 (8)

Arithematic Means: (A.M.)

"A number "A" is called an arithemetic Mean between a and b if a, A, b are in A.P."

$\because$ a, A, b are in A.P.

$d = A - a \ \_\_(1) \wedge \ d = b - A \ \_\_(2)$

By Comparing (1) and (2)

$A - a = b - A$

$A + A = b + a$

$2A = a + b$

$$\boxed{A = \frac{a+b}{2}}$$

Exercise: 6.3

Q.1 Find A.M between

(i) $3\sqrt{5}$ and $5\sqrt{5}$

$\because A = \frac{a+b}{2}$

$A = \frac{3\sqrt{5}+5\sqrt{5}}{2} = \frac{(3+5)\sqrt{5}}{2}$

$A = \frac{8\sqrt{5}}{2} = 4\sqrt{5}$ Ans.

(ii) $x-3$ and $x+5$

$\because A = \frac{a+b}{2}$

$A = \frac{(x-3)+(x+5)}{2} = \frac{2x+2}{2}$

$A = x+1$ Ans.

(iii) $1-x+x^2$ and $1+x+x^2$

$\because A = \frac{a+b}{2}$

$A = \frac{(1-\cancel{x}+x^2)+(1+\cancel{x}+x^2)}{2}$

$A = \frac{2+2x^2}{2}$

$A = \frac{\cancel{2}(1+x^2)}{\cancel{2}}$

$A = 1+x^2$ Ans.

Q.2 $\because$ 5, 8 are two AM between a, b

so a, 5, 8, b

$\because$ 5 is A.M between a and 8

so $5 = \frac{a+8}{2}$

$\Rightarrow a+8 = 10 \Rightarrow a = 10-8$

$\Rightarrow \boxed{a = 2}$

Now 8 is the AM between b and 5

so $8 = \frac{5+b}{2}$

$\Rightarrow 16 = 5+b$

$\Rightarrow b = 16-5 = 11 \Rightarrow \boxed{b=11}$

Q.3 Let $A_1, A_2, A_3, A_4, A_5, A_6$ are A.Ms b/w 2 and 5

So $2, A_1, A_2, A_3, A_4, A_5, A_6, 5$ are in A.P.

$a_1 = 2 \quad a_7 = 5 \quad n = 8$

$\because a_n = a_1 + (n-1)d$

$5 = 2 + (8-1)d \Rightarrow 5 = 2 + 6d$

$5 - 2 = 7d \Rightarrow 7d = 3$

$d = \frac{3}{7} \Rightarrow d = \frac{3}{7}$

$A_1 = a_1 + d = 2 + \frac{3}{7} = \frac{14+3}{7} = \frac{17}{7}$

$A_2 = a_1 + 2d = 2 + 2\left(\frac{3}{7}\right) = \frac{14+6}{7} = \frac{20}{7}$

$A_3 = a_1+3d = 2+3\left(\frac{3}{7}\right) = \frac{14+9}{7} = \frac{23}{7}$

$A_4 = a_1+4d = 2+4\left(\frac{3}{7}\right) = \frac{14+12}{7} = \frac{26}{7}$

$A_5 = a_1+5d = 2+5\left(\frac{3}{7}\right) = \frac{14+15}{7} = \frac{29}{7}$

$A_6 = a_1+6d = 2+6\left(\frac{3}{7}\right) = \frac{14+18}{7} = \frac{32}{7}$

Q.4 Let A_1, A_2, A_3, A_4 are 4 AMs b/w $\sqrt{2}$ and $\frac{12}{\sqrt{2}}$

$\therefore \sqrt{2}, A_1, A_2, A_3, A_4, \frac{12}{\sqrt{2}}$ are in A.P.

$a_1 = \sqrt{2} \quad a_6 = \frac{12}{\sqrt{2}} \quad n = 6$

$a_n = a_1 + (n-1)d$

$\frac{12}{\sqrt{2}} = \sqrt{2} + (6-1)d$

$\frac{12}{\sqrt{2}} - \sqrt{2} = 5d$

$5d = \frac{12-2}{\sqrt{2}} = \frac{10}{\sqrt{2}}$

$d = \frac{\cancel{10}\,2}{\cancel{5}\sqrt{2}} \quad d = \frac{2}{\sqrt{2}} = \sqrt{2}$

$A_1 = a_1 + d = \sqrt{2}+\sqrt{2} = 2\sqrt{2}$

$A_2 = a_1 + 2d = \sqrt{2}+2\sqrt{2} = 3\sqrt{2}$

$A_3 = a_1+3d = \sqrt{2}+3\sqrt{2} = 4\sqrt{2}$

$A_4 = a_1+3d = \sqrt{2}+4\sqrt{2} = 5\sqrt{2}$

Q.5 Let $A_1, A_2, A_3, A_4, A_5, A_6, A_7$ are AMs between 4 and 8

so $4, A_1, A_2, A_3, A_4, A_5, A_6, A_7, 8$ are in A.P.

