Series:

"Sum of the terms of a sequence is known as Series."

Finite Series:-

"Sum of the terms of a finite Sequence is called Finite Series."

Infinite Series:-

"Sum of the infinite Sequence terms is called Infinite Series."

Arithmetic Series:-

"The sum of an arithmetic Sequence terms is called Arithmetic Series."

Sum of the n terms of A.P.

We know that A.P. $= a_1, a_2, a_3, \ldots, a_n$

Adding all the terms

$S_n = a_1 + a_2 + a_3 + \cdots + a_n$

$S_n = a_1 + (a_1+d) + (a_1+2d) + \cdots + (a_1+(n-1)d)$ —(1)

In reverse order (1) can be written

$S_n = (a_1+(n-1)d) + (a_1+(n-2)d) + \cdots + (a_1+2d) + (a_1+d) + (a_1)$ —(2)

Adding (1) and (2), we get

$2S_n = [a_1+(a_1+(n-1)d)] + [a_1+(a_1+(n-1)d)] + \cdots$

$\cdots + [a_1+(a_1+(n-1)d)]$ —(A)

(11)

$\because a_n = a_1+(n-1)d$

$\therefore 2S_n = (a_1+a_n) + (a_1+a_n) + \cdots + (a_1+a_n)$ upto n terms.

$2S_n = n[a_1+a_n]$

$\boxed{S_n = \frac{n}{2}[a_1+a_n]}$

which is the Sum of n terms of an arithemetic Series."

This formula can also be written as

$S_n = \frac{n}{2}[a_1+(a_1+(n-1)d)]$

$\boxed{S_n = \frac{n}{2}[2a_1+(n-1)d]}$

Imp. Exercise: 6.4

Q.1 The terms multiplies by 3 b/w 4 and 97 are $6, 9, 12, 15, \ldots, 96$ are in A.P.

$a_1 = 6 \quad d = 9-6 = 3 \quad a_n = 96$

$\because a_n = a_1+(n-1)d$

$96 = 6+(n-1)3$

$\Rightarrow 96-6 = 3n-3$

$90+3 = 3n \Rightarrow 3n = 93$

$\Rightarrow n = \frac{93}{3} = 31 \Rightarrow (n=31)$

$\because S_n = \frac{n}{2}[2a_1+(n-1)d]$

$S_n = \frac{31}{2}[2(6)+(31-1)3]$

$S_n = \frac{31}{2}(12+93-3)$

$S_n = \frac{31}{2}(102) = (31)(51)$

$S_n = 1581$ Ans.

Q.2 Sum the Series:

(i) $(-3)+(-1)+(7)+(3)+\cdots+a_{16}$

$a_1=-3 \quad d=-1-(-3)=-1+3=2 \quad n=16$

$\because S_n=\frac{n}{2}[2a_1+(n-1)d]$

$S_{16}=\frac{16}{2}[2(-3)+(16-1)2]$

$S_{16}=8[-6+30]=8(24)$

$S_{16}=192$ Ans

(ii) $\frac{3}{\sqrt{2}}+2\sqrt{2}+\frac{5}{\sqrt{2}}+\cdots+a_{13}$

$a_1=\frac{3}{\sqrt{2}} \quad d=2\sqrt{2}-\frac{3}{\sqrt{2}}=\frac{4-3}{\sqrt{2}}=\frac{1}{\sqrt{2}} \quad n=13$

$\because S_n=\frac{n}{2}[2a_1+(n-1)d]$

$S_{13}=\frac{13}{2}\left(2\left(\frac{3}{\sqrt{2}}\right)+(13-1)\frac{1}{\sqrt{2}}\right)$

$S_{13}=\frac{13}{2}\left(\frac{6}{\sqrt{2}}+\frac{12}{\sqrt{2}}\right)=\frac{13}{2}\left(\frac{18}{\sqrt{2}}\right)$

