Q.5 According to Statement

$\frac{1}{2}$ hours, 1 hours, $\frac{3}{2}$ hours, 2 hours and so on

the Complete becteria 2A, 4A, 8A, 16A...

In Complete hours.

1, 2, 3, 4 and so on Complete

bekterias will be 4A, 16A, 64A, ...

$a = 4A \quad r = \frac{16A}{4A} = 4 \quad n = n$

$a_n = ar^{n-1}$

$a_n = (4A).(4)^{n-1}$

$a_n = A.4^{n-1+1} \Rightarrow A4^n$

$a_n = A\,2^{2n}$ beterias.

Q.6 Parameter of origonal $\Delta = 3/2$

Parameter of equiletral Triangles are

$\frac{3}{2}, \frac{3/2}{2}, \frac{3/2}{2\times 2}, \frac{3/2}{2\times 2\times 2}, \ldots$

$\frac{3}{2}, \frac{3}{4}, \frac{3}{8}, \frac{3}{16}, \ldots$ G.P.

$a = 3/2 \quad r = \frac{3/4}{3/2} = \frac{3}{4}\times\frac{2}{3} = \frac{1}{2} \quad S_\infty = ?$

$S_\infty = \frac{a}{1-r}$

$S_\infty = \frac{3/2}{1-1/2} = \frac{3/\cancel{2}}{\frac{2-1}{\cancel{2}}} = \frac{3}{1}$

$S_\infty = 3$

Hence the Sum of all the

the equilateral triangles parameters

is 3

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Hormonic Sequence (H.P)

"A sequence whose number reciprocals form an Arithmetic Sequence is called Hormonic Sequence or Hormonic Progression"

we can expand H.P by using

$$H_n = \frac{1}{a+(n-1)d}$$

Note: Zero cannot be the term of Hormonic Progression.

Hormonic Mean: (H.M.)

A number H is called a Hormonic Mean between a and b if a, H, b are in H.P.

So $\frac{1}{a}, \frac{1}{H}, \frac{1}{b}$ are in A.P.

$$\frac{1}{H} = \frac{\frac{1}{a}+\frac{1}{b}}{2} = \frac{\frac{a+b}{ab}}{2}$$

$$\frac{1}{H} = \frac{a+b}{2ab}$$

So $H = \frac{2ab}{a+b}$

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Relation b/w (A.M) and (G.M), (H.M)

For any two numbers a, b

$A = \frac{a+b}{2}$

$G = \pm\sqrt{ab} \Rightarrow G^2 = ab$

$H = \frac{2ab}{a+b}$

$A\times H = \frac{\cancel{a+b}}{\cancel{2}}\times\frac{\cancel{2}ab}{\cancel{a+b}} = ab$

$\boxed{A\times H = G^2}$

Prove $A > H$

For any two distinct real numbers

$A > H$ if $\frac{a+b}{2} > \frac{2ab}{a+b}$

$$\frac{a+b}{2} > \frac{2ab}{a+b}$$

Multiplying both sides by $(a+b)2$

$$\frac{\cancel{2}(a+b)^2}{\cancel{2}} > \frac{2ab}{\cancel{a+b}} 2(\cancel{a+b})$$

$$(a+b)^2 > 4ab$$

$$(a+b)^2 - 4ab > 0$$

$$a^2+b^2+2ab-4ab > 0$$

$$a^2+b^2-2ab > 0$$

$(a-b)^2 > 0$ which is true

so proved that

$\boxed{A > H}$ ——(1)

Prove $A > G$:

$A > G$ if $\frac{a+b}{2} > \pm\sqrt{ab}$

$$\frac{a+b}{2} > \pm\sqrt{ab}$$

$$\frac{a+b}{2} \mp \sqrt{ab} > 0$$

Multiplying both sides by 2

$$a+b \mp 2\sqrt{ab} > 0$$

$$(\sqrt{a})^2+(\sqrt{b})^2 \mp 2\sqrt{ab} > 0$$

$$(\sqrt{a} \mp \sqrt{b})^2 > 0$$

So $A > G$ which is true so

$\boxed{A > G}$ ——(2)

