Q.17

$\because a_6 + a_8 = 40$

$(a+5d)+(a+7d) = 40$

$2a + 12d = 40$

$\frac{\cancel{2}(a+6d)}{\cancel{2}} = \frac{40}{\cancel{2}} 20$

$a + 6d = 20$

$a = 20 - 6d$ ——— (1)

$\because a_4 a_7 = 220$

$(a+3d)(a+6d) = 220$

$a^2 + 3ad + 6ad + 18d^2 = 220$

$a^2 + 9ad + 18d^2 = 220$

$(20-6d)^2 + 9(20-6d)d + 18d^2 = 220$

$400 + \cancel{36}d^2 - 240d + 180d - \cancel{54}d^2 + \cancel{18}d^2 = 220$

$400 - 60d = 220$

$400 - 220 = 60d$

$\frac{\cancel{60}d}{\cancel{60}} = \frac{\cancel{180}}{\cancel{60}} 3$

$d = 3$

$a = 20 - 6d$

$a = 20 - 6(3) = 20 - 18 = 2$

$a_1 = 2$

$a_2 = 2 + 3 = 5$

$a_3 = 2 + 2(3) = 8$

$a_4 = 2 + 3(3) = 11$

and so on

Hence the Sequence is

$2, 5, 8, 11, \ldots$

Q.18

$\frac{1}{b+c}, \frac{1}{c+a}, \frac{1}{a+b}$ will be A.P. if

$\frac{1}{c+a} - \frac{1}{b+c} = \frac{1}{a+b} - \frac{1}{c+a}$

$\frac{b+\cancel{c}-\cancel{c}-a}{(c+a)(b+c)} = \frac{c+\cancel{a}-\cancel{a}-b}{(a+b)(c+a)}$

Multiplying by $(c+a)$

$\cancel{(c+a)} \times \frac{b-a}{\cancel{(c+a)}(b+c)} = \frac{(c-b)}{(a+b)\cancel{(c+a)}} \times \cancel{(c+a)}$

$\frac{b-a}{b+c} = \frac{c-b}{a+b}$

$(a+b)(b-a) = (c-b)(b+c)$

$b^2 - a^2 = c^2 - b^2$ (Proved)

This is the common difference of the A.P a^2, b^2, c^2

So $\frac{1}{a+b}, \frac{1}{b+c}, \frac{1}{c+a}$ is in A.P

Exercise: 6.5

Q.1 $10, 15, 20, \ldots, a_9$

$a_1 = 10 \quad d = 15 - 10 = 5 \quad n = 9$

$S_9 = \frac{9}{2}\{2(10) + 8(5)\}$

$S_9 = \frac{9}{2}[20 + 40]$

$S_9 = \frac{9}{\cancel{2}}[\cancel{60}^{30}]$

$S_9 = 270$ rupees.

Q.2 Some of trees $= 378$

$AP = 1, 2, 3, 4, \ldots, 378$

$a_1 = 1 \quad d = 2 - 1 = 1 \quad n = ?$

$S_n = \frac{n}{2}[2a + (n-1)d]$

$378 = \frac{n}{2}[2(1) + (n-1)(1)]$

$378 \times 2 = n[2 + n - 1]$

$756 = n[n+1]$

$n^2 + n - 756 = 0$

$n^2 + 28n - 27n - 756 = 0$

$n[n+28] - 27[n+28] = 0$

$(n-27)(n+28) = 0$

$n - 27 = 0 \quad \wedge \quad n + 28 = 0$

$n = 27 \quad \wedge \quad n = -28$

(which is not possible)

so $n = 27$

So total number of rows = 27

$a_{27} = a_1 + (27-1)d$

$a_{27} = 1 + 26(1)$

$a_{27} = 1 + 26$

$a_{27} = 27$

Thus 27 trees are in the base of △

Q.3 Total Money to pay = 1100 + 230

$S_n = 1330$

He has to repay 1330 in 14 installment each installment being less than up coming by Rs 10 so

$n = 10$ and $d = -10$

$S_n = \frac{n}{2}[2a_1 + (n-1)d]$

$S_{14} = \frac{14}{2}[2a_1 + 13(-10)]$

$1330 = 7[2a_1 - 130]$

$\frac{1330}{7} = 2a_1 - 130$

$190 = 2a_1 - 130$

$\frac{190+130}{2} = a_1$

$a_1 = \frac{320}{2} = 160$

$a_1 = 160$

His first installment is Rs. 160

Q.4 The Clock Strikes according to the Sequence

$1, 2, 3, 4, \dots, a_{12}$

$a_1 = 1 \quad d = 2 - 1 = 1 \quad n = 12$

$S_n = \frac{n}{2}[2a_1 + (n-1)d]$

$S_{12} = \frac{12}{2}[2(1) + (12-1)1]$

$S_{12} = 6[2+11] = 6[13] = 78$

So it strikes 78 times in 12 hours.