$S_{13}=13\times\frac{9}{\sqrt{2}}=\frac{117}{\sqrt{2}}$ Ans

(iii) $1.11+1.41+1.71+\cdots+a_{10}$

$a_1=1.11 \quad d=1.41-1.11=0.30 \quad n=10$

$\because S_n=\frac{n}{2}[2a_1+(n-1)d]$

$S_{10}=\frac{10}{2}\left(2(1.11)+(10-1)(0.30)\right)$

$S_{10}=5[2.22+9(0.30)]$

$S_{10}=5[2.22+2.70]=5[4.92]$

$S_{10}=24.60$

(iv) $-8-3\frac{1}{2}+1+\cdots+a_{11}$

$a_1=-8 \quad d=\frac{-7}{2}-(-8)=\frac{-7}{2}+8=\frac{-7+16}{2}=\frac{9}{2} \quad n=11$

$S_n=\frac{n}{2}[2a_1+(n-1)d]$

$S_{11}=\frac{11}{2}\left[2(-8)+(11-1)\frac{9}{2}\right]$

$S_{11}=\frac{11}{2}[-16+45]=\frac{11}{2}(29)=\frac{319}{2}=159.9$ Ans.

(v) $(x-a)+(x+a)+(x+3a)+\cdots+n$ terms.

$a_1=x-a \quad d=x+a-x+a=2a \quad n=n$

(12)

$\because S_n=\frac{n}{2}[2(a_1)+(n-1)d]$

$S_n=\frac{n}{2}[2(x-a)+(n-1)2a]$

$S_n=\frac{n}{2}[2(x-a)+2an-2a]$

$S_n=\frac{n}{2}[2x-2a+2an-2a]$

$S_n=n[x-2a+an]$

$S_n=n[x+a(n-2)]$

(vi) $\frac{1}{1-\sqrt{x}}+\frac{1}{1-x}+\frac{1}{1+\sqrt{x}}+\cdots n$ terms.

$a_1=\frac{1}{1-\sqrt{x}} \quad d=\frac{1}{1-x}-\frac{1}{1-\sqrt{x}}=\frac{1-(1+\sqrt{x})}{1-x}$

$d=\frac{1-(1+\sqrt{x})}{1-x}=\frac{\cancel{1}-\cancel{1}-\sqrt{x}}{1-x}$

$d=\frac{-\sqrt{x}}{1-x}$

$\because S_n=\frac{n}{2}[2a_1+(n-1)d]$

$S_n=\frac{n}{2}\left[2\left(\frac{1}{1-\sqrt{x}}\right)+(n-1)\left(\frac{-\sqrt{x}}{1-x}\right)\right]$

$S_n=\frac{n}{2}\left[\frac{2}{1-\sqrt{x}}+\frac{\sqrt{x}}{1-x}-\frac{n\sqrt{x}}{1-x}\right]$

$S_n=\frac{n}{2}\left[\frac{2(1+\sqrt{x})+\sqrt{x}-n\sqrt{x}}{1-x}\right]$

$S_n=\frac{n}{2}\left[\frac{2+2\sqrt{x}+\sqrt{x}-n\sqrt{x}}{1-x}\right]$

$S_n=\frac{n}{2}\left[\frac{2+3\sqrt{x}-n\sqrt{x}}{1-x}\right]$

$S_n=\frac{n}{2}\left[\frac{2+(-n+3)\sqrt{x}}{1-x}\right]$

$S_n=\frac{n}{2}\left[\frac{2+(3-n)\sqrt{x}}{1-x}\right]$

Ans.

(vii) $\frac{1}{1+\sqrt{x}}+\frac{1}{1-x}+\frac{1}{1-\sqrt{x}}+\dots$ upto n terms.

$a_1=\frac{1}{1+\sqrt{x}}$ $\quad d=\frac{1}{1-x}-\frac{1}{1+\sqrt{x}}=\frac{1-(1-\sqrt{x})}{1-x}$

$d=\frac{\cancel{1}-\cancel{1}+\sqrt{x}}{1-x}=\frac{\sqrt{x}}{1-x}$

$\because S_n=\frac{n}{2}[2a_1+(n-1)d]$

$S_n=\frac{n}{2}\left[2\left(\frac{1}{1+\sqrt{x}}\right)+(n-1)\frac{\sqrt{x}}{1-x}\right]$