Prove $G > H$

$G > H$ if $\sqrt{ab} > \frac{2ab}{a+b}$

$$\sqrt{ab} > \frac{2ab}{a+b}$$

Multiplying both sides by $\frac{a+b}{\sqrt{ab}}$

$$\frac{a+b}{\cancel{\sqrt{ab}}}\cancel{\sqrt{ab}} > \frac{2ab}{\cancel{(a+b)}} \frac{\cancel{a+b}}{\sqrt{ab}}$$

$$a+b > 2\sqrt{ab}$$

$$a+b-2\sqrt{ab} > 0$$

$$(\sqrt{a})^2+(\sqrt{b})^2-2\sqrt{ab} > 0$$

$(\sqrt{a}-\sqrt{b})^2 > 0$ which is true.

so $\boxed{G > H}$ ——(3)

From (1), (2) and (3)

$\boxed{A > G > H}$

Exercise 6.10

Q.1 (i) Find $a_9 = ?$

$\frac{1}{3}, \frac{1}{5}, \frac{1}{7}, \ldots$ are in H.P

so $3, 5, 7, \ldots$ are in A.P.

$a = 3 \quad d = 5-3 = 2$

$a_9 = a+(9-1)d$

$a_9 = 3+8(2) = 3+16 = 19$ (A.P)

$a_9 = \frac{1}{19}$ (in H.P)

(ii) $\frac{-1}{5}, \frac{-1}{3}, -1, \ldots$ are in H.P

$-5, -3, -1, \ldots$ are in A.P.

$a = -5 \quad d = -3-(-5) = -3+5 = 2$

$a_9 = a + 8d$

$a_9 = -5 + 8(2) = -5 + 16 = 11$ (A.P)

so $a_9 = \frac{1}{11}$ (in H.P)

Q.2 $a_{12} = ?$

(i) $\frac{1}{2}, \frac{1}{5}, \frac{1}{8}, \ldots$ are in H.P

$2, 5, 8, \ldots$ are in A.P.

$a = 2 \quad d = 5 - 2 = 3$

$a_{12} = a + 11d$

$a_{12} = 2 + 11(3) = 2 + 33 = 35$ (A.P)

$a_{12} = \frac{1}{35}$ (in H.P)

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(ii) $\frac{1}{3}, \frac{2}{9}, \frac{1}{6}, \ldots$ are in H.P

$3, \frac{9}{2}, 6, \ldots$ are in A.P

$a = 3 \quad d = \frac{9}{2} - 3 = \frac{9-6}{2} = \frac{3}{2}$

$a_{12} = a + 11d$

$a_{12} = 3 + 11\left(\frac{3}{2}\right) = \frac{6+33}{2} = \frac{39}{2}$ (A.P)

$a_{12} = \frac{2}{39}$ (in H.P)

Q.3 Find 5 H.P b/w

(i) $\frac{-2}{5}$ and $\frac{2}{13}$

Let H_1, H_2, H_3, H_4, H_5 are 5 H.M. b/w $-\frac{2}{5}$ and $\frac{2}{13}$

so $\frac{-2}{5}, H_1, H_2, H_3, H_4, H_5, \frac{2}{13}$ are in H.P

and $\frac{-5}{2}, \frac{1}{H_1}, \frac{1}{H_2}, \frac{1}{H_3}, \frac{1}{H_4}, \frac{1}{H_5}, \frac{13}{2}$ are in (A.P)

$a = -\frac{5}{2} \quad n = 7 \quad a_7 = \frac{13}{2}$

$\because a_7 = a + 6d$

$\frac{13}{2} = \frac{-5}{2} + 6d$

$\frac{13}{2} + \frac{5}{2} = 6d \Rightarrow \frac{13+5}{2} = 6d$

$6d = 9 \Rightarrow d = \frac{3}{2}$

$\frac{1}{H_1} = a + d = \frac{-5}{2} + \frac{3}{2} = \frac{-2}{2} = -1$

$\frac{1}{H_2} = a + 2d = -\frac{5}{2} + 2\left(\frac{3}{2}\right) = \frac{-5+6}{2} = \frac{1}{2}$

