

Sequences And Series

(1)

Sequence:-

"A Set of numbers written under some rule is called Sequence."

OR "An ordered set of numbers is called Sequence."

A sequence is also named as Progression and denoted as $\{a_n\}$ for $n \in \mathbb{N}$.

e.g. $\{1, 3, 5, 7, 9, 11, \dots\}$

$\{1, 4, 7, 10, 13, 16, 19\}$

$\{5, 10, 15, 20, 25, \dots\}$

Finite Sequence:-

"A Sequence which has finite (limited) terms is called

Finite Sequence.

e.g. $\{1, 5, 9, 13, 17\}$

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Infinite Sequence:-

"A sequence which has not finite (limited) terms is called an Infinite Sequence."

$\{5, 10, 15, 20, 25, \dots\}$

NOTE:

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(i) The first term of a sequence is called leading term.

(ii) n th term of a sequence is called General term.

Exercise: 6.1

Q.1 Write the first 4 terms:

(i) $a_n = 2n - 3$

$$a_1 = 2(1) - 3 = 2 - 3 = -1$$

$$a_2 = 2(2) - 3 = 4 - 3 = 1$$

$$a_3 = 2(3) - 3 = 6 - 3 = 3$$

$$a_4 = 2(4) - 3 = 8 - 3 = 5$$

(ii) $a_n = (-1)^n n^2$

$$a_1 = (-1)^1 (1)^2 = (-1)(1) = -1$$

$$a_2 = (-1)^2 (2)^2 = (1)(4) = 4$$

$$a_3 = (-1)^3 (3)^2 = (-1)(9) = -9$$

$$a_4 = (-1)^4 (4)^2 = (1)(16) = 16$$

(iii) $a_n = (-1)^n (2n - 3)$

$$a_1 = (-1)^1 (2(1) - 3) = (-1)(2 - 3) = (-1)(-1) = 1$$

$$a_2 = (-1)^2 (2(2) - 3) = (1)(4 - 3) = (1)(1) = 1$$

$$a_3 = (-1)^3 (2(3) - 3) = (-1)(6 - 3) = (-1)(3) = -3$$

$$a_4 = (-1)^4 (2(4) - 3) = (1)(8 - 3) = (1)(5) = 5$$

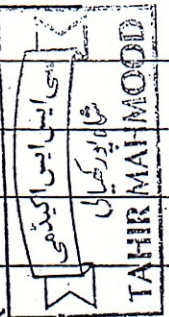
(iv) $a_n = 3n - 5$

$$a_1 = 3(1) - 5 = 3 - 5 = -2$$

$$a_2 = 3(2) - 5 = 6 - 5 = 1$$

$$a_3 = 3(3) - 5 = 9 - 5 = 4$$

$$a_4 = 3(4) - 5 = 12 - 5 = 7$$



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$$(v) a_n = \frac{n}{2^{n+1}}$$

$$a_1 = \frac{1}{2^{(1)+1}} = \frac{1}{2+1} = \frac{1}{3}$$

$$a_2 = \frac{2}{2(2)+1} = \frac{2}{4+1} = \frac{2}{5}$$

$$a_3 = \frac{3}{2(3)+1} = \frac{3}{6+1} = \frac{3}{7}$$

$$a_4 = \frac{4}{2(4)+1} = \frac{4}{8+1} = \frac{4}{9}$$

$$a_4 = 4a_3 \Rightarrow a_4 = 4\left(\frac{6}{7}\right) = \frac{24}{7}$$

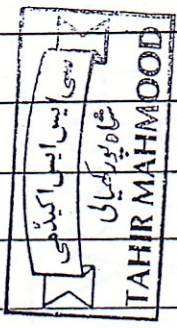
$$(vi) a_n = \frac{1}{2^n}$$

$$a_1 = \frac{1}{2^1} = \frac{1}{2}$$

$$a_2 = \frac{1}{2^2} = \frac{1}{4}$$

$$a_3 = \frac{1}{2^3} = \frac{1}{8}$$

$$a_4 = \frac{1}{2^4} = \frac{1}{16}$$



$$(ix) a_n = (n+1)a_{n-1} \quad (a_1 = 1)$$

$$a_1 = (1+1)a_0 \Rightarrow a_0 = \frac{1}{2}$$

$$a_2 = (2+1)a_1 \Rightarrow a_2 = 3(1) = 3$$

$$a_3 = (3+1)a_2 \Rightarrow a_3 = 4(3) = 12$$

$$a_4 = (4+1)a_3 \Rightarrow a_4 = 5(12) = 60$$

$$(x) a_n = \frac{1}{a+(n-1)d}$$

$$a_1 = \frac{1}{a+(1-1)d} = \frac{1}{a+0} = \frac{1}{a}$$

$$a_2 = \frac{1}{a+(2-1)d} = \frac{1}{a+d}$$

$$a_3 = \frac{1}{a+(3-1)d} = \frac{1}{a+2d}$$

$$a_4 = \frac{1}{a+(4-1)d} = \frac{1}{a+3d}$$

$$(vii) a_n - a_{n-1} = n+2 \quad (a_1 = 2)$$

$$a_1 - a_0 = (1)+2 \Rightarrow 2 - a_0 = 3$$

$$\Rightarrow a_0 = 2 - 3 \Rightarrow a_0 = -1$$

$$a_2 - a_1 = 2+2 \Rightarrow a_2 - 2 = 4$$

$$\Rightarrow a_2 = 6$$

$$a_3 - a_2 = 3+2 \Rightarrow a_3 - 6 = 5$$

$$\Rightarrow a_3 = 11$$

$$a_4 - a_3 = 4+2 \Rightarrow a_4 - 11 = 6$$

$$\Rightarrow a_4 = 17$$

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$$(viii) a_n = n a_{n-1} \quad (a_1 = 1)$$