$S_n=\frac{n}{2}\left[\frac{2}{1+\sqrt{x}}+\frac{n\sqrt{x}-\sqrt{x}}{1-x}\right]$

$S_n=\frac{n}{2}\left[\frac{2(1-\sqrt{x})+n\sqrt{x}-\sqrt{x}}{1-x}\right]$

$S_n=\frac{n}{2}\left[\frac{2-2\sqrt{x}+n\sqrt{x}-\sqrt{x}}{1-x}\right]$

$S_n=\frac{n}{2}\left[\frac{2+n\sqrt{x}-3\sqrt{x}}{1-x}\right]$

$S_n=\frac{n}{2}\left[\frac{2+(n-3)\sqrt{x}}{1-x}\right]$ Ans.

Q.3 number of terms = ?

(i) $-7+(-5)+(-3)+\dots$ $\quad S_n=65$

$a_1=-7$ $\quad d=-5-(-7)=-5+7=2$ $\quad n=?$

$\because S_n=\frac{n}{2}[2a_1+(n-1)d]$

$65=\frac{n}{2}[2(-7)+(n-1)2]$

$65=\frac{n}{2}[-14+2n-2]$

$65=\frac{n}{2}[2(n-8)] \Rightarrow 65=[n^2-8n]$

$n^2-8n-65=0$

$n^2-13n+5n-65=0$

$n[n-13]+5[n-13]=0$

$(n-13)(n+5)=0$

$n-13=0 \quad \wedge \quad n+5=0$

$n=13 \quad \wedge \quad n=-5$ (Not Possible)

Thus $S_n=S_{13}=65$

(ii) $(-7)+(-4)+(-1)+\dots$ $\quad S_n=114$

$a_1=-7$ $\quad d=-4-(-7)=-4+7=3$

$S_n=\frac{n}{2}[2a_1+(n-1)d]$

$114=\frac{n}{2}[2(-7)+(n-1)3]$

$114=\frac{n}{2}[-14+3n-3]$

$2\times114=n[3n-17]$

$228=3n^2-17n$

$3n^2-17n-228=0$

$3n^2-36n+19n-228=0$

$3n[n-12]+19[n-12]=0$

$(n-12)(3n+19)=0$

$n-12=0 \quad \wedge \quad 3n+19=0$

$n=12 \quad \wedge \quad n=\frac{-19}{3}$ (not Possible)

So $n=12$

$S_{12}=114$

Q.4 Sum upto 3n terms.

(i) $3+5-7+9+11-13+15+17-19+\dots$ 3n terms.

Now $(3+5-7)+(9+11-13)+(15+17-19)+\dots$ n terms

$(8-7)+(20-13)+(32-19)+\dots$ n terms.

$1+7+13+\dots$ n terms.

$a_1=1$ $\quad d=7-1=6$

$\because S_n=\frac{n}{2}[2a_1+(n-1)d]$

$S_n=\frac{n}{2}[2(1)+(n-1)6]=\frac{n}{2}(2+6n-6)$

$S_n=\frac{n}{\cancel{2}}\cdot[\cancel{2}(3n-2)]=n(3n-2)$

So $S_n=n(3n-2)$

(ii) $1+4-7+10+13-16+19+22-25+\dots$ $3n$ terms.

$(1+4-7)+(10+13-16)+(19+22-25)+\dots$ n terms.

$-2+7+16+\dots$ n terms.

$a_1=-2 \quad d=7-(-2)=9$

$S_n=\frac{n}{2}[2a_1+(n-1)d]$

$S_n=\frac{n}{2}[2(-2)+(n-1)9]$

$S_n=\frac{n}{2}[-4+9n-9]$

$S_n=\frac{n}{2}[9n-13]$

Q.5 $n=20 \quad S_{20}=? \quad a_r=3r+1$

$a_1=3(1)+1=4$

$a_2=3(2)+1=7$

$3(3)+1=10$

so on

Thus Sequence $4,7,10,\dots$ 20 terms.

$a_1=4 \quad d=7-4=3 \quad n=20$

$\because S_n=\frac{n}{2}[2a_1+(n-1)d]$

$S_{20}=\frac{20}{2}[2(4)+(20-1)3]$

$S_{20}=10[8+19\times3]$

$S_{20}=10[8+57]=10\times65$

$S_{20}=650$

Q.6 If $S_n=n(2n-1)$ find Series?

