$a_8 = a + 7d$

$a_8 = 4000 + 7(1000)$

$a_8 = 4000 + 7000$

$a_8 = 11000$

The first team will recieve Rs. 11,000

Q.10 Let us denote the Sum of the balls of 1st, 2nd, 3rd, ... 8th layer by $S_1, S_2, S_3, \ldots, S_8$

$\because S_n = \frac{n}{2}(a + a_n)$

$S_8 = 8+7+6+\cdots+1 = \frac{8}{2}((8)+1)$

$= 4(8+1) = 36$

$S_7 = 7+6+5+\cdots+1 = \frac{7}{2}(7+1) = 28$

$S_6 = 6+5+4+\cdots+1 = \frac{\cancel{6}^3}{\cancel{2}}(6+1) = 21$

$S_5 = 5+4+3+2+1 = \frac{5}{2}(5+1) = 15$

$S_4 = 4+3+2+1 = \frac{\cancel{4}^2}{\cancel{2}}(4+1) = 10$

$S_3 = 3+2+1 = \frac{3}{2}(3+1) = 6$

$S_2 = 2+1 = 3$

$S_1 = 1$

Thus Total number of balls

$1+3+6+10+15+21+28+36 = 120$ balls

Tahir Mahmood.
M.Sc. Govt. College Gujranwala.

Geometric Sequence (G.P.):

"The Sequence in which a Common ratio is exist between two consecutive terms is called Geometric Sequence or Geometric Progression."

Common ratio is denoted by r

$r = \frac{a_n}{a_{n-1}}$

Using the following formula we can expand the (G.P)

$a_n = ar^{n-1}$

Exercise 6.6

Q.1 G.P. 3, 6, 12, ...

$a = 3 \quad r = \frac{6}{3} = 2$

$a_n = ar^{n-1}$

$a_5 = (3)(2)^4 = 3 \times 16 = 48$

Q.2 G.P. $= 1+i, 2, \frac{4}{1+i}$

$a = 1+i \quad r = \frac{2}{1+i}$

$a_n = ar^{n-1}$

$a_{11} = ar^{10}$

$a_{11} = (1+i)\left[\frac{2}{1+i}\right]^{10}$

$a_{11} = (1+i)\,\frac{1024}{(1+i)^{10}}$

$a_{11} = \frac{1024}{(1+i)^9}$

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Q.3

G.P. $= 1+i, 2i, -2+2i, \ldots$

$a = 1+i \quad r = \frac{2i}{1+i}$

$\because a_n = ar^{n-1}$

$a_{12} = (1+i)\left(\frac{2i}{1+i}\right)^{11}$

$a_{12} = (1+i)\frac{2^{11} \cdot i^{11}}{(1+i)^{11}}$

$a_{12} = \frac{2048\, i\, (i)^{10}}{(1+i)^{10}} = \frac{2048\, i\, (-1)^5}{(1+i)^{10}}$

$a_{12} = -\frac{2048\, i}{(1+i)^{10}}$

Q.4 G.P. $= 1+i, 2, 2(1-i), \ldots$

$a = 1+i \quad r = \frac{2}{1+i}$

$a_{11} = ar^{10}$

$a_{11} = (1+i)\left(\frac{2}{1+i}\right)^{10}$

$a_{11} = (1+i)\frac{1024}{(1+i)^{10}}$

$a_{11} = \frac{1024}{(1+i)^9}$

TAHIR

Q.5 $a = 12{,}000$

Depreciation 1^{st} year $= a \times \frac{5}{100} = a\, 0.05$

Cost after 1 year $= a - 0.05a$

$= 0.95a$

G.P $= a, 0.95a, \ldots$

$r = \frac{0.95\cancel{a}}{\cancel{a}} = 0.95$

Price after 4 years.

$a_5 = ar^4 \Rightarrow 12000 \times (0.95)^4$

$a_5 = 12000 \times 0.8145 = Rs.\ 9774$

Hence price after 4 years $= 9774$ rupees.

Q.6 $x^2-y^2, x+y, \frac{x+y}{x-y}, \ldots, \frac{x+y}{(x-y)^9}$ **(23)**

$a = x^2-y^2 \quad r = \frac{x+y}{x^2-y^2} = \frac{\cancel{(x+y)}}{\cancel{(x+y)}(x-y)} = \frac{1}{x-y}$

$\because a_n = \frac{x+y}{(x-y)^9}$

$\because a_n = ar^{n-1}$

$\frac{\cancel{x+y}}{(x-y)^9} = [\cancel{(x+y)}(x-y)]\left[\frac{1}{x-y}\right]^{n-1}$

$\left(\frac{1}{x-y}\right)^9 = \left(\frac{1}{x-y}\right)^{n-1} \cdot \left(\frac{1}{x-y}\right)^{-1}$

$\left(\frac{1}{x-y}\right)^9 = \left(\frac{1}{x-y}\right)^{n-1-1}$

$\left(\frac{1}{x-y}\right)^{n-2} = \left(\frac{1}{x-y}\right)^9$

Equating the powers

$n-2 = 9 \Rightarrow n = 9+2$

$n = 11$ Hence 11th term.