Q.5 $S_n = 2100$

$a_1 = 12 \qquad d = 4$

$S_n = \frac{n}{2}[2a_1 + (n-1)d]$

$2100 = \frac{n}{2}[2(12) + (n-1)4]$

$2100 = \frac{\cancel{2}n}{\cancel{2}}[12 + (n-1)2]$

$2100 = n[12 + 2n - 2]$

$2100 = 2n^2 + 10n$

$\frac{\overset{1050}{\cancel{2100}}}{\cancel{2}} = \frac{\cancel{2}(n^2 + 5n)}{\cancel{2}}$

$1050 = n^2 + 5n$

$n^2+5n-1050=0$

$n^2+35n-30n-1050=0$

$n(n+35)-30(n+35)=0$

$(n+35)(n-30)=0$

$n-30=0 \quad \wedge \quad n+35=0$

$n=30 \quad \wedge \quad n=-35$

$n=30$ (which is not possible)

After 30 weeks he will Collect 2100 rupees.

Q.6 Sequence of falling object.

$9, 27, 45, \ldots$

(ii) $a_1 = 9 \quad d = 27-9 = 18 \quad n=5$

$S_n = \frac{n}{2}[2a_1 + (n-1)d]$

$S_5 = \frac{5}{2}[2(9) + (5-1)18]$

$S_5 = \frac{\cancel{2}\cdot 5}{\cancel{2}}[9 + 4(9)]$

$S_5 = 5[9+36]$

$S_5 = 5[45]$

$S_5 = 225$ meters

(i) $a_5 = ?$

$a_5 = a_1 + 4d$

$a_5 = 9 + 4(18)$

$a_5 = 9 + 72 = 81$ m

It will be 81 meter away in 5th second.

Q.7 $a_1 = 6000 \quad n = 11$ (years)

$a_{11} = 12000 \quad S_{11} = ?$

$a_n = a_1 + (n-1)d$

$12000 = 6000 + 10d$

$\frac{12000-6000}{10} = \frac{\cancel{10}d}{\cancel{10}}$

$d = \frac{6000}{10}$

$d = 600$

So income in past 11 years.

$S_{11} = \frac{11}{2}(2(6000) + 10 \times 600)$

$S_{11} = \frac{11}{2}(12000 + 6000)$

$S_{11} = \frac{11}{\cancel{2}}(\overset{9000}{\cancel{18000}})$

$S_{11} = 99000$ rupees

Q.8. Sum of the angles of 3 sided polygon $= \pi$

" " " " " 4 " " $= 2\pi$

" " " " " 5 " " $= 3\pi$

So A.P. $= \pi, 2\pi, 3\pi, \ldots, a_{14}$

$a = \pi \quad d = 2\pi - \pi = \pi \quad n = 14$

$a_n = a_1 + (n-1)d$

$a_{14} = \pi + 13\pi$

$a_{14} = 14\pi$

Q.9 $S_8 = 60{,}000 \quad n = 8$

$a = 4000 \quad a_8 = ?$

$\because S_n = \frac{n}{2}[2a + (n-1)d]$

$60000 = \frac{\cancel{8}4}{\cancel{2}}[2(4000) + (8-1)d]$

$\frac{\overset{15000}{\cancel{60000}}}{\cancel{4}} = \frac{\cancel{4}}{\cancel{4}}[8000 + 7d]$

$15000 = 8000 + 7d$

$7d = 15000 - 8000 = 7000$

$d = 1000$

$a_8 = a + 7d$

$a_8 = 4000 + 7(1000)$

$a_8 = 4000 + 7000$

$a_8 = 11\,000$

The first team will recieve Rs. 11,000

Q.10 Let us denote the Sum of the balls of Ist, 2nd, 3rd, ... 8th Layer by $S_1, S_2, S_3, \ldots, S_8$

$\because S_n = \frac{n}{2}[a + a_n]$

$S_8 = 8+7+6+\cdots+1 = \frac{8}{2}[(8)+1]$

$= 4(8+1) = 36$

$S_7 = 7+6+5+\cdots+1 = \frac{7}{2}[7+1] = 28$

$S_6 = 6+5+4+\cdots+1 = \frac{\cancel{6}^3}{\cancel{2}}[6+1] = 21$

$S_5 = 5+4+3+2+1 = \frac{5}{2}[5+1] = 15$

$S_4 = 4+3+2+1 = \frac{\cancel{4}^2}{\cancel{2}}[4+1] = 10$

$S_3 = 3+2+1 = \frac{3}{2}[3+1] = 6$

$S_2 = 2+1 = 3$

$S_1 = 1$

Thus Total number of balls

$1+3+6+10+15+21+28+36 = 120$ balls

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Geometric Sequence (G.P.):

"The Sequence in which a Common ratio is exist between two consecutive terms is called Geometric Sequence or Geometric Progression."

Common ratio is denoted by r

$r = \frac{a_n}{a_{n-1}}$

Using the following formula we can expand the (G.P)

$a_n = ar^{n-1}$

Exercise 6.6

Q.1 G.P. 3, 6, 12, ...

$a = 3 \quad r = \frac{6}{3} = 2$

$a_n = ar^{n-1}$

$a_5 = (3)(2)^4 = 3 \times 16 = 48$

Q.2 G.P. $= 1+i, 2, \frac{4}{1+i}$

$a = 1+i \quad r = \frac{2}{1+i}$

$a_n = ar^{n-1}$

$a_{11} = ar^{10}$

$a_{11} = (1+i)\left[\frac{2}{1+i}\right]^{10}$

$a_{11} = (1+i)\,\frac{1024}{(1+i)^{10}}$

$a_{11} = \frac{1024}{(1+i)^9}$