$\because S_n=n(2n-1)$

Put $(n-1)$ in place of n

$S_{n-1}=(n-1)[2n-2-1]=(n-1)(2n-3)$

$S_{n-1}=2n^2-3n-2n+3=2n^2-5n+3$

$a_n=S_n-S_{n-1}$

$a_n=(2n^2-n)-(2n^2-5n+3)$

$a_n=\cancel{2n^2}-n-\cancel{2n^2}+5n-3$

$a_n=4n-3$

$a_1=4(1)-3=4-3=1$

$a_2=4(2)-3=8-3=5$

$a_3=4(3)-3=12-3=9$

and so on

So the Series $1+5+9+13+\dots+(4n-3)$

Q.7 Let a, d are the first term and Common difference for first Series and $\bar{a}$ and $\bar{d}$ are for Second Series.

$\therefore \frac{S_n}{\bar{S}_n}=\frac{3n+2}{n+1}$

$\frac{\cancel{\frac{n}{2}}[2a+(n-1)d]}{\cancel{\frac{n}{2}}[2\bar{a}+(n-1)\bar{d}]}=\frac{3n+2}{n+1}$

$\frac{\cancel{2}[a+(\frac{n-1}{2})d]}{\cancel{2}[\bar{a}+(\frac{n-1}{2})\bar{d}]}=\frac{3n+2}{n+1}$

We know that

$a_8=a+7d$

$\because$ if $\frac{n-1}{2}=7$ Then $n-1=14 \Rightarrow n=15$

$\frac{a+7d}{\bar{a}+7\bar{d}}=\frac{3(15)+2}{15+1}$

$\frac{a+7d}{\bar{a}+7\bar{d}}=\frac{45+2}{15+1}=\frac{47}{16}=\frac{47}{16}$

$\frac{a_8}{\bar{a}_8}=\frac{47}{16}$

$a_8:\bar{a}_8=47:16$

Q.8 $\because S_2$ is the Sum of $2n$ terms so

$$S_2 = \frac{2n}{2}[2a+(2n-1)d]$$

$$S_2 = n[2a+(2n-1)d] \quad \text{——} (1)$$

$\because S_3$ is the Sum of $3n$ terms so

$$S_3 = \frac{3n}{2}[2a+(3n-1)d] \quad \text{——} (2)$$

$\because S_5$ is the Sum of $5n$ terms so

$$S_5 = \frac{5n}{2}[2a+(5n-1)d] \quad \text{——} (3)$$

Eq(2) − Eq (1), we get

$$S_3 - S_2 = \frac{3n}{2}[2a+(3n-1)d] - \frac{2n}{2}[2a+(2n-1)d]$$

$$S_3 - S_2 = \frac{n}{2}[6a+3(3n-1)d - 2(2a+(2n-1)d]$$

$$S_3 - S_2 = \frac{n}{2}[6a+\underline{9nd}-3d-4a-\underline{4nd}+2d]$$

$$S_3 - S_2 = \frac{n}{2}[2a+5nd-d]$$

$$S_3 - S_2 = \frac{n}{2}[2a+(5n-1)d]$$

Multiplying both Sides by 5,

$$5(S_3 - S_2) = \frac{5n}{2}[2a+(5n-1)d]$$

$$5(S_3 - S_2) = S_5 \quad \text{(Proved.)}$$

Q.9 Sum of all terms (integers) in First 1000 integers neither divisible on 5 or 2 are

$1+3+7+9+11+13+17+19+21+\dots$ 400 terms.

$(1+3+7+9)+(11+13+17+19)+\dots$ 100 terms.

$20+60+100+\dots$ 100 terms.