Q.7 If a, b, c, d are in G.P. Prove

$\because r = \frac{b}{a} = \frac{c}{b} = \frac{d}{c}$

$b = ar$ — (1) $\quad c = br \quad d = cr$

$c = ar^2$ — (2) $\quad d = ar^3$ — (3)

(i) $a-b, b-c, c-d$ are in G.P.

$\because \frac{b-c}{a-b} = \frac{c-d}{b-c}$

$(b-c)^2 = (a-b)(c-d)$

LHS $= (b-c)^2$

$= (ar - ar^2)^2 = a^2r^2(1-r)^2$

RHS $= (a-b)(c-d)$

$= (a-ar)(ar^2-ar^3)$

$= a(1-r)\, ar^2(1-r)$

$= a^2r^2(1-r)^2$

$\because$ LHS = RHS

So $a-b, b-c, c-d$ are in G.P.

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(ii) $a^2-b^2,\ b^2-c^2,\ c^2-d^2$ are in G.P.

so $\frac{b^2-c^2}{a^2-b^2} = \frac{c^2-d^2}{b^2-c^2}$

$$(b^2-c^2)^2 = (c^2-d^2)(a^2-b^2)$$

LHS $= (b^2-c^2)^2$

$= (a^2r^2-a^2r^4)^2$

$= (a^2r^2)^2(1-r^2)^2$

$= a^4r^4(1-r^2)^2$

RHS $= (c^2-d^2)(a^2-b^2)$

$= (a^2-a^2r^2)(a^2r^4-a^2r^6)$

$= a^2(1-r^2)\,a^2r^4(1-r^2)$

$= a^4r^4(1-r^2)^2$

LHS = RHS.

So $a^2-b^2,\ b^2-c^2,\ c^2-d^2$ are in G.P

(iii) $a^2+b^2,\ b^2+c^2,\ c^2+d^2$ are in G.P.

so $\frac{b^2+c^2}{a^2+b^2} = \frac{c^2+d^2}{b^2+c^2}$

$$(b^2+c^2)^2 = (a^2+b^2)(c^2+d^2)$$

LHS $= (b^2+c^2)^2$

$= (a^2r^2+a^2r^4)^2$

$= a^4r^4(1+r^2)^2$

RHS $= (a^2+b^2)(c^2+d^2)$

$= (a^2+a^2r^2)(a^2r^4+a^2r^6)$

$= a^2(1+r^2)\,a^2r^4(1+r^2)$

$= a^4r^4(1+r^2)^2$

LHS = RHS

So $a^2+b^2,\ b^2+c^2,\ c^2+d^2$ are in G.P.

Q.8

G.P $= a_1,\ a_1r^2,\ a_1r^4, \dots$

Reciprocal of the Given G.P

$\frac{1}{a_1},\ \frac{1}{a_1r^2},\ \frac{1}{a_1r^4},\ \dots$

Here

$r = \frac{\frac{1}{a_1r^2}}{\frac{1}{a_1}} = \frac{a_1}{a_1r^2} = \frac{1}{r^2}$

also

$r = \frac{\frac{1}{a_1r^4}}{\frac{1}{a_1r^2}} = \frac{a_1r^2}{a_1r^4} = \frac{1}{r^2}$

Since common ratio exists so

$\frac{1}{a_1},\ \frac{1}{a_1r^2},\ \frac{1}{a_1r^4} \dots$ are in G.P.

Q.9 $\because \frac{a_5}{a_3} = \frac{4}{9}$ $\qquad a_2 = 4/9$

$\frac{ar^4}{ar^2} = 4/9$ $\qquad ar = 4/9$

$r^2 = \frac{4}{9}$ $\qquad \because a\left(\pm\frac{2}{3}\right) = \frac{4}{9}$

$r = \pm\frac{2}{3}$ $\qquad a = \frac{4}{9}\times \pm\frac{3}{2}$

$a = \pm\frac{2}{3}$

$a=\frac{2}{3},\ r=\frac{2}{3}$

$a_n = ar^{n-1}$

$a_n = \frac{2}{3}\left(\frac{2}{3}\right)^{n-1}$

$a_n = \left(\frac{2}{3}\right)^{n-1+1}$

$a_n = \left(\frac{2}{3}\right)^{n}$ —(1)

$a=-\frac{2}{3},\ r=-\frac{2}{3}$

$a_n = ar^{n-1}$

$a_n = -\frac{2}{3}\left(-\frac{2}{3}\right)^{n-1}$

$a_n = \left(-\frac{2}{3}\right)^{n-1+1}$

$a_n = (-1)^n\left(\frac{2}{3}\right)^n$ —(2)

Hence nth terms are (1) and (2)