$a_1 = 4 \quad n = 9 \quad a_9 = 8$

$\therefore a_9 = a_1 + (9-1)d$

$8 = 4 + 8d$

$8 - 4 = 8d$

$4 = 8d$

$d = \frac{4}{8} = \frac{1}{2}$

$A_1 = a_1 + d = 4 + \frac{1}{2} = \frac{8+1}{2} = \frac{9}{2}$

$A_2 = a_1+2d = 4+2\left(\frac{1}{2}\right) = 4+1 = 5$

$A_3 = a_1+3d = 4+3\left(\frac{1}{2}\right) = \frac{8+3}{2} = \frac{11}{2}$

$A_4 = a_1+4d = 4+\cancel{4}\,2\left(\frac{1}{\cancel{2}}\right) = 4+2 = 6$

$A_5 = a_1+5d = 4+5\left(\frac{1}{2}\right) = \frac{8+5}{2} = \frac{13}{2}$

$A_6 = a_1+6d = 4+\cancel{6}\,3\left(\frac{1}{\cancel{2}}\right) = 7$

$A_7 = a_1+7d = 4+7\left(\frac{1}{2}\right) = \frac{8+7}{2} = \frac{15}{2}$

Q.6 Let A_1, A_2, A_3 are 3 AM between 3 & 11

so $3, A_1, A_2, A_3, 11$ are in A.P.

$a_1 = 3 \quad n = 5 \quad a_5 = 11$

$\therefore a_5 = a_1 + (5-1)d$

$11 = 3 + 4d$

$11 - 3 = 4d$

$8 = 4d \implies d = \frac{8}{4}$

$d = 2$ ✓

$A_1 = a_1 + d = 3+2 = 5$

$A_2 = a_1+2d = 3+2(2) = 3+4 = 7$

$A_3 = a_1+3d = 3+3(2) = 3+6 = 9$

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Q.7 $n = ?$

$\because \dfrac{a^n+b^n}{a^{n-1}+b^{n-1}}$ is any A.M. b/w a & b

so

$$\frac{a^n+b^n}{a^{n-1}+b^{n-1}} = \frac{a+b}{2}$$

$$2(a^n+b^n) = (a+b)(a^{n-1}+b^{n-1})$$

$$2a^n+2b^n = a^{n-1}\cdot a + ab^{n-1} + a^{n-1}b + b^{n-1}\cdot b$$

$$2a^n+2b^n = a^n + ab^{n-1} + a^{n-1}b + b^n$$

$$2a^n+2b^n - a^n - a^{n-1}b - ab^{n-1} - b^n = 0$$

$$a^n + b^n - a^{n-1}b - ab^{n-1} = 0$$

$$a^n - a^{n-1}b + b^n - ab^{n-1} = 0$$

$$a^{n-1}\cdot a - a^{n-1}b + b^{n-1}\cdot b - ab^{n-1} = 0$$

$$a^{n-1}(a-b) + b^{n-1}(b-a) = 0$$

or $a^{n-1}(a-b) - b^{n-1}(a-b) = 0$

$$(a-b)(a^{n-1} - b^{n-1}) = 0$$

$a^{n-1} - b^{n-1} = 0 \qquad \because a-b \neq 0$

$a^{n-1} = b^{n-1} \qquad a \neq b$

$$\frac{a^{n-1}}{b^{n-1}} = \frac{b^{n-1}}{b^{n-1}}$$

$$\left(\frac{a}{b}\right)^{n-1} = 1 \Rightarrow \left(\frac{a}{b}\right)^{n-1} = \left(\frac{a}{b}\right)^0$$

$$\Rightarrow n-1 = 0 \Rightarrow n = 1$$

So $\boxed{n=1}$

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Q.8

Let $A_1, A_2, A_3, \ldots, A_n$ be n A.Ms b/w a & b

so $a, A_1, A_2, A_3, \ldots, A_n, b$ are in A.P.

(10)

$a_1 = a \qquad n = n+2 \qquad a_{n+2} = b$

$\because a_{n+2} = a_1 + (n+2-1)d$

$$b = a + (n+1)d$$

$$b = a + nd + d$$

$$b - a = nd + d$$

$$(n+1)d = b - a$$

$$d = \frac{b-a}{n+1}$$

Thus sum of n A.Ms b/w a & b

$$S_n = A_1 + A_2 + A_3 + \cdots + A_n$$

$$\left(\because S_n = \frac{n}{2}[a_1 + a_n]\right)$$

$$\because S_n = \frac{n}{2}[A_1 + A_n]$$

$$S_n = \frac{n}{2}[(a+d) + (a + (n+2-1)d)]$$

$$S_n = \frac{n}{2}[a+d + a+nd]$$

$$S_n = \frac{n}{2}[2a + (n+1)d]$$

$$S_n = \frac{n}{2}\left(2a + \cancel{(n+1)}\cdot\frac{b-a}{\cancel{(n+1)}}\right)$$

$$S_n = \frac{n}{2}[2a + b - a]$$

$$S_n = \frac{n}{2}[a+b]$$

$$S_n = n\left(\frac{a+b}{2}\right)$$

$\left(\because \text{A.M} = \frac{a+b}{2}. \text{ between } a \,\&\, b\right)$

$\because S_n = n[\text{A.M b/w } a \text{ and } b]$

(Proved.)

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