$a=20 \quad d=60-20=40 \quad n=100$

$$S_n = \frac{n}{2}[2a+(n-1)d]$$

$$S_{100} = \frac{100}{2}[2(20)+(100-1)40]$$

$$S_{100} = 50[40+99\times 40]$$

TAHIR

$$S_{100} = 50[40+3960]$$

$$S_{100} = 50[4000]$$

$$S_{100} = 2,00,000$$

Q.10 $\because S_8$ is the Sum of first 8 terms

$$\because S_8 = \frac{8}{2}[2a+7d] \quad (\because a_1 = a = 2)$$

$$S_8 = 4[2(2)+7d]$$

$$S_8 = 4[4+7d]$$

$$S_8 = 16+28d \quad \text{——} (1)$$

$\because S_9$ is the Sum of First 9 terms

$$\because S_9 = \frac{9}{2}[2a+8d]$$

$$S_9 = \frac{9}{2}[4+8d] = \frac{9}{\cancel{2}}\cdot\cancel{2}[2+4d]$$

$$S_9 = 18+36d \quad \text{——} (2)$$

$$\because 50S_9 = 63S_8$$

$$50[18+36d] = 63[16+28d]$$

$$900+1800d = 1008+1764d$$

$$-1764d+1800d = 1008-900$$

$$1800d-1764d = 1008-900$$

$$36d = 108$$

$$d = \frac{108}{36} = 3$$

$$\because S_9 = 18+36d$$

$$S_9 = 18+36[3]$$

$$S_9 = 18+108 = 126$$

$$\Rightarrow S_9 = 126$$

TAHIR

Q.11 $S_9 = 171$ $a_8 = 31$

$\because S_n = \frac{n}{2}[2a+(n-1)d]$

$S_9 = \frac{9}{2}[2a+8d]$

$171 = \frac{9}{\cancel{2}}.\cancel{2}[a+4d]$

$171 = 9a+36d$ ——(1)

$a_8 = 31$

$31 = a+7d$ ——(2)

$a = 31-7d$

Putting in (1), we get

$171 = 9[31-7d]+36d$

$171 = 279-63d+36d$

$171-279 = -27d$

$-108 = -27d$

$27d = 108$

$d = \frac{108}{27} = 4$

$d = 4$

Now $a = 31-7d$

$a = 31-7(4) = 31-28 = 3$

$a_1 = 3$

$a_2 = 3+4 = 7$

$a_3 = 3+2(4) = 11$

$a_4 = 3+3(4) = 15$

and so one

So Series: $3+7+11+15+\ldots$

TAHIR

Q.12 The Sum of $S_9+S_7 = 203$

(16)

$\cdot\ S_9 - S_7 = 49$

$\because S_9 = \frac{9}{2}[2a+8d]$

$S_9 = 9[a+4d]$

$S_9 = 9a+36d$ ——(1)

$\because S_7 = \frac{7}{2}[2a+6d]$

$S_7 = 7[a+3d]$

$S_7 = 7a+21d$ ——(2)

Adding (1) & (2)

$S_9+S_7 = 9a+36d+7a+21d$

$203 = 16a+57d$ ——(3)

Subtracting (2) from (1)

$S_9-S_7 = 9a+36d-7a-21d$

$49 = 2a+15d$ ——(4)

TAHIR

$\frac{49-15d}{2} = a$

Putting in (3), we get

$203 = \overset{8}{\cancel{16}}\left(\frac{49-15d}{\cancel{2}}\right)+57d$

$203 = 392-120d+57d$

$-392+203 = -120d+57d$

$-189 = -63d$

$d = \frac{189}{63} = 3$

Now $a = \frac{49-15d}{2} = \frac{49-15(3)}{2}$

$a = \frac{49-45}{2} = \frac{4}{\cancel{2}}2$

$a = 2$

TAHIR

$a_1 = a = 2$

$a_2 = a+d = 2+3 = 5$

$a_3 = a+2d = 2+2(3) = 8$

$a_4 = a+3d = 2+(3)3 = 11$

and so on

So Series $= 2+5+8+11+\cdots$

Q.13 $\because \dfrac{S_9}{S_7} = \dfrac{18}{11}$

$$\frac{\frac{9}{2}(2a+8d)}{\frac{7}{2}(2a+6d)} = \frac{18}{11}$$

$$\frac{9(a+4d)}{7(a+3d)} = \frac{18}{11}$$

$$\frac{9a+36d}{7a+21d} = \frac{18}{11}$$

$$11(9a+36d) = 18(7a+21d)$$

$$99a+396d = 126a+378d$$

$$126a-99a = 396d-378d$$

$$27a = 18d$$

$$3a = 2d \implies a = \frac{2}{3}d$$

$\because a_7 = 20$

$20 = a+6d$ —— (1)