$\frac{1}{H_3} = a + 3d = \frac{-5}{2} + 3\left(\frac{3}{2}\right) = \frac{-5+9}{2} = \frac{4}{2} = \frac{2}{1}$

$\frac{1}{H_4} = a + 4d = \frac{-5}{2} + 4\left(\frac{3}{2}\right) = \frac{-5+12}{2} = \frac{7}{2}$

$\frac{1}{H_5} = a + 5d = \frac{-5}{2} + 5\left(\frac{3}{2}\right) = \frac{-5+15}{2} = \frac{10}{2} = 5$

So H.Ms are, $-1, 2, \frac{1}{2}, \frac{2}{7}, \frac{1}{5}$

(ii) $\frac{1}{4}$ and $\frac{1}{24}$

Let H_1, H_2, H_3, H_4, H_5 are 5 H.Ms between $\frac{1}{4}$ and $\frac{1}{24}$ so

$\frac{1}{4}, H_1, H_2, H_3, H_4, H_5, \frac{1}{24}$ are in H.P

$4, \frac{1}{H_1}, \frac{1}{H_2}, \frac{1}{H_3}, \frac{1}{H_4}, \frac{1}{H_5}, 24$ are in A.P

$a = 4 \quad n = 7 \quad a_7 = 24$

$\because a_7 = a + 6d$

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$24 = 4 + 6d$

$24 - 4 = 6d \Rightarrow 6d = 20$

$d = \frac{20}{6} = \frac{10}{3}$

$\frac{1}{H_1} = a + d = 4 + \frac{10}{3} = \frac{12+10}{3} = \frac{22}{3}$

$\frac{1}{H_2} = a + 2d = 4 + 2\left(\frac{10}{3}\right) = \frac{12+20}{3} = \frac{32}{3}$

$\frac{1}{H_3} = a+3d = 4+3\left(\frac{10}{3}\right) = \frac{12+30}{3} = \frac{42}{3} = 14$

$\frac{1}{H_4} = a+4d = 4+4\left(\frac{10}{3}\right) = \frac{12+40}{3} = \frac{52}{3}$

$\frac{1}{H_5} = a+5d = 4+5\left(\frac{10}{3}\right) = \frac{12+50}{3} = \frac{62}{3}$

Hence H.Ms are $\frac{3}{22}, \frac{3}{32}, \frac{1}{14}, \frac{3}{52}, \frac{3}{62}$

Q.4// Find 4 HMs b/w

(i) $\frac{1}{3}$ and $\frac{1}{23}$

Let H_1, H_2, H_3, H_4 HMs b/w $\frac{1}{3}$ and $\frac{1}{23}$

so $\frac{1}{3}, H_1, H_2, H_3, H_4, \frac{1}{23}$ (are in H.P)

$3, \frac{1}{H_1}, \frac{1}{H_2}, \frac{1}{H_3}, \frac{1}{H_4}, 23$ (are in A.P)

$a = 3 \quad n = 6 \quad a_6 = 23$

$a_6 = a+5d$

$23 = 3+5d \Rightarrow 5d = 23-3$

$5d = 20 \quad d = \frac{20}{5} = 4$

$\frac{1}{H_1} = a+d = 3+4 = 7$

$\frac{1}{H_2} = a+2d = 3+2(4) = 11$

$\frac{1}{H_3} = a+3d = 3+3(4) = 15$

$\frac{1}{H_4} = a+4d = 3+4(4) = 19$

so H.Ms are $\frac{1}{7}, \frac{1}{11}, \frac{1}{15}, \frac{1}{19}$

(ii) $\frac{7}{3}$ and $\frac{7}{11}$. **TAHIR**

Let H_1, H_2, H_3, H_4 are 4 HMs b/w $\frac{7}{3}$ & $\frac{7}{11}$

so $\frac{7}{3}, H_1, H_2, H_3, H_4, \frac{7}{11}$ are in H.P

$\frac{3}{7}, \frac{1}{H_1}, \frac{1}{H_2}, \frac{1}{H_3}, \frac{1}{H_4}, \frac{11}{7}$ are in A.P.

