(vi) $1, -3, 5, -7, 9, -11$

$a_1 = 1$ $\quad a_2 = -3$

$a_3 = 1+4 = 5$ $\quad a_4 = -3-4 = -7$

$a_5 = 5+4 = 9$ $\quad a_6 = -7-4 = -11$

$a_7 = 9+4 = 13$ $\quad a_8 = -11-4 = -15$

Hence Sequence:

$1, -3, 5, -7, 9, -11, 13, -15, \ldots$

Arithmetic Sequence: (A.P)

"The sequence in which a common difference is exist between two consecutive terms." It is also called Arithmetic progression. Common difference between two terms is denoted by d and $d = a_n - a_{n-1}$

We can expand this sequence using the following formula.

$l = a_n = a_1 + (n-1)d$

where a_n or l is nth term and n is an integer. (+ve)

NOTE:- TAHIR

(i) Arithmetic Sequence is also called Arithmetic progression and is denoted by (A.P.) abbreviately (اے پی)

Exercise: 6.2

(4)

Q.1 Find First Four terms if:

(i) $a_1 = 5$ and 3 Consecutive terms 23, 26, 29

∵ 23, 26, 29 are Consecutive terms so

$d = 26 - 23 = 3$

$a_2 = 5+(2-1)3 = 5+3 = 8$

$a_3 = 5+(3-1)3 = 5+6 = 11$

$a_4 = 5+(4-1)3 = 5+9 = 14$

$a_5 = 5+(5-1)3 = 5+12 = 17$

(ii) $a_5 = 17$ $\quad a_9 = 37$

$17 = a_1 + 4d$ —(1) $\quad 37 = a_1 + 8d$ —(2)

$37 = a_1 + 8d$

$-17 = -a_1 - 4d$

$20 = 4d$

$d = 5$

∵ $a_1 + 4(5) = 17$

$a_1 = 17 - 20$ TAHIR

$a_1 = -3$

$a_2 = -3+(2-1)5 = -3+5 = 2$

$a_3 = -3+(3-1)5 = -3+10 = 7$

$a_4 = -3+(4-1)5 = -3+15 = 12$

$a_5 = -3+(5-1)5 = -3+20 = 17$

(iii) $7a_4 = 3a_7$ $\quad a_{10} = 33$

$33 = a_1 + 9d$ —(1)

$7(a_1+3d) = 3(a_1+6d)$

$7a_1 + 21d = 3a_1 + 18d$

$7a_1 - 3a_1 + 21d - 18d = 0$

$4a_1 + 3d = 0 \Rightarrow 4a_1 = -3d$

$\Rightarrow a_1 = \frac{-3d}{4}$

Putting in (1), we get

$33 = \left(\frac{-3d}{4}\right) + 9d$

$33 = \frac{-3d + 36d}{4}$

$33 \times 4 = -3d + 36d$

$33d = 132 \Rightarrow d = 4$

Now $a_1 = \frac{-3(4)}{4} = -3$

$\Rightarrow a_1 = -3$

$a_2 = -3 + (2-1)4 \Rightarrow a_2 = -3 + 4 = 1$

$a_3 = -3 + (3-1)4 = -3 + 8 = 5$

$a_4 = -3 + (4-1)4 = -3 + 12 = 9$

$a_5 = -3 + (5-1)4 = -3 + 16 = 13$

Q.2 $a_{n-3} = 2n - 5$ $\quad a_n = ?$

$2n - 5 = a_{n-3}$

Put $n+3$ instead of n

$a_{n+3-3} = 2(n+3) - 5$

$a_n = 2n + 6 - 5$

$a_n = 2n + 1$

which is nth term.

Q.3 $a_5 = 16$ $\quad a_{20} = 46$ $\quad a_{12} = ?$

$16 = a_1 + 4d$ —(1) $\quad 46 = a_1 + 19d$ —(2)

$46 = a_1 + 19d$

$\underline{-16 = -a_1 - 4d}$

$30 = 15d$ $\quad d = \frac{30}{15} = 2$

$\therefore d = 2$

$16 = a_1 + 4(2)$

$16 = a_1 + 8 \Rightarrow a_1 = 16 - 8$

$a_1 = 8$

$a_{12} = a_1 + (12-1)d$

$a_{12} = 8 + 11(2)$

$a_{12} = 8 + 22$

$a_{12} = 30$

Q.4 $a_{13} = ?$

Sequence $= x, 1, 2-x, 3-2x, \ldots$

$a_1 = x$ $\quad d = 1 - x$

$a_{13} = a_1 + (13-1)d$

$a_{13} = x + 12(1-x)$

$a_{13} = x + 12 - 12x$

$a_{13} = 12 - 11x$

Q.5 $a_{18} = ?$ $\quad a_6 = 19$ $\quad a_9 = 31$

$19 = a_1 + 5d$ —(1) $\quad 31 = a_1 + 8d$ —(2)

$31 = a_1 + 8d$

$\underline{-19 = -a_1 - 5d}$

$12 = 3d$

$3d = 12 \Rightarrow d = 4$

$\therefore 19 = a_1 + 5d$

$a_1 = 19 - 5d$

$a_1 = 19 - 5(4) \Rightarrow a_1 = 19 - 20 = -1$

$\therefore a_{18} = a_1 + 17d$

$a_{18} = -1 + 17(4)$

$a_{18} = -1 + 68 = 67$

(6)

Q.6

$A.P. = 5, 2, -1, \ldots, -85$

$\because a_n = a_1 + (n-1)d$

$a = 5 \quad d = 2 - 5 = -3 \quad a_n = -85$

$\because -85 = 5 + (n-1)(-3)$

$-85 - 5 = (n-1)(-3)$

$-3n + 3 = -90$

$-3n = -90 - 3$

$+3n = +93$

$n = \frac{93}{3} = 31$

$n = 31$

So $a_{31} = -85$ term.

Q.7 $A.P. = -2, 4, 10, \ldots, 148$

$a_1 = -2 \quad d = 4 - (-2) = 6 \quad a_n = 148$

$\because a_n = a_1 + (n-1)d$

$148 = -2 + (n-1)6$

$148 + 2 = 6n - 6$

$150 + 6 = 6n$

$n = \frac{156}{6} = 26$

$n = 26$

$a_{26} = 148$ term.

Q.8 $a_1 = 11 \quad a_n = 68 \quad d = 3$

$n = ?$

$\because a_n = a_1 + (n-1)d$

$68 = 11 + (n-1)3$

$68 - 11 = 3n - 3$

$57 = 3n - 3$

$3n = 57 + 3$

$3n = 60$

$n = \frac{60}{3} = 20$

Hence there are 20 term in A.P.

Q.9 $a_n = 3n - 1$

$a_1 = 3(1) - 1 = 2 \quad$ at $n = 1$

$a_2 = 3(2) - 1 = 6 - 1 = 5 \quad$ at $n = 2$

$a_3 = 3(3) - 1 = 9 - 1 = 8 \quad$ at $n = 3$

$a_4 = 3(4) - 1 = 12 - 1 = 11 \quad$ at $n = 4$

— — — —

Hence

$A.P. = 2, 5, 8, 11, \ldots, 3n-1$

Q.10 $A.P. = 17, 13, 9, \ldots$

(i) Is -19 the term of A.P. = ?

$a_n = -19 \quad a_1 = 17 \quad d = 13 - 17 = -4$

$\because -19 = 17 + (n-1)(-4)$

$-19 - 17 = -4n + 4$

$-38 - 4 = -4n$

$+4n = +40$

$n = 10 \quad$ n is an integer

So $a_{10} = -19$ and is the term of A.P.

(ii) Is 2 the term of A.P.

$a_n = 2 \quad a_1 = 17 \quad d = -4$

$\because 2 = 17 + (n-1)(-4)$

$2 - 17 = -4n + 4$

$-15 - 4 = -4n \Rightarrow +4n = +19$

$n = \frac{19}{4}$ which is not integer 2 ∉ A.P.

Q.11

$a_p = l \qquad a_q = m \qquad a_r = n$

$\because\ l = a_1 + (p-1)d = a_1 + pd - d$ — ①

$m = a_1 + (q-1)d = a_1 + qd - d$ — ②

$n = a_1 + (r-1)d = a_1 + rd - d$ — ③

(i) Eq ① − Eq ② ∧ Eq ② − Eq ③

$l - m = a_1 + pd - d - a_1 - qd + d$

$l - m = (p-q)d$ — ④

$m - n = a_1 + qd - d - a_1 - rd + d$

$m - n = (q-r)d$ — (5)

Dividing ④ by ⑤

$$\frac{l-m}{m-n} = \frac{(p-q)d}{(q-r)d}$$

$(p-q)(m-n) = (l-m)(q-r)$

$pm - pn - qm + qn = lq - lr - qm + mr$

$pm - pn + qn - lq + lr - mr = 0$

$p(m-n) + q(n-l) + r(l-m) = 0$

(Proved)

(ii) $\because\ pm - pn + qn - lq + lr - mr = 0$

Multiplying both sides by (−1), we get.

$-pm + pn - qn + lq - lr + mr = 0$

$lq - lr + mr - mp + np - nq = 0$

$l(q-r) + m(r-p) + n(p-q) = 0$

(Proved)

Q.12 $a_n = ?$

sequence $\left(\frac{4}{3}\right)^2, \left(\frac{7}{3}\right)^2, \left(\frac{10}{3}\right)^2, \ldots$

Let 4, 7, 10, ... is A.P.

$\because\ a_1 = 4 \quad d = 7 - 4 = 3$

$a_n = a_1 + (n-1)d$

$a_n = 4 + (n-1)3$

$a_n = 4 + 3n - 3$

$a_n = 3n + 1$

$\therefore\ a_n$ of the Given Sequence is

$$a_n = \left(\frac{3n+1}{3}\right)^2$$

Q.13 $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in A.P.

So $\frac{1}{b} - \frac{1}{a} = \frac{1}{c} - \frac{1}{b}$

$\frac{a-b}{ab} = \frac{b-c}{bc}$

$\Rightarrow\ c(a-b) = a(b-c)$

$ac - bc = ab - ac$

$ac + ac = ab + bc$

$b(a+c) = 2ac$

$\therefore\ b = \frac{2ac}{a+c}$ (Proved).

Q.14 $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in A.P. So

$d = \frac{1}{b} - \frac{1}{a} = \frac{a-b}{ab}$

$d = \frac{1}{c} - \frac{1}{b} = \frac{b-c}{bc}$

By Adding them

$2d = \frac{1}{c} - \frac{1}{a} = \frac{a-c}{ac}$

$d = \frac{a-c}{2ac}$ (Proved)