Putting the value a in (1)

$$20 = \frac{2}{3}d + 6d$$

$$\cancel{3}\times 20 = \frac{2d+18d}{\cancel{3}}\times \cancel{3}$$

$$20d = 60 \implies d = 3$$

Now $a = \frac{2}{\cancel{3}}(\cancel{3}) \implies a = 2$

Now

$a_1 = a = 2$

$a_2 = a+d = 2+3 = 5$

$a_3 = a+2d = 2+2(3) = 8$

and so on

So Series: $2+5+8+11,\ldots$

Q.14 Let the numbers are

$a-d,\ a,\ a+d$

$\because (a-d)+a+(a+d) = 24$

$3a = 24 \implies a = 8$ —— (1)

$\because (a-d)(a)(a+d) = 440$

$a(a^2-d^2) = 440$

$8(64-d^2) = 440$

$64-d^2 = \frac{\cancel{440}^{55}}{\cancel{8}} = 55$

$64-d^2 = 55$

$64-55 = d^2 \implies d^2 = 9$

$d = \pm 3$

So the Numbers are

$a=8 \quad d=3$	$a=8 \quad d=-3$
$a-d = 8-3 = 5$	$a-d = 8-(-3) = 11$
$a = 8$	$a = 8$
$a+d = 8+3 = 11$	$a+d = 8+(-3) = 5$

Hence the numbers in both Cases are $5, 8, 11$

Q.15 Let the numbers are

$a-3d,\ a-d,\ a+d,\ a+3d$

$\because (a-3d)+(a-d)+(a+d)+(a+3d) = 32$

$4a = 32 \implies a = 8$

Now Sum of Squares of number

$(a-3d)^2+(a-d)^2+(a+d)^2+(a+3d)^2 = 276$

$$\left.\begin{array}{l} a^2+9d^2-\cancel{6da}+a^2+d^2-\cancel{2ad}+a^2+d^2+\cancel{2ad} \\ +a^2+9d^2+\cancel{6ad} \end{array}\right\} = 276$$

$4a^2+20d^2 = 276$

$\dfrac{\cancel{4}(a^2+5d^2)}{\cancel{4}} = \dfrac{\cancel{276}^{\,69}}{\cancel{4}}$

$a^2+5d^2 = 69$

$(8)^2+5d^2 = 69$

$64+5d^2 = 69$

$5d^2 = 69-64$

$\dfrac{\cancel{5}d^2}{\cancel{5}} = \dfrac{\cancel{5}}{\cancel{5}}\ 1$

$d^2 = 1 \implies d = \pm 1$

So the numbers

$a=8 \quad d=1$	$a=8 \quad d=-1$
$a-3d = 8-3(1) = 5$	$a-3d = 8-3(-1) = 11$
$a-d = 8-1 = 7$	$a-d = 8-(-1) = 9$
$a+d = 8+1 = 9$	$a+d = 8+(-1) = 7$
$a+3d = 8+3 = 11$	$a+3d = 8+3(-1) = 5$
5, 7, 9, 11	11, 9, 7, 5

In both Cases, the numbers are

5, 7, 9, 11

Q.16

Let the numbers are

$a+2d,\ a+d,\ a,\ a-d,\ a-2d$

Sum of the numbers.

$(a+\cancel{2d})+(a+\cancel{d})+a+(a-\cancel{d})+(a-\cancel{2d}) = 25$

$5a = 25 \implies a = 5$

The Sum of Squares of the numbers.