$a = \frac{3}{7} \quad n = 6 \quad a_6 = \frac{11}{7}$

$a_6 = a+5d$

$\frac{11}{7} = \frac{3}{7}+5d \Rightarrow \frac{11}{7} - \frac{3}{7} = 5d$

$5d = \frac{8}{7} \Rightarrow d = \frac{8}{35}$

$\frac{1}{H_1} = a+d = \frac{3}{7}+\frac{8}{35} = \frac{15+8}{35} = \frac{23}{35}$

$\frac{1}{H_2} = a+2d = \frac{3}{7}+2\left(\frac{8}{35}\right) = \frac{15+16}{35} = \frac{31}{35}$

$\frac{1}{H_3} = a+3d = \frac{3}{7}+3\left(\frac{8}{35}\right) = \frac{15+24}{35} = \frac{39}{35}$

$\frac{1}{H_4} = a+4d = \frac{3}{7}+4\left(\frac{8}{35}\right) = \frac{15+32}{35} = \frac{47}{35}$

Hence HMs are $\frac{35}{23}, \frac{35}{31}, \frac{35}{39}, \frac{35}{47}$

(iii) 4 and 20

Let H_1, H_2, H_3, H_4 are 4 HMs b/w 4 & 20

so $4, H_1, H_2, H_3, H_4, 20$ are in H.P

$\frac{1}{4}, \frac{1}{H_1}, \frac{1}{H_2}, \frac{1}{H_3}, \frac{1}{H_4}, \frac{1}{20}$ are in A.P.

$a = \frac{1}{4} \quad n = 6 \quad a_6 = \frac{1}{20}$

$a_6 = a+5d$

$\frac{1}{20} = \frac{1}{4}+5d \Rightarrow 5d = \frac{1}{20} - \frac{1}{4}$

$5d = \frac{1-5}{20} = -\frac{4}{20}$ **TAHIR**

$5d = \frac{-1}{5} \Rightarrow d = \frac{-1}{25}$

$\frac{1}{H_1} = a+d = \frac{1}{4} - \frac{1}{25} = \frac{25-4}{100} = \frac{21}{100}$

$\frac{1}{H_2} = a+2d = \frac{1}{4} - \frac{2}{25} = \frac{25-8}{100} = \frac{17}{100}$

$\frac{1}{H_3} = a+3d = \frac{1}{4} - \frac{3}{25} = \frac{25-12}{100} = \frac{13}{100}$

$\frac{1}{H_4} = a+4d = \frac{1}{4} - \frac{4}{25} = \frac{25-16}{25} = \frac{9}{100}$

so HMs are $\frac{100}{21}, \frac{100}{17}, \frac{100}{13}, \frac{100}{9}$

Q.5 $a_7 = 1/3$ $a_{10} = 5/21$

$a_{14} = ?$

In A.P $a_7 = 3$ $a_{10} = \frac{21}{5}$

$3 = a + 6d$ — ① $\frac{21}{5} = a + 9d$ — ②

$a = 3 - 6d$

Putting in (2), we get

$\frac{21}{5} = 3 - 6d + 9d$

$\frac{21}{5} - 3 = 3d \Rightarrow 3d = \frac{21-15}{5}$

$3d = \frac{6}{5} \Rightarrow d = \frac{6}{15} = \frac{2}{5}$

$d = 2/5$

$a = 3 - 6(2/5) = \frac{15-12}{5} = 3/5$

$a_{14} = a + 13d$

$a_{14} = 3/5 + 13(2/5) = \frac{3+26}{5} = \frac{29}{5}$ (AP)

$a_{14} = \frac{5}{29}$ in H.P

Q.6 $a = a_1 = -1/3$ $a_5 = 1/5$ (in HP)

$a_1 = a = -3$ $a_5 = 5$ (in AP)