$$a_1 = (1)a_0 \Rightarrow a_0 = 1$$

$$a_2 = 2a_1 \Rightarrow a_2 = 2(1) = 2$$

$$a_3 = 3a_2 \Rightarrow a_3 = 3(2) = 6$$

Q.2 Find indicated term:

$$(i) 2, 6, 11, 17, \dots, a_7$$

$$a_1 = 2$$

$$a_2 = 2+4 = 6$$

$$a_3 = 6+5 = 11$$

$$a_4 = 11+6 = 17$$

$$a_5 = 17+7 = 24$$

$$a_6 = 24+8 = 32$$

$$a_7 = 32+9 = 41$$

General term

$$a_n = a_{n-1} + (n-1) \quad ; n > 1$$

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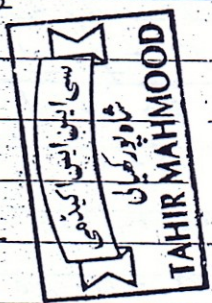
(ii) $1, 3, 12, 60, \dots, a_8$

$a_1 = 1$
 $a_2 = 1(3) = 3$
 $a_3 = 3(4) = 12$
 $a_4 = 12(5) = 60$
 $a_5 = 60(6) = 360$
 $a_6 = 360(7) = 2520$

$a_n = a_{n-1}(n+1)$
 $n \geq 1$
 $a_1 = 1$

(iii) $1, \frac{3}{2}, \frac{5}{4}, \frac{7}{8}, \dots, a_8$

$a_1 = 1$
 $a_2 = \frac{1+2}{1(2)} = \frac{3}{2}$
 $a_3 = \frac{3+2}{2(2)} = \frac{5}{4}$
 $a_4 = \frac{5+2}{4(2)} = \frac{7}{8}$
 $a_5 = \frac{7+2}{8(2)} = \frac{9}{16}$
 $a_6 = \frac{9+2}{16(2)} = \frac{11}{32}$
 $a_7 = \frac{11+2}{32(2)} = \frac{13}{64}$
 $a_8 = \frac{13+2}{64(2)} = \frac{15}{128}$



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(iv) $1, -3, 5, -7, \dots, a_8$

$a_1 = 1$
 $a_2 = -3$
 $a_3 = 1 - 4 = -3$
 $a_4 = 1 + 4 = 5$
 $a_5 = -3 - 4 = -7$
 $a_6 = 5 + 4 = 9$
 $a_7 = -7 - 4 = -11$
 $a_8 = 9 + 4 = 13$

Hence $a_8 = 13$

(v) $1, -3, 5, -7, 9, -11, \dots, a_8$ (3)

$a_1 = 1$
 $a_2 = -3$
 $a_3 = 1 + 4 = 5$
 $a_4 = -3 - 4 = -7$
 $a_5 = 5 + 4 = 9$
 $a_6 = -7 - 4 = -11$
 $a_7 = 9 + 4 = 13$
 $a_8 = -11 - 4 = -15$

Q.3 Find next two terms:

(i) $7, 9, 12, 16, \dots$

$a_1 = 7$
 $a_2 = 7 + 2 = 9$
 $a_3 = 9 + 3 = 12$
 $a_4 = 12 + 4 = 16$
 $a_5 = 16 + 5 = 21$
 $a_6 = 21 + 6 = 27$

Hence Sequence with two next terms

$7, 9, 12, 16, 21, 27, \dots$

(ii) $1, 3, 7, 15, 31, \dots$

$a_1 = 1$
 $a_2 = 1 + 2 = 3$
 $a_3 = 3 + 4 = 7$
 $a_4 = 7 + 8 = 15$
 $a_5 = 15 + 16 = 31$
 $a_6 = 31 + 32 = 63$
 $a_7 = 63 + 64 = 127$

Hence the sequence after two

$1, 3, 7, 15, 31, 63, 127, \dots$

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(iii) $-1, 2, 12, 40, \dots$

$a_1 = -1 \times 2^0 = -1$
 $a_2 = 2^1 \times 1 = 2$
 $a_3 = 2^2 \times 3 = 12$
 $a_4 = 2^3 \times 5 = 40$
 $a_5 = 2^4 \times 7 = 112$
 $a_6 = 2^5 \times 9 = 288$

Seq = $-1, 2, 12, 40, 112, 288, \dots$