrt{ab} + 2$

$a+b = 2\sqrt{ab} + 4$ ——(1)

Sum of number $a+b = 20$

$a = 20 - b$

Putting in (1) we get

$(20-b)+b = 2\sqrt{(20-b)b} + 4$

$20 = 2\sqrt{20b-b^2} + 4$

$$10 = \sqrt{20b - b^2} + 2$$

$$10 - 2 = \sqrt{20b - b^2}$$

$$8 = \sqrt{20b - b^2}$$

Squaring on both sides

$$64 = 20b - b^2$$

$$b^2 - 20b + 64 = 0$$

$$b^2 - 16b - 4b + 64 = 0$$

$$b(b-16) - 4(b-16) = 0$$

$$(b-16)(b-4) = 0$$

$b = 16$	$b = 4$
so $a = 20 - b$	$a = 20 - b$
$a = 20 - 16 = 4$	$a = 20 - 4 = 16$
$a = 4,\ b = 16$	$a = 16,\ b = 4$

So the numbers are 4, 16.

Q.8 Let the numbers are a & b

so $\dfrac{a+b}{2} = 5$

$a + b = 10$ —— (1) $\quad 10 - a = b$

also $\sqrt{ab} = 4$

$ab = 16$ (On Squaring)

$$a(10 - a) = 16$$

$$10a - a^2 = 16$$

$$a^2 - 10a + 16 = 0$$

$$a^2 - 8a - 2a + 16 = 0$$

$$a(a-8) - 2(a-8) = 0$$

$$(a-8)(a-2) = 0$$

$$(a-8)(a-2) = 0$$

$a = 8$	$a = 2$
$b = 10 - a$	$b = 10 - a$
$b = 10 - 8$	$b = 10 - 2$
$b = 2$	$b = 8$
so $a = 8,\ b = 2$	so $a = 2\ \ b = 8$

Hence the numbers are 2, 8

TAHIR

Sum of n terms of G.P.

We know that

$$S_n = a + ar + ar^2 + \dots + ar^{n-1} \quad \text{—— (1)}$$

Multiplying both sides by r, we get

$$rS_n = ar + ar^2 + ar^3 + \dots + ar^n \quad \text{—— (2)}$$

Subtracting (2) from (1), we get

$$S_n - rS_n = a - ar^n$$

$$S_n(1 - r) = a(1 - r^n)$$

$$S_n = \frac{a(1 - r^n)}{1 - r} \quad \text{(Proved.)}$$

Now

$$S_n = \frac{a(1 - r^n)}{1 - r} \quad \text{if } |r| < 1$$

$$S_n = \frac{a(r^n - 1)}{r - 1} \quad \text{if } |r| > 1$$

For infinite terms

TAHIR

if $|r| < 1$ then $S_\infty = \dfrac{a}{1 - r}$

*Infinite G. Series Converges if $|r| < 1$