$a_5 = a + 4d$

$5 = -3 + 4d \Rightarrow 5 + 3 = 4d$

$4d = 8 \Rightarrow d = 2$

$a_9 = a + 8d$ (in AP)

$a_9 = -3 + 8(2) = -3 + 16 = 13$ (in A.P)

$a_9 = \frac{1}{13}$ (in H.P)

Q.7 5 is H.Ms b/w 2 and b $b = ?$

$H = \frac{2ab}{a+b} \Rightarrow 5 = \frac{2(2)(b)}{2+b}$

$\frac{4b}{2+b} = 5$

$4b = 10 + 5b$

$-10 = 5b - 4b$

$b = -10$

Q.8 $\frac{1}{K}, \frac{1}{2K+1}, \frac{1}{4K-1}$ are in H.P.

$K, 2K+1, 4K-1$ are in A.P

Since these are in A.P so

$d = 2K-1-K = 4K-1-2K-1$

$2K-1-K = 4K-2K-2$

$K-1 = 2(K-1)$

$2K-2-K-1 = 0$

$K-3 = 0 \Rightarrow K = 3$

Q.9 $\frac{a^{n+1}+b^{n+1}}{a^n+b^n}$ is H.M b/w a & b

$\frac{a^{n+1}+b^{n+1}}{a^n+b^n} = \frac{2ab}{a+b}$

$(a+b)(a^{n+1}+b^{n+1}) = 2ab(a^n+b^n)$

$aa^{n+1}+ab^{n+1}+a^{n+1}b+bb^{n+1} = 2a^{n+1}b+2ab^{n+1}$

$a^{n+2}+ab^{n+1}+a^{n+1}b+b^{n+2} = 2a^{n+1}b+2ab^{n+1}$

$a^{n+2}+a^{n+1}b-2a^{n+1}b = 2ab^{n+1}-ab^{n+1}-b^{n+2}$

$a^{n+1}(a+b-2b) = b^{n+1}(2a-a-b)$

$a^{n+1}\cancel{(a-b)} = b^{n+1}\cancel{(a-b)}$

$\frac{a^{n+1}}{b^{n+1}} = 1$

$\left(\frac{a}{b}\right)^{n+1} = \left(\frac{a}{b}\right)^0$

Equating Powers

$n+1 = 0 \Rightarrow n = -1$

Q.10 a^2, b^2, c^2 are in A.P

$\therefore b^2-a^2 = c^2-b^2$ —— (1)

$a+b,\ b+c,\ c+a$ are in H.P

$\frac{1}{a+b},\ \frac{1}{c+a},\ \frac{1}{b+c}$ are in A.P

$$\frac{1}{c+a}-\frac{1}{a+b} = \frac{1}{b+c}-\frac{1}{c+a}$$

$$\frac{a+b-c-a}{(a+b)(c+a)} = \frac{c+a-b-c}{(b+c)(c+a)}$$

$$\frac{(b-c)}{(a+b)\cancel{(c+a)}} = \frac{(a-b)}{(b+c)\cancel{(c+a)}}$$

$$\frac{(b-c)}{(a+b)} = \frac{(a-b)}{(b+c)}$$

$$(b-c)(b+c) = (a+b)(a-b)$$

$$b^2-c^2 = a^2-b^2$$

which is the common difference of a^2, b^2, c^2 in AP so

$a+b,\ c+a,\ b+c$ are in H.P.

Q.11 $a_1+a_5 = 4/7$ in H.P

$\therefore a_1 = 1/2$ in H.P $\quad a_1 = 2$ in A.P.