$(a+2d)^2+(a+d)^2+a^2+(a-d)^2+(a-2d)^2 = 135$

$$\left.\begin{array}{l} a^2+4d^2+\cancel{4ad}+a^2+d^2+\cancel{2ad}+a^2+a^2+d^2-\cancel{2ad} \\ +a^2+4d^2-\cancel{4da} \end{array}\right\} = 135$$

$5a^2+10d^2 = 135$

$\dfrac{\cancel{5}(a^2+2d^2)}{\cancel{5}} = \dfrac{\cancel{135}}{\cancel{5}}\ 27$

$a^2+2d^2 = 27$

$(5)^2+2d^2 = 27$

$25+2d^2 = 27$

$2d^2 = 27-25 = 2$

$d^2 = 1 \implies d = \pm 1$

So the numbers are

$a=5 \quad d=1$	$a=5 \quad d=-1$
$a+3d = 5+3 = 8$	$a+3d = 5+3(-1) = 2$
$a+d = 5+1 = 6$	$a+d = 5+(-1) = 4$
$a = 5$	$a = 5$
$a-d = 5-1 = 4$	$a-d = 5-(-1) = 6$
$a-3d = 5-3 = 2$	$a-3d = 5-3(-1) = 8$
8, 6, 5, 4, 2	2, 4, 5, 6, 8

So the Numbers are 2, 4, 5, 6, 8

Q.17

$\because a_6 + a_8 = 40$

$(a+5d)+(a+7d) = 40$

$2a + 12d = 40$

$\frac{\cancel{2}(a+6d)}{\cancel{2}} = \frac{40}{\cancel{2}} 20$

$a + 6d = 20$

$a = 20 - 6d$ —— (1)

$\because a_4 a_7 = 220$

$(a+3d)(a+6d) = 220$

$a^2 + 3ad + 6ad + 18d^2 = 220$

$a^2 + 9ad + 18d^2 = 220$

$(20-6d)^2 + 9(20-6d)d + 18d^2 = 220$

$400 + \cancel{36d^2} - 240d + 180d - \cancel{54d^2} + \cancel{18d^2} = 220$

$400 - 60d = 220$

$400 - 220 = 60d$

$\frac{\cancel{60}d}{\cancel{60}} = \frac{\cancel{180}}{\cancel{60}} 3$

$d = 3$

$a = 20 - 6d$

$a = 20 - 6(3) = 20 - 18 = 2$

$a_1 = 2$

$a_2 = 2 + 3 = 5$

$a_3 = 2 + 2(3) = 8$

$a_4 = 2 + 3(3) = 11$

and so on

Hence the Sequence is

$2, 5, 8, 11, \ldots$

Q.18

$\frac{1}{b+c}, \frac{1}{c+a}, \frac{1}{a+b}$ will be A.P. if

$\frac{1}{c+a} - \frac{1}{b+c} = \frac{1}{a+b} - \frac{1}{c+a}$

$\frac{b+\cancel{c}-\cancel{c}-a}{(c+a)(b+c)} = \frac{c+\cancel{a}-\cancel{a}-b}{(a+b)(c+a)}$

Multiplying by $(c+a)$

$\cancel{(c+a)} \times \frac{b-a}{\cancel{(c+a)}(b+c)} = \frac{(c-b)}{(a+b)\cancel{(c+a)}} \times \cancel{(c+a)}$

$\frac{b-a}{b+c} = \frac{c-b}{a+b}$

$(a+b)(b-a) = (c-b)(b+c)$

$b^2 - a^2 = c^2 - b^2$ (Proved)

This is the common difference of the A.P a^2, b^2, c^2

So $\frac{1}{a+b}, \frac{1}{b+c}, \frac{1}{c+a}$ is in A.P

Exercise: 6.5

Q.1 $10, 15, 20, \ldots, a_9$

$a_1 = 10 \quad d = 15 - 10 = 5 \quad n = 9$

$S_9 = \frac{9}{2}\{2(10) + 8(5)\}$

$S_9 = \frac{9}{2}[20 + 40]$

$S_9 = \frac{9}{\cancel{2}}[\cancel{60}\, 30]$

$S_9 = 270$ rupees.

Q.2 Some of trees $= 378$

A.P. $= 1, 2, 3, 4, \ldots, 378$

$a_1 = 1 \quad d = 2 - 1 = 1 \quad n = ?$

$S_n = \frac{n}{2}[2a + (n-1)d]$