Tahir Mahmood

Q.10 Let the numbers are $\frac{a}{r}, a, ar$

$\because \frac{a}{r} + a + ar = 26$

$\frac{a+ar+ar^2}{r} = 26$

$a+ar+ar^2 = 26r$ —(1)

Now $\frac{a}{r} \cdot a \cdot ar = 216$

$a^3 = 216$

$a = 6$

Putting in (1) we get

$6+6r+6r^2 = 26r$

$6r^2+6r-26r+6 = 0$

$2[3r^2-10r+3] = 0 \quad 2 \neq 0$

$3r^2-10r+3 = 0$

$3r^2-9r-r+3 = 0$

$3r[r-3]-1[r-3] = 0$

$(r-3)(3r-1) = 0$

$r = 3 \quad r = 1/3$

$a=6 \;\; r=3$	$a=6 \;\; r=1/3$
$\frac{a}{r} = \frac{6}{3} = 2$	$\frac{a}{r} = \frac{6}{1/3} = 18$
$a = 6$	$a = 6$
$ar = 6(3) = 18$	$ar = 6 \cdot \frac{1}{3} = 2$
2, 6, 18	18, 6, 2

Hence in both the cases

Numbers are, 2, 6, 18

Q.11 Let the numbers are a, ar, ar^2, ar^3

$a+ar+ar^2+ar^3 = 80$ —(1)

Now A.M of 2nd and 4th number

$\frac{ar+ar^3}{2} = 30$

$ar+ar^3 = 60$ —(2)

Putting in (1), we get

$a+ar^2+(60) = 80$

$a+ar^2 = 80-60 = 20$

$a+ar^2 = 20$ —(A)

Multiplying by r.

$ar+ar^3 = 20r$ —(3)

Subtracting (2) from (3)

$\cancel{ar}+\cancel{ar^3} = 20r$

$\cancel{ar}+\cancel{ar^3} = 60$

$0 = 20r-60$

$20r = 60 \Rightarrow r = \frac{60}{20}$

$r = 3$

Putting in (A) we get

$a+a(3)^2 = 20$

$a+9a = 20 \Rightarrow 10a = 20$

$a = 2$

So the numbers are

$a = 2$

$ar = 2(3) = 6$

$ar^2 = 2(3)^2 = 18$

$ar^3 = 2(3)^3 = 54$

Q.12 $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in G.P

Let r is the common ratio.

$r = \frac{1/b}{1/a} = \frac{a}{b}$ —— (1)

also $r = \frac{1/c}{1/b} = b/c$ —— (2)

Multiplying (1) and (2), we get

$r \times r = \frac{a}{b} \times \frac{b}{c}$

$r^2 = \frac{a}{c}$

$r = \pm\sqrt{a/c}$ which is common ratio.

Q.13 Let the numbers in A.P are

$a-d,\ a,\ a+d$

$(a-d)+a+(a+d) = 21$

$3a = 21 \Rightarrow a = 7$

Subtracting 1, 4, 3, we get G.P

$a-d-1,\ a-4,\ a+d-3$

so $\frac{a-4}{a-d-1} = \frac{a+d-3}{a-4}$

$(a-4)^2 = (a+d-3)(a-d-1)$

Put $a = 7$

$(7-4)^2 = (7+d-3)(7-d-1)$

$9 = (4+d)(6-d)$

$9 = 24+6d-4d-d^2$

$d^2-2d-24+9 = 0$

$d^2-2d-15 = 0$

$d^2-5d+3d-15 = 0$

$d(d-5)+3(d-5) = 0$

$(d-5)(d+3) = 0$

$d=5 \qquad d=-3$

Hence the numbers.

$a=7 \quad d=5$	$a=7 \quad d=-3$
$a-d = 7-5 = 2$	$a-d = 7+3 = 10$
$a = 7$	$a = 7$
$a+d = 7+5 = 12$	$a+d = 7-3 = 4$
2, 7, 12	10, 7, 4

Q.14 Let the numbers in A.P are

$a-d,\ a,\ a+d$

$(a-d)+a+(a+d) = 6$

$3a = 6 \Rightarrow a = 2$

Adding 1, 4, 15, we get G.P

$a-d+1,\ a+4,\ a+d+15$ are in G.P.

$\therefore \frac{(a+4)}{(a-d+1)} = \frac{(a+d+15)}{(a+4)}$

$(a+4)^2 = (a+d+15)(a-d+1)$

putting $a=2$ we get

$(6)^2 = (2+d+15)(2-d+1)$

$36 = (17+d)(3-d)$

$36 = 51-17d+3d-d^2$

$d^2+14d+36-51 = 0$

$d^2+14d-15 = 0$

$d^2+15d-d-15 = 0$

$d(d+15)-1(d+15) = 0$

$(d-1)(d+15) = 0$

$d = 1, -15$

So the numbers are.

$a-d = 2-1 = 1$	$a-d = 2+15 = 17$
$a = 2$	$a = 2$
$a+d = 2+1 = 3$	$a+d = 2+(-15) = -13$
1, 2, 3	17, 2, -13