$a_5 = 4/7 - 1/2 = \frac{8-7}{14} = 1/14$

$a_5 = 1/14$ in H.P

$a_5 = 14$ in A.P

$14 = a+4d \Rightarrow 14 = 2+4d$

$4d = 12 \Rightarrow d = 3$

so

$a_2 = a+d = 2+3 = 5$

$a_3 = a+2d = 2+2(3) = 8$

$\vdots$

so H.P $= \frac{1}{2}, \frac{1}{5}, \frac{1}{8}, \ldots$

(Q12 is already solved)

Q.14 Show that $A > G > H$

(i) $a = 2 \quad b = 8$

$A = \frac{a+b}{2} = \frac{2+8}{2} = 5$

$H = \frac{2ab}{a+b} = \frac{2(2)(8)}{2+8} = \frac{32}{10} = 3.2$

$G = \sqrt{ab} = \sqrt{8\times 2} = \sqrt{16} = 4$

$\therefore A > G > H = 5 > 4 > 3.2$

(ii) $a = 2/5 \quad b = 8/5$

$A = \frac{2/5+8/5}{2} = \frac{10}{2\times 5} = 1$

$G = \sqrt{\frac{2}{5}\times\frac{8}{5}} = \frac{\sqrt{16}}{\sqrt{25}} = 4/5 = 0.8$

$H = \frac{2ab}{a+b} = \frac{2(8/5)(2/5)}{8/5+2/5}$

$H = \frac{\frac{32}{25}}{10/5} = \frac{32}{25}\times\frac{5}{10}$

$H = \frac{32}{50} = 0.64$

$A > G > H = 1 > 0.8 > 0.64$

Q.13 Find A, H, G and $G^2 = AH$

(i) $a = -2 \quad b = -6$

$A = \frac{(-2)+(-6)}{2} = \frac{-8}{2} = -4$

$G = \sqrt{(-2)\times(-6)} = \sqrt{12} = 2\sqrt{3}$

$H = \frac{2ab}{a+b} = \frac{2(-2)(-6)}{(-2)+(-6)} = \frac{24}{-8} = -3$

$G^2 = AB$

$G^2 = (2\sqrt{3})^2 = 4.3 = 12$

$A\times H = -3\times -4 = 12$

So $\boxed{G^2 = AH}$

(ii) $a = 2i$ $b = 4i$

$A = \frac{a+b}{2} = \frac{2i+4i}{2} = \frac{6i}{2} = 3i$

$G = \sqrt{2i \cdot 4i} = \sqrt{8i^2} = 2\sqrt{2}\, i$

$H = \frac{2ab}{a+b} = \frac{2(2i)(4i)}{2i+4i} = \frac{8\cancel{16}i^2}{3\cancel{6i}}$

$H = \frac{8}{3} i$

$G^2 = (2\sqrt{2}\, i)^2 = 4 \cdot 2 \cdot i^2 = 8i^2$

$A \times H = \cancel{3}i \times \frac{8}{\cancel{3}} i = 8i^2$

so

$G^2 = AH$ (Proved)

(iii) $a = 9$ $b = 4$

$A = \frac{a+b}{2} = \frac{9+4}{2} = 13/2$

$G = \sqrt{9 \times 4} = \sqrt{36} = 6$

$H = \frac{2(9)(4)}{9+4} = \frac{72}{13}$

$G^2 = (6)^2 = 36$

$A \times H = \frac{\cancel{13}}{\cancel{2}} \times \frac{\cancel{72}\,36}{\cancel{13}} = 36$

$G^2 = AH$ (Proved)

Q.15 Show $A < G < H$ if $(G < 0)$

(i) $a = -2$ $b = -8$

$A = \frac{-2+(-8)}{2} = \frac{-10}{2} = -5$

$G = -\sqrt{-2 \times -8} = -\sqrt{16} = -4$

$H = \frac{2(-2)(-8)}{(-2)+(-8)} = \frac{32}{-10} = -3.2$

$A < G < H = -5 < -4 < -3.2$

(ii) $a = -2/5$ $b = -8/5$

$A = \frac{(-2/5)+(-8/5)}{2} = \frac{-2-8}{5 \times 2} = \frac{-10}{10} = -1$

$G = -\sqrt{(-2/5)(-8/5)} = -\sqrt{16/25} = -4/5$

$= -0.8$

$H = \frac{2(-2/5)(-8/5)}{(-2/5)+(-8/5)} = \frac{32/25}{-10/5}$

$H = \frac{32}{5\,\cancel{25}} \times \frac{\cancel{5}}{-10} = \frac{-32}{50} = -0.64$

$A < G < H = -1 < -0.8 < -0.64$

Q.16 Let the numbers are a and b

By the Given Conditions

$A = \frac{a+b}{2} = \frac{9}{2} \Rightarrow a+b = 9$ — (1)

$a = 9 - b$

$H = \frac{2ab}{a+b} = 4$

$2ab = 4(a+b)$ — (2)

Putting $a = 9-b$ in (2)

$2(9-b)b = 4(9-\cancel{b}+\cancel{b})$

$18b - 2b^2 = 36$

$\cancel{2}(9b - b^2) = \cancel{36}\ 18$

$9b - b^2 = 18$

$b^2 - 9b + 18 = 0$

$b^2 - 6b - 3b + 18 = 0$

$b(b-6) - 3(b-6) = 0$

$(b-3)(b-6) = 0$

$b = 3$	$b = 6$
$a = 9-3 = 6$	$a = 9-6 = 3$

Hence Numbers are 3, 6

Q.17

Let the numbers are a, b

By the Given Conditions.

$\sqrt{ab} = 4 \Rightarrow ab = 16$

$\frac{2ab}{a+b} = \frac{16}{5}$ $\quad a = \frac{16}{b}$

$10ab = 16a + 16b$

$10\left(\frac{16}{b}\right)b = 16\left(\frac{16}{b}\right) + 16b$

$160 = 16\left[\frac{16}{b} + b\right]$ Dividing by 16

$10 = \left(\frac{16+b^2}{b}\right)$

$10b = 16 + b^2$

$b^2 - 10b + 16 = 0$

$b^2 - 8b - 2b + 16 = 0$

$b(b-8) - 2(b-8) = 0$

$(b-8)(b-2) = 0$

$b = 8$	$b = 2$
$a = \frac{16}{b} = \frac{16}{8}$	$a = \frac{16}{b} = \frac{16}{2}$
$a = 2$	$b = 8$

So in both the Cases numbers are 2, 8

Q.18 Let the three Consecutive terms are in G.P. $\frac{a}{r}, a, ar$

Product $= \frac{a}{r} \cdot a \cdot ar \Rightarrow a^3 = \frac{1}{27}$

$\Rightarrow a = \frac{1}{3}$

By Subtracting $\frac{1}{2}, \frac{4}{21}, \frac{1}{36}$

$\frac{a}{r} - \frac{1}{2}, a - \frac{4}{21}, ar - \frac{1}{36}$ are in H.P

Putting $a = \frac{1}{3}$

$\frac{1}{3r} - \frac{1}{2}, \frac{1}{3} - \frac{4}{21}, \frac{r}{3} - \frac{1}{36}$

$\frac{2-3r}{6r}, \frac{7-4}{21}, \frac{12r-1}{36}$ in H.P

$\frac{2-3r}{6r}, \frac{3}{21}, \frac{12r-1}{36}$ in H.P

$\frac{6r}{2-3r}, \frac{21}{3}, \frac{36}{12r-1}$ in A.P

Since these are in A.P so

$\frac{\cancel{21}\,7}{\cancel{3}} - \frac{6r}{2-3r} = \frac{36}{12r-1} - \frac{\cancel{21}\,7}{\cancel{3}}$

$\frac{7(2-3r) - 6r}{2-3r} = \frac{36 - 7(12r-1)}{12r-1}$

$\frac{14 - 21r - 6r}{2-3r} = \frac{36 - 84r + 7}{12r-1}$

$\frac{14 - 27r}{2-3r} = \frac{43 - 84r}{12r-1}$

$(14-27r)(12r-1) = (43-84r)(2-3r)$

$168r - 14 - 324r^2 + 27r = 86 - 129r - 168r + 252r^2$

$252r^2 + 324r^2 - 297r - 195r + 86 + 14 = 0$

$576r^2 - 492r + 100 = 0$

$4\left[144r^2 - 123r + 25\right] = 0$

$144r^2 - 123r + 25 = 0 \qquad 4 \neq 0$

$144r^2 - 75r - 48r + 